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Understanding the Fourth Dimension Through a New Mathematical Approach

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Abstract: This paper establishes a new concept of dimensions in an effort to understand the fourth dimension. From school days, we are accustomed to a square as two dimensional primitive, a cube as three dimensional primitive. We will try to imagine the fourth dimension in the same lines and so it is difficult to imagine and visualize it. As we know, point is having zero dimensions and a line is having one dimension. After that, the two dimensional primitive is not a square but it is a triangle. Precisely, it is an equilateral triangle. So if we move in this direction, it will be easier for us to visualize and understand the 4^{th} dimension. After the equilateral triangle, we will think of the next solid which is a tetrahedron, in which all the sides are equal and there are four equilateral triangular faces. This is the solid primitive of three dimensions and not the cube. On these lines, if we move on to the next dimension, we can easily analyse that the fourth dimensional point will appear inside this tetrahedron having five volumes like tetrahedron, which will be discussed in detail in this paper. This paper logically arrives at equations for finding vertices, edges, surfaces, centre of masses and volumes of 'n' dimensional object.

MSC: 51A50, 51F05, 51M20, 51P99

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1. Introduction

The conventional way of representing the dimensions is shown in the Figure 1. Initially, we start with a point for zero dimension, then two points for a line as one dimension, a square for two dimensions and a cube representing three dimensions. From line with two vertices, we have skipped the primitive of two dimensions, which is a triangle and jumped to a square!.

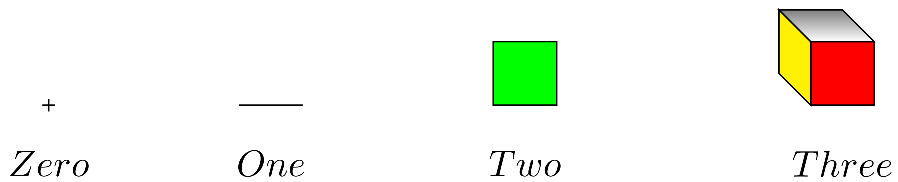

Figure 1. **Conventional way of representing dimensions.**

This is the logical error because, a line has two end points and next dimension should be having three vertices and not four. Since we started with square as two dimensions, next will have to be cube with eight vertices. But logically, every next

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dimension must be represented by adding just one more vertex. Since we mark zero dimension as one point, one dimension with two end points, the general equation must be

$$v = n + 1 \tag{1}$$

Where 'v' is the number of vertices and 'n' is the number of dimensions.

1.1. New way of representation

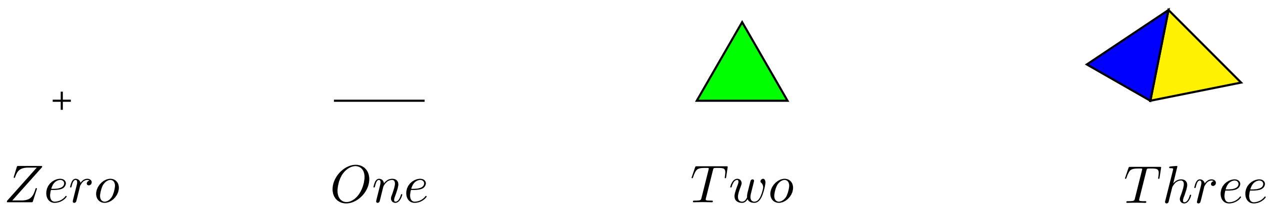

Figure 2. **New method of representation**

In this method, the sequence will be as follows: point, line, triangle, tetrahedron and fourth dimensional object (shown later in this paper). The constructional details and logically locating next vertex will be explained as below:
Let us start from drawing a point 'A', which is having zero dimensions.

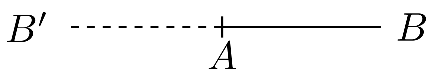

Figure 3. **Point 'A' and possible positions of Point 'B'.**

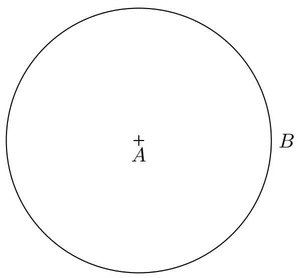

Figure 4. **Point A and possible positions of point B in two dimensions**

After that, we will be having two possibilities for locating the next point B or B' to draw a line of say, unit length on either direction in one dimension, as shown in Figure 3.
But form a two dimensional perspective, we will have infinite possibilities along the circle and from a three dimensional perspective, infinite possibilities on the surface of a sphere!. A line is drawn by selecting the point B. So, This is one dimension. Up to here, the old and new conventions are same. From here, instead of square, we will have to draw an equilateral triangle.

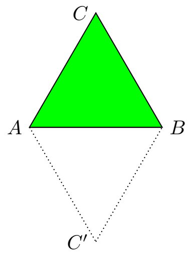

Figure 5. **Two possible positions of point C in two dimensions.**

Then in the two dimensions, there are two possibilities drawing c or c' as shown in the Figure 5. So let us locate it as C. AB, AC and BC are called as sides or edges of the triangle. But if we go to the third dimension, then from the same line AB, there are infinite possibilities of locating c which is shown by the circle around the line Figure 6.

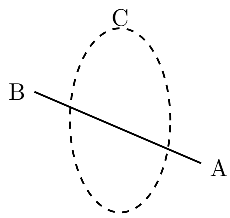

Figure 6. **Possible positions of point C in three dimensions.**

From the points a, b, c in two dimensions, we have again two choices in three dimension to locate D or D'. Let us take it as D to make the solid primitive, which is a tetrahedron as shown in Figure 7.

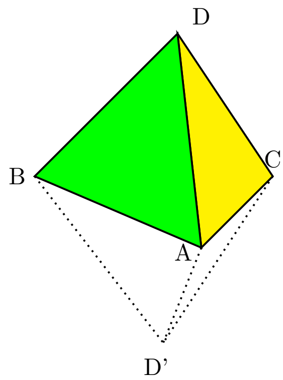

Figure 7. **Possible positions of point D in three dimensions.**

According to the previous logic, if we go to the fourth dimension and see, then we must be able to get infinite possibilities of locating the point D which probably will appear like a circle equidistant from all the three points ABC!.

Extending the same logic, we can locate the point E or E' equidistant from all these four vertices ABCD. So, the 4^{th} dimensional vertex therefore must appear as centroidal point, equidistant from all the four corners in every triangular face. It is like the position of the carbon atom in a methane molecule. The logic of locating this vertex as centroid of tetrahedron has been discussed in Article 3.2 and 3.3 of this paper.

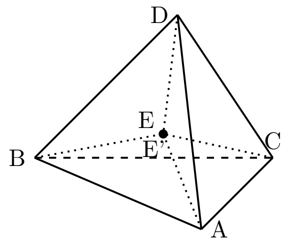

Figure 8. **Three dimensional model of fourth dimensional object**

2. Calculation of Vertices, Edges, Faces, Volumes and Centroid in Each Dimension

2.1. Number of vertices in different dimensions

Number of vertices is given by the equation (1). So, from the equation (1), we get

0 dimension = 1 vertex

1 dimension = 2 vertices

2 dimensions = 3 vertices

3 dimensions = 4 vertices

4 dimensions = 5 vertices

So, for 'n' = 5, there will be six vertices.

2.2. Number of edges in different dimensions

1 dimension = 1 edge

2 dimensions = 3 edges

3 dimensions = 6 edges

4 dimensions = 10 edges

Logic:

In each dimension, there will be a new vertex. This will be connected to previous vertices. Hence one dimensional line will have two end points connected to one new vertex. So, number of edges in the next dimension will be

$$\textit{Previous number of edges} + \textit{existing number of vertices}$$

That is, $1 + 2 = 3$, for two dimensions.

Similarly, from two dimension to three dimension $3 + 3 = 6$ edges. It is evident from the above logic that the equation for finding number of edges of 'n' dimensional object must be summation of number of dimensions. So it is given by

$$\textit{Number of edges} = \sum n \tag{2}$$

So, for $n = 4$, $\sum 4 = 1 + 2 + 3 + 4 = 10$ edges.

Similarly, for $n = 5$, $\sum 5 = 1 + 2 + 3 + 4 + 5 = 15$ edges.

2.3. Number of surfaces in different dimensions

1 dimension = zero surface

2 dimensions = 1 surface

3 dimensions = 4 surfaces

4 dimensions = 10 surfaces

Every edge in any dimension will form a surface in the next dimension.

Example 2.1. *Number of surfaces in two dimension is 1. So, total surface in the next dimension will be*

$$Existing\ surface\ in\ present\ dimension + new\ surfaces = 1 + 3 = 4.$$

Similarly, each of the 6 edges in three dimensional, solid the tetrahedron will make 6 new surfaces. So, total surfaces in 4^{th} dimension = existing surfaces in present dimension + new surfaces = $4 + 6 = 10$.

$$Number of surfaces in "n" dimensions = \sum 1 + \sum 2 + ... + \sum (n-1) \tag{3}$$

So, for $n = 4$, it is $= 1 + (1 + 2) + (1 + 2 + 3) = 10$.

2.4. Number of volumes in different dimensions

1 dimension = zero volume

2 dimensions = zero volume

3 dimensions = 1 volume

4 dimensions = 5 volumes

Every surface in any dimension will make a volume in the next dimension.

Example 2.2. *Number of surfaces in two dimension is 1. So, total volume in the next dimension will be*

$$Total\ number\ of\ surfaces\ up\ to\ the\ previous\ dimension = 0 + 1 = 1.$$

So, total volumes in 4^{th} dimension = Total number of surfaces up to the previous dimension = $1 + 4 = 5$. No big equation is required for this. This can be calculated using the equation (3) and summing up all the surfaces up to the previous dimension. So, for 5^{th} dimension, $n = 5$ and Number of volumes = Total number of surfaces up to the 4^{th} dimension = $1 + 4 + 10 = 15$ volumes.

3. Discussion About the New Method

3.1. Two dimensional observer

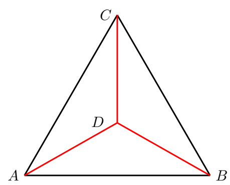

Figure 9. **Image of a tetrahedron as seen by two dimensional observer**

- For a two dimensional observer, the top view of the wire-frame model of a tetrahedron looks like an equilateral triangle with three vertices joined to the centroid, as shown in the Figure 9.
- He analyses that the three triangles formed by ADC, ADB and BDC are equilateral triangles identical to the triangle ABC. But he will be seeing them as isosceles triangles.
- Vertices of all higher dimensions will appear to coincide with D.
- He can see all the edges of three dimensional tetrahedron but higher dimensional edges will coincide and he cannot differentiate or count them.

A note on existing three dimensional model

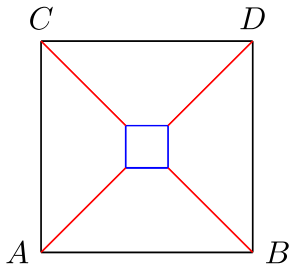

Figure 10. **Existing three dimensional wire frame image**

- In the existing three dimensional model,in the top view of the wire-frame model, two dimensional observer will see the image as a square inside a square as shown in Figure 10, but the ratio between the sizes of squares depends upon the distance of the observer from the object and the size of the object, which makes it more confusing. This type of confusion is automatically eliminated in the new method because, the new vertex appears as a point inside the triangle.

3.2. Three dimensional observer

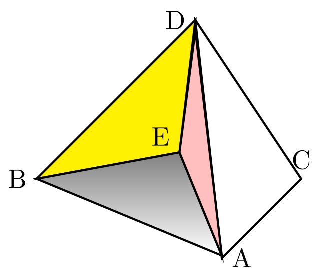

Figure 11. **The surfaces in the fourth dimension, as seen by three dimensional observer**

- For a Three dimensional observer, a 4^{th} dimensional model looks like an tetrahedron with all four vertices joined to the centroid, as shown in the Figure 11.
- He analyses that the four triangular pyramidal volumes formed by ABCE, ADCE, BDCE and ABDE are identical to the tetrahedron ABCD.

- Vertices of all higher dimensions will appear to coincide with E.
- He can see all the edges of four dimensional object but higher dimensional edges will coincide and he cannot differentiate or count them.
- He can see all the faces of four dimensional object but higher dimensional faces will coincide and he cannot differentiate or count them.

3.3. Centroid or center of mass calculation

It has been observed that for a lower dimensional observer, all higher dimensional lines coincide. But if the observation is done perpendicular to the reference surface, then an interesting fact is noticed.

- Let us start from one dimensional entity, the line. The centre of mass is the mid point of the line (here, it is assumed that line is having some very small thickness and uniform). Hence the centre of mass for one dimension = 1/2 of the length.
- Similarly, the centre of mass for two dimensional entity, the equilateral triangle = $1/3^{\text{rd}}$ the height h, where 'h' is the vertical height from the reference line.
- In three dimension, the centre of mass = $1/4^{\text{th}}$ the height
- In the same logic, for 4^{th} dimension, the centre of mass = $1/5^{\text{th}}$ the height from reference plane.
- So, for 'n' dimensional entity, centre of mass = $1/(\text{n}+1)^{\text{th}}$ the height from reference. Hence the general equation for n^{th} dimension is

$$Distance from reference line is = \frac{1}{n+1} = \frac{1}{v} \tag{4}$$

Where 'v' is the number of vertices = $(n+1)$ from Equation (1). It is pictorially represented in Figure 12, as calculated and visualised by a two dimensional observer.

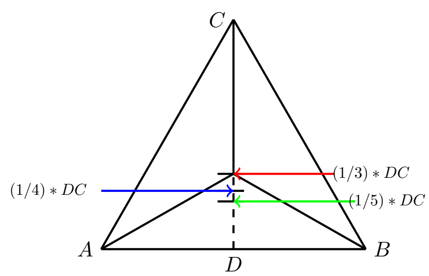

Figure 12. **Centre of mass in different dimensions**

The final thing that is yet to be analysed is that as 'n' reaches a large value, the centre of mass appears to move very close to the reference line!

4. Conclusions

- This new approach makes the visualization of 4^{th} dimensional entity very easy.
- It clearly illustrates that the angle between the axes of different dimensions need not necessarily be 90 degrees.

- It is more logical and simple, as each new dimension requires only one additional vortex.
- This approach simplifies calculation of edges, surfaces, volumes and centroid in each dimension.

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