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On Some Properties of Metric F-Structure Satisfying $F^{2k+1}+F=0$

Research Article

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Abstract: In this paper, we have studied various properties of the F-structure satisfying $F^{2k+1}+F=0$. Where k is positive integer The metric F- structure, f induced on each integral manifold of tangent bundle l^* have also been discussed.

Keywords: Differentiable manifold, projection operators, tangent bundles and metric.

1. Introduction

Let V_n be a C^∞ differentiable manifold and F be a $C^\infty(1,1)$ tensor on V_n such that

$$F^{2k+1}+F=0 \tag{1}$$

we define the the projection operators l and m on V_n by

$$l=-F^{2k}, \qquad m=I+F^{2k} \tag{2}$$

from (1) and (2), we get

$$l+m=I,\ \ l^2=l,\ \ m^2=m,\ \ lm=ml=0,\ \ lF=Fl=F,\ \ Fm=mF=0, \tag{3}$$

where I denotes the identify operator.

Theorem 1.1. *If $rank((F))=n$ then*

$$l=I, m=0. \tag{4}$$

Proof. from the fact

$$rank((F))+nulity((F))=\dim V_n=n \tag{5}$$

Thus

$$nulity((F))=0 \Rightarrow \ ker((F))=\{0\} \tag{6}$$

Thus $FX=0 \Rightarrow X=0$. Then $FX_1=FX_2 \Rightarrow F(X_1-X_2)=0 \Rightarrow X_1=X_2$ or F is $1-1$. Moreover V_n being finite dimensional F is onto also F is invertible operating F^{-1} on $Fl=F$ and $mF=0$, we get (4). □

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Theorem 1.2. *If* $rank((F)) = n-1$ *then*

$$l = I - A \otimes T,\ m = A \otimes T,\ AoF = 0,\ FT = 0. \tag{7}$$

Proof. From (1)

$$F(F^{2k}+I) = 0 \tag{8}$$

Let

$$F^{2k}+I = A \otimes T \tag{9}$$

From (8) and (9)

$$FT = 0 \tag{10}$$

Also from (2) and (9)

$$l = -F^{2k} = I - A \otimes T$$
$$m = -F^{4} + I = A \otimes T$$

From (5) and (6)

$$F^{2k}X + X = A \times T$$
$$F^{2k+1}X + FX = A(FX)T$$
$$0 = A(FX)T$$

Thus $AoF = 0$. □

Theorem 1.3. *Let the operator* m *and* F *satisfying*

$$m^2 = m,\ Fm = mF = 0,\ (m+F^k)(m-F^k) = I \tag{11}$$

Then we get (1).

Proof. From $(m+F^k)(m-F^k) = I$

$$m^2 - mF^k + F^k m - F^{2k} = I$$
$$m - 0 + 0 - F^{2k} = I$$
$$F^{2k+1} + F = 0$$

□

2. Metric F-Structure

If we define

$$'F(X,Y) = g\ (FX,Y) \tag{12}$$

is skew-symmetric. Then

$$g(FX,Y) = -g(X,FY). \tag{13}$$

Theorem 2.1. *The definitions in (12) and (13), we have*

$$g\left(F^kX,F^kY\right) = (-1)^k\left[g(X,Y) - {`m}(X,Y)\right] \tag{14}$$

where

$$'m(X,Y) = g(mX,Y) = g(X,mY). \tag{15}$$

Proof. From (2), (3) and (13), (15), we have

$$g\left(F^kX,F^kY\right) = (-1)^k g\left(X,F^{2k}Y\right) \tag{16}$$

$$= (-1)^k g(X,-lY)$$

$$= (-1)^{k+1} g(X,(I-m)Y)$$

$$= (-1)^{k+1}\left[g(X,Y) - g(X,mY)\right] \tag{17}$$

$$= (-1)^{k+1}\left[g(X,Y) - g(mX,Y)\right]$$

$$= (-1)^{k+1}\left[g(X,Y) -{'}\,m(X,Y)\right]$$

□

Theorem 2.2. $\{F,g\}$ *is not unique.*

Proof. Let

$$\mu F' = F\mu,\quad {'g}(X,Y) = g(\mu X,\mu Y) \tag{18}$$

Then from (1) and (2), (3), (18)

$$\mu F'^{2k+1} = F^{2k+1}\mu = -F\mu = \mu F' \tag{19}$$

or

$$F'^{2k+1} + F' = 0. \tag{20}$$

Also

$$'g(F'^kX,F'^kY) = g(\mu F'^kX,\mu F'^kY) \tag{21}$$

$$= g(F^k\mu X,F^k\mu Y)$$

$$= (-1)^k g(\mu X,F^{2k}\mu Y)$$

$$= (-1)^k g(\mu X,-l\mu Y)$$

$$= (-1)^{k+1} g(\mu X,l\mu Y)$$

$$= (-1)^{k+1} g(\mu X,(I-m)\mu Y)$$

$$= (-1)^{k+1}\left[g(\mu X,\mu Y) - g(\mu X,m\mu Y)\right]$$

$$= (-1)^{k+1}\left[{'g}(X,Y) - {'m}(X,Y)\right]$$

□

3. Induced Structure f

Define

$$fX' = FX' \text{ for } X' \in l^* \tag{22}$$

Theorem 3.1. *If f satisfying (22) and (1) then $\{f^k\}$ is an almost complex structure.*

Proof. From (2), (3) and (22)

$$\begin{aligned} f^{2k}lX' &= F^{2k}lX' \\ &= -l^2X' \\ &= -lX' \end{aligned} \tag{23}$$

Thus $\{f^k\}$ as an almost complex structure on l^* Also

$$\begin{aligned} \mu l' &= -\mu F'^{2k} \\ &= -F^{2k}\mu \\ &= l\mu \end{aligned} \tag{24}$$

$$\begin{aligned} \mu m' &= \mu(I + F'^{2k}) \\ &= \mu + \mu F'^{2k} \\ &= \mu + F^{2k}\mu \\ &= (I + F^{2k})\mu \\ &= m\mu \end{aligned} \tag{25}$$

□

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