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General Solution, Stability and Non-Stability of Quattuorvigintic Functional Equation in Multi-Banach Spaces

Research Article

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Abstract: In this current work, we estabilish the general solution and Hyers-Ulam stability for a new form of Quattuorvigintic functional equation in Multi-Banach Spaces.

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Keywords: Hyers-Ulam stability, Multi-Banach Spaces, Quattuorvigintic Functional Equations, Fixed Point Method.

1. Introduction

The issue of stability of functional equations has appeared in connection with a question that Ulam [22] asked in 1940. Hyers [7], by using direct method, brilliantly gave a partial answer for the case of the additive Cauchy functional equation for mappings between Banach Spaces. This result was then improved by Aoki [1] and Rassias [13], who weakened the condition for the bound of the norm of Cauchy difference. The stability phenomenons proved in [7] and [13] were named Hyers-Ulam and Hyers-Ulam-Rassias stability duo to the high influence of Hyers and Rassias on this area of research. Some results regarding to the stability of various forms of the quartic [8], quintic [20], sextic [20], septic and octic [12], decic [3], undecic [15] and quattuordecic [16] functional equations have been investigated by a number of authors with more general domains and co-domains.

Definition 1.1 ([5]). *A Multi- norm on* $\left\{\mathcal{A}^k : k \in \mathbb{N}\right\}$ *is a sequence* $(\|.\|) = \left(\|.\|_k : k \in \mathbb{N}\right)$ *such that* $\|.\|_k$ *is a norm on* $\mathcal{A}^k$ *for each* $k \in \mathbb{N}, \|x\|_1 = \|x\|$ *for each* $x \in \mathcal{A}$*, and the following axioms are satisfied for each* $k \in \mathbb{N}$ *with* $k \geq 2:$

(1). $\left\|\left(x_{\sigma(1)}, ..., x_{\sigma(k)}\right)\right\|_k = \|(x_1 ... x_k)\|_k$ *, for* $\sigma \in \Psi_k, x_1, ..., x_k \in \mathcal{A}$;

(2). $\|(\alpha_1 x_1, ..., \alpha_k x_k)\|_k \leq (\max_{i \in \mathbb{N}_k} |\alpha_i|) \|(x_1 ... x_k)\|_k$ *for* $\alpha_1 ... \alpha_k \in \mathbb{C}, x_1, ..., x_k \in \mathcal{A}$;

(3). $\|(x_1, ..., x_{k-1}, 0)\|_k = \|(x_1, ..., x_{k-1})\|_{k-1}$ *, for* $x_1, ..., x_{k-1} \in \mathcal{A}$;

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(4). $\|(x_1,...,x_{k-1},x_{k-1})\|_k = \|(x_1,...,x_{k-1})\|_{k-1}$ *for* $x_1,...,x_{k-1} \in \mathcal{A}$.

In this case, we say that $\left((\mathcal{A}^k, \|.\|_k) : k \in \mathbb{N}\right)$ *is a multi - normed space. Suppose that* $\left((\mathcal{A}^k, \|.\|_k) : k \in \mathbb{N}\right)$ *is a multi - normed spaces, and take* $k \in \mathbb{N}$. *We need the following two properties of multi - norms. They can be found in [5].*

(a). $\|(x,...,x)\|_k = \|x\|, for x \in \mathcal{A}$,

(b). $\max_{i\in\mathbb{N}_k} \|x_i\| \leq \|(x_1,...,x_k)\|_k \leq \sum_{i=1}^{k} \|x_i\| \leq k \max_{i\in\mathbb{N}_k} \|x_i\|, \forall x_1,...,x_k \in \mathcal{A}$.

It follows from (b) that if $(\mathcal{A}, \|.\|)$ *is a Banach space, then* $(\mathcal{A}^k, \|.\|_k)$ *is a Banach space for each* $k \in \mathbb{N}$; *In this case,* $\left((\mathcal{A}^k, \|.\|_k) : k \in \mathbb{N}\right)$ *is a multi - Banach space.*

In last few years, some authors have been estabilished the stability of different type of functional equations in Multi-Banach spaces [2], [6], [9],[10], [17], [18],[19], [21]. In this current work, we acquire the general solution and Hyers-Ulam stability for a new form of Quattuorvigintic functional equation in Multi-Banach Spaces.

$$\begin{aligned}\mathcal{D}\phi(v,w) &= \phi(v+12w) - 24\phi(v+11w) + 276\phi(v+10w) - 2024\phi(v+9w) + 10626\phi(v+8w) - 42504\phi(v+7w)\\ &+ 134596\phi(v+6w) - 346104\phi(v+5w) + 735471\phi(v+4w) - 1307504\phi(v+3w) + 1961256\phi(v+2w)\\ &- 2496144\phi(v+w) + 2704156\phi(v) - 2496144\phi(v-w) + 1961256\phi(v-2w) - 1307504\phi(v-3w)\\ &+ 735471\phi(v-4w) + 134596\phi(v-6w) - 42504\phi(v-7w) + 10626\phi(v-8w) - 2024\phi(v-9w)\\ &+ 276\phi(v-10w) - 346104\phi(v-5w) - 24\phi(v-11w) + \phi(v-12w) - 24!\phi(w) \end{aligned} \tag{1}$$

2. General Solution of Quattuorvigintic Functional Equation in (1)

Theorem 2.1. *Let* $\mathcal{F}$ *and* $\mathcal{G}$ *be the vector spaces. If* $\phi : \mathcal{F} \to \mathcal{G}$ *be a function (1) for all* $v, w \in \mathcal{F}$ *is*

Proof. Doing $v = 0$ and $w = 0$ in (1), we obtain that $\phi(0) = 0$. Substituting (v, w) with (v, v) and $(v, -v)$ in (1), respectively, and subtracting two resulting equations, we can arrive at $\phi(-v) = \phi(v)$, that is to say, ϕ is an even function. Doing (v, w) by $(12v, v)$ and $(0, 2v)$ respectively, and subtracting the two resulting equations, we arrive at

$$\begin{aligned}&24\phi(23v) - 300\phi(22v) + 2024\phi(21v) - 10350\phi(20v) + 42504\phi(19v)\\ &- 136620\phi(18v) + 346104\phi(17v) - 724845\phi(16v) + 1307504\phi(15v)\\ &- 2003760\phi(14v) + 2496144\phi(13v) - 2569560\phi(12v) + 2496144\phi(11v)\\ &- 2307360\phi(10v) + 1307504\phi(9v) + 346104\phi(7v) - 1442100\phi(6v)\\ &+ 42504\phi(5v) + 1950630\phi(4v) + 2024\phi(3v) - \frac{24!}{2}\phi(2v) + 24!\phi(v) = 0\end{aligned} \tag{2}$$

$\forall v \in \mathcal{F}$. Doing $v = 11v$ and $w = v$ in (1), one gets

$$\begin{aligned}&\phi(23v) - 24\phi(22v) + 276\phi(21v) - 2024\phi(20v) + 10626\phi(19v)\\ &- 42504\phi(18v) + 134596\phi(17v) - 346104\phi(16v) + 735471\phi(15v)\\ &- 1307504\phi(14v) + 1961256\phi(13v) - 2496144\phi(12v) + 2704156\phi(11v)\\ &- 2496144\phi(10v) + 1961256\phi(9v) - 1307504\phi(8v) + 735471\phi(7v) - 346104\phi(6v)\\ &+ 134596\phi(5v) - 42504\phi(4v) + 10626\phi(3v) - 2024\phi(2v) - 24!\phi(v) = 0\end{aligned} \tag{3}$$

$\forall v \in \mathcal{F}$. Multiplying (3) by 24, and then subtracting (2) from the resluting equation, we get

$$\begin{aligned} &276\phi(22v) - 4600\phi(21v) + 38226\phi(20v) - 212520\phi(19v) + 883476\phi(18v) \\ &- 2884200\phi(17v) + 7581651\phi(16v) - 16343800\phi(15v) + 29376336\phi(14v) \\ &- 44574000\phi(13v) + 57337896\phi(12v) - 62403600\phi(11v) + 57600096\phi(10v) \\ &- 45762640\phi(9v) - 31380096\phi(8v) - 17305200\phi(7v) + 6864396\phi(6v) \\ &- 3187800\phi(5v) + 2970726\phi(4v) - 253000\phi(3v) + \frac{24!}{2}\phi(2v) + 24!(25)\phi(v) = 0 \end{aligned} \tag{4}$$

$\forall v \in \mathcal{F}$. Doing $v = 10v$ and $w = v$ in (1), one gets

$$\begin{aligned} &\phi(22v) - 24\phi(21v) + 276\phi(20v) - 2024\phi(19v) + 10626\phi(18v) - 42504\phi(17v) \\ &+ 134596\phi(16v) - 346104\phi(15v) + 735471\phi(14v) - 1307504\phi(13v) + 1961256\phi(12v) \\ &- 2496144\phi(11v) + 2704156\phi(10v) - 2496144\phi(9v) + 1961256\phi(8v) \\ &- 1307504\phi(7v) + 735471\phi(6v) - 346104\phi(5v) + 134596\phi(4v) \\ &- 42504\phi(3v) + 10627\phi(2v) - 24!\phi(v) = 0 \end{aligned} \tag{5}$$

$\forall v \in \mathcal{F}$. Multiplying (5) by 276, and then subtracting (4) from the reslting equation, we get

$$\begin{aligned} &2024\phi(21v) - 37950\phi(20v) + 346104\phi(19v) - 2049300\phi(18v) + 8846904\phi(17v) \\ &- 29566845\phi(16v) + 79180904\phi(15v) - 173613660\phi(14v) + 316297104\phi(13v) \\ &- 483968760\phi(12v) + 626532144\phi(11v) - 688746960\phi(10v) + 643173104\phi(9v) \\ &- 509926560\phi(8v) + 343565904\phi(7v) - 196125600\phi(6v) + 92336904\phi(5v) \\ &- 34177770\phi(4v) + 11478104\phi(3v) - \frac{24!}{2}\phi(2v) - 24!(301)\phi(v) = 0 \end{aligned} \tag{6}$$

$\forall v \in \mathcal{F}$. Doing $v = 9v$ and $w = v$ in (1), one gets

$$\begin{aligned} &\phi(21v) - 24\phi(20v) + 276\phi(19v) - 2024\phi(18v) + 10626\phi(17v) - 42504\phi(16v) \\ &+ 134596\phi(15v) - 346104\phi(14v) + 735471\phi(13v) - 1307504\phi(12v) \\ &+ 1961256\phi(11v) - 2496144\phi(10v) + 2704156\phi(9v) - 2496144\phi(8v) \\ &+ 1961256\phi(7v) - 1307504\phi(6v) + 735471\phi(5v) - 346104\phi(4v) \\ &+ 134597\phi(3v) - 42528\phi(2v) - 24!\phi(v) = 0 \end{aligned} \tag{7}$$

$\forall v \in \mathcal{F}$. Multiplying (7) by 2024, and then subtracting (6) from the reslting equation, we get

$$\begin{aligned} &10626\phi(20v) - 212520\phi(19v) + 2047276\phi(18v) - 12660120\phi(17v) + 56461251\phi(16v) \\ &- 193241400\phi(15v) + 526900836\phi(14v) - 1172296200\phi(13v) + 2162419336\phi(12v) \\ &- 3343050000\phi(11v) + 4363448496\phi(10v) - 4830038640\phi(9v) + 4542268896\phi(8v) \\ &- 3626016240\phi(7v) + 2450262496\phi(6v) - 1396256400\phi(5v) + 666336726\phi(4v) \\ &- 260946224\phi(3v) + \frac{24!}{2}\phi(2v) - 24!(2325)\phi(v) = 0 \end{aligned} \tag{8}$$

$\forall v \in \mathcal{F}$. Doing $v = 8v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(20v) - 24\phi(19v) + 276\phi(18v) - 2024\phi(17v) + 10626\phi(16v) \\
&- 42504\phi(15v) + 134596\phi(14v) - 346104\phi(13v) + 735471\phi(12v) \\
&- 1307504\phi(11v) + 1961256\phi(10v) - 2496144\phi(9v) + 2704156\phi(8v) \\
&- 2496144\phi(7v) + 1961256\phi(6v) - 1307504\phi(5v) + 735472\phi(4v) \\
&- 346128\phi(3v) + 134872\phi(2v) - 24!\phi(v) = 0
\end{aligned} \tag{9}$$

$\forall v \in \mathcal{F}$. Multiplying (9) by 10626, and then subtracting (8) from the reslting equation, we get

$$\begin{aligned}
&42504\phi(19v) - 885500\phi(18v) + 8846904\phi(17v) - 56450625\phi(16v) \\
&+ 258406104\phi(15v) - 903316260\phi(14v) + 2505404904\phi(13v) - 5652695510\phi(12v) \\
&+ 10550487500\phi(11v) - 16476857760\phi(10v) + 21693987500\phi(9v) - 24192092760\phi(8v) \\
&+ 22898009900\phi(7v) - 18390043760\phi(6v) + 12497281100\phi(5v) - 7148788746\phi(4v) \\
&+ 3417009904\phi(3v) - \frac{24!}{2}\phi(2v) - 24!(12951)\phi(v) = 0
\end{aligned} \tag{10}$$

$\forall v \in \mathcal{F}$. Doing $v = 7v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(19v) - 24\phi(18v) + 276\phi(17v) - 2024\phi(16v) + 10626\phi(15v) \\
&- 42504\phi(14v) + 134596\phi(13v) - 346104\phi(12v) + 735471\phi(11v) \\
&- 1307504\phi(10v) + 1961256\phi(9v) - 2496144\phi(8v) + 270415\phi(7v) \\
&- 2496144\phi(6v) + 1961257\phi(5v) - 1307528\phi(4v) + 735747\phi(3v) \\
&- 348128\phi(2v) - 24!\phi(v) = 0
\end{aligned} \tag{11}$$

$\forall v \in \mathcal{F}$. Multiplying (11) by 42504, and then subtracting (10) from the resulting equation, we get

$$\begin{aligned}
&134596\phi(18v) - 2884200\phi(17v) + 29577471\phi(16v) - 193241400\phi(15v) \\
&+ 903273756\phi(14v) - 3215463480\phi(13v) + 9058108906\phi(12v) - 20709971880\phi(11v) \\
&+ 39097292260\phi(10v) - 61667237520\phi(9v) + 81904011820\phi(8v) - 92039436720\phi(7v) \\
&+ 87706060820\phi(6v) - 70863986420\phi(5v) + 48426381370\phi(4v) \\
&- 27855180580\phi(3v) - \frac{24!}{2}\phi(2v) - 24!(55455)\phi(v) = 0
\end{aligned} \tag{12}$$

$\forall v \in \mathcal{F}$. Doing $v = 6v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(18v) - 24\phi(17v) + 276\phi(16v) - 2024\phi(15v) + 10626\phi(14v) \\
&- 42504\phi(13v) + 134596\phi(12v) - 346104\phi(11v) + 735471\phi(10v) - 1307504\phi(9v) \\
&+ 1961256\phi(8v) - 2496144\phi(7v) + 2704157\phi(6v) - 2496168\phi(5v) \\
&+ 1961532\phi(4v) - 1309528\phi(3v) + 746097\phi(2v) - 24!\phi(v) = 0
\end{aligned} \tag{13}$$

$\forall v \in \mathcal{F}$. Multiplying (13) by 134596, and then subtracting (12) from the resulting equation, we get

$$\begin{aligned}
&346104\phi(17v) - 7571025\phi(16v) + 79180904\phi(15v) - 526943340\phi(14v)\\
&+ 2505404904\phi(13v) - 9057974310\phi(12v) + 25874242100\phi(11v) - 59894162460\phi(10v)\\
&+ 114317570900\phi(9v) - 182073200800\phi(8v) + 243931561100\phi(7v)\\
&- 276262654800\phi(6v) + 265110241700\phi(5v) - 215587979700\phi(4v)\\
&+ 148402050100\phi(3v) - \frac{24!}{2}\phi(2v) + 24!(190051)\phi(v) = 0 \qquad (14)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Doing $v = 5v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(17v) - 24\phi(16v) + 276\phi(15v) - 2024\phi(14v) + 10626\phi(13v)\\
&- 42504\phi(12v) + 134596\phi(11v) - 346104\phi(10v) + 735471\phi(9v)\\
&- 1307504\phi(8v) + 1961257\phi(7v) - 2496168\phi(6v) + 2704432\phi(5v)\\
&- 2498168\phi(4v) + 1971882\phi(3v) - 1350008\phi(2v) - 24!\phi(v) = 0 \qquad (15)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Multiplying (15) by 346104, and then subtracting (14) from the resulting equation, we get

$$\begin{aligned}
&735471\phi(16v) - 16343800\phi(15v) + 173571156\phi(14v) - 1172296200\phi(13v)\\
&+ 5652830106\phi(12v) - 20709971880\phi(11v) + 59893816360\phi(10v) - 140231884100\phi(9v)\\
&+ 270459163700\phi(8v) - 434867331600\phi(7v) + 587671074700\phi(6v) - 670904491200\phi(5v)\\
&+ 649037957800\phi(4v) - 534074197600\phi(3v) - \frac{24!}{2}\phi(2v) + 24!(536155)\phi(v) = 0 \qquad (16)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Doing $v = 4v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(16v) - 24\phi(15v) + 276\phi(14v) - 2024\phi(13v) + 10626\phi(12v)\\
&- 42504\phi(11v) + 134596\phi(10v) - 346104\phi(9v) + 735472\phi(8v)\\
&- 1307528\phi(7v) + 1961532\phi(6v) - 2498168\phi(5v) + 2714782\phi(4v)\\
&- 2538648\phi(3v) + 2095852\phi(2v) - 24!\phi(v) = 0 \qquad (17)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Multiplying (17) by 735471, and then subtracting (16) from the resulting equation, we get

$$\begin{aligned}
&1307504\phi(15v) - 29418840\phi(14v) + 316297104\phi(13v) - 2162284740\phi(12v)\\
&+ 10550487500\phi(11v) - 39097638360\phi(10v) - 114317570900\phi(9v) + 270459163700\phi(8v)\\
&+ 526781594100\phi(7v) - 854978826900\phi(6v) + 1166425626000\phi(5v) - 1347605475000\phi(4v)\\
&+ 1333027786000\phi(3v) - \frac{24!}{2}\phi(2v) + 24!(1271626)\phi(v) = 0 \qquad (18)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Doing $v = 3v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(15v) - 24\phi(14v) + 276\phi(13v) - 2024\phi(12v) + 10626\phi(11v)\\
&- 42504\phi(10v) + 134597\phi(9v) - 346128\phi(8v) + 735747\phi(7v)\\
&- 1309528\phi(6v) + 1971882\phi(5v) - 2538648\phi(4v) + 2838752\phi(3v)\\
&- 2842248\phi(2v) - 24!\phi(v) = 0 \qquad (19)
\end{aligned}$$

$\forall v \in \mathcal{F}$. Multiplying (19) by 1307504, and then subtracting (18) from the resulting equation, we get

$$\begin{aligned}
&1961256\phi(14v) - 44574000\phi(13v) + 484103356\phi(12v) - 3343050000\phi(11v) \\
&+ 16476511660\phi(10v) - 61668545020\phi(9v) + 182104580900\phi(8v) \\
&- 435210551400\phi(7v) + 857234271300\phi(6v) - 1411817977000\phi(5v) + 1971686940000\phi(4v) \\
&- 2378651809000\phi(3v) + \frac{24!}{2}\phi(2v) - 24!(2579130)\phi(v) = 0
\end{aligned} \tag{20}$$

$\forall v \in \mathcal{F}$. Doing $v = 2v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(14v) - 24\phi(13v) + 276\phi(12v) - 2024\phi(11v) + 10627\phi(10v) \\
&- 42528\phi(9v) + 134872\phi(8v) - 348128\phi(7v) + 746097\phi(6v) \\
&- 1350008\phi(5v) + 2095852\phi(4v) - 2842248\phi(3v) + 3439627\phi(2v) - 24!\phi(v) = 0
\end{aligned} \tag{21}$$

$\forall v \in \mathcal{F}$. Multiplying (21) by 1961256, and then subtracting (20) from the resulting equation, we get

$$\begin{aligned}
&2496144\phi(13v) - 57203300\phi(12v) + 626532144\phi(11v) - 4365755856\phi(10v) \\
&+ 21739750140\phi(9v) - 82413938380\phi(8v) + 247557577300\phi(7v) - 606052946600\phi(6v) \\
&+ 1235893313000\phi(5v) - 2138815370000\phi(4v) + 3195724134000\phi(3v) \\
&- \frac{24!}{2}\phi(2v) + 24!(4540386)\phi(v) = 0
\end{aligned} \tag{22}$$

$\forall v \in \mathcal{F}$. Doing $v = v$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(13v) - 24\phi(12v) + 277\phi(11v) - 2048\phi(10v) + 10902\phi(9v) \\
&- 44528\phi(8v) + 145222\phi(7v) - 388608\phi(6v) + 870067\phi(5v) \\
&- 1653608\phi(4v) + 2696727\phi(3v) - 3803648\phi(2v) - 24!\phi(v) = 0
\end{aligned} \tag{23}$$

$\forall v \in \mathcal{F}$. Multiplying (23) by 2496144, and then subtracting (22) from the resulting equation, we get

$$\begin{aligned}
&2704156\phi(12v) - 64899744\phi(11v) + 746347056\phi(10v) - 5473211744\phi(9v) \\
&+ 28734361660\phi(8v) - 114937446600\phi(7v) + 363968581000\phi(6v) \\
&- 935919208200\phi(5v) + 1988828317000\phi(4v) - 3535694787000\phi(3v) \\
&+ \frac{24!}{2}\phi(2v) - 24!(7036530)\phi(v) = 0
\end{aligned} \tag{24}$$

$\forall v \in \mathcal{F}$. Doing $v = 0$ and $w = v$ in (1), one gets

$$\begin{aligned}
&\phi(12v) - 24\phi(11v) + 276\phi(10v) - 2024\phi(9v) + 10626\phi(8v) \\
&- 42504\phi(7v) + 134596\phi(6v) - 346104\phi(5v) + 735471\phi(4v) \\
&- 1307504\phi(3v) + 1961256\phi(2v) - 3.102242009 \times 10^{23}\phi(v) = 0
\end{aligned} \tag{25}$$

$\forall v \in \mathcal{F}$. On simplification we arrive at

$$\frac{1}{16777216}\phi(2v) - \phi(v) = 0 \tag{26}$$

□

3. Hyers-Ulam Stability of Functional Equation (1) in Multi-Banach Spaces

First, we prove a lemma, which gives a useful strictly contractive mapping.

Lemma 3.1. *Let $\mathcal{F}$ be a linear space and $(\mathcal{G}^n, \|.\|)$ be a Banach space for all $n \in N$. Let $0 < \beta < 2^{24}$ and a mapping $\Psi : \mathcal{G}^n \to [0, \infty)$ such that*

$$\Psi(2v_1, 2v_2, ..., 2v_k) \leq \beta\Psi(v_1, v_2, ..., v_k)$$

for all $v_1, ..., v_k \in \mathcal{F}$. Let $P = \{l : \mathcal{F} \to \mathcal{G} : h(0) = 0\}$, and the generalized metric d defined on P by

$$d(l, m) = \inf \left\{ \mu \in (0, \infty) : \sup_{k \in \mathbb{N}} \|l(v_1) - m(v_1), ..., l(v_k) - m(v_k)\|_k \leq \mu\Psi(v_1, v_2, ..., v_k), \right\}$$

$\forall v_1, ..., v_k \in \mathcal{F}$. Then it is easy to prove that (P, d) is complete generalized metric on P [11]. Define a mapping $\mathcal{J} : P \to P$ by

$$\mathcal{J}l(v) = \frac{l(2^n v)}{2^{24n}}$$

for all $l \in P$ is strictly contractive mapping.

Proof. It is easy to show that d is a complete metric on X. Given $l, m \in P$, let $\mu \in (0, \infty)$ be an arbitary constant with $d(l, m) \leq \mu$. Then from the definition of d, it follows for each $v_1, v_2...v_k \in \mathcal{F}$ that

$$\sup_{k \in \mathbb{N}} \|(l(v_1) - m(v_1), ..., l(v_k) - m(v_k))\| \leq \mu\Psi(v_1, .., v_k)$$

and so

$$\begin{aligned} \sup_{k \in \mathbb{N}} \|(\mathcal{J}l(v_1) - \mathcal{J}m(v_1), ..., \mathcal{J}l(v_k) - \mathcal{J}m(v_k))\| &\leq \left\| \left(\frac{l(2^n v_1)}{2^{24n}} - \frac{m(2^n v_1)}{2^{24n}}, ..., \frac{l(2^n v_k)}{2^{24n}} - \frac{m(2^n v_k)}{2^{24n}} \right) \right\| \\ &\leq \frac{1}{2^{24n}} \|(l(2^n v_1) - m(2^n v_1), ..., l(2^n v_k) - m(2^n v_k))\| \\ &\leq \frac{\beta^n}{2^{24n}} \mu\Psi(v_1, ..., v_k) \end{aligned}$$

for all $v_1, ..., v_k \in \mathcal{F}$. Hence, it holds that

$$d(\mathcal{J}l, \mathcal{J}m) \leq \frac{\beta^n}{2^{24n}} d(l, m)$$

for all $l, m \in P$. Hence $\mathcal{J}$ is a strictly contractive mapping with Lipschitz constant $\frac{\beta^n}{2^{24n}}$. □

Theorem 3.2. *Let $\mathcal{F}$ be an linear space and let $\left((\mathcal{G}^k, \|.\|_k) : K \in \mathbb{N}\right)$ be a multi-Banach space. Suppose that δ is a non-negative real number and $\phi : \mathcal{F} \to \mathcal{G}$ be a function fulfills*

$$\sup_{k \in \mathbb{N}} \|(\mathcal{D}\phi(v_1, w_1), ..., \mathcal{D}\phi(v_k, w_k))\|_k \leq \delta \tag{27}$$

$\forall v_1, ..., v_k, w_1, ..., w_k \in \mathcal{F}$. Then there exists a unique Quattuorvigintic mapping $\mathcal{Q}_{24} : \mathcal{F} \to \mathcal{G}$ such that

$$\sup_{k \in \mathbb{N}} \|(\phi(v_1) - \mathcal{Q}_{24}(v_1), ..., \phi(v_k) - \mathcal{Q}_{24}(v_k))\|_k \leq \frac{16777218}{24!(16777216)}\delta. \tag{28}$$

$\forall v_i \in \mathcal{F}$, where $i = 1, 2, ..., k$.

Proof. Doing (v_i, w_i) by $(12v_i, v_i)$ and $(0, 2v_i)$ where $i = 1, 2...k$ in (27), and subtracting the two resulting equations, we arrive at

$$\sup_{k\in\mathbb{N}} \|(24\phi(23v_1) - 300\phi(22v_1) + 2024\phi(21v_1) - 10350\phi(20v_1) + 42504\phi(19v_1)$$
$$-136620\phi(18v_1) + 346104\phi(17v_1) - 724845\phi(16v_1) + 1307504\phi(15v_1)$$
$$-2003760\phi(14v_1) + 2496144\phi(13v_1) - 2569560\phi(12v_1) + 2496144\phi(11v_1)$$
$$-2307360\phi(10v_1) + 1307504\phi(9v_1) + 346104\phi(7v_1) - 1442100\phi(6v_1)$$
$$+42504\phi(5v_1) + 1950630\phi(4v_1) + 2024\phi(3v_1) - \frac{24!}{2}\phi(2v_1) + 24!\phi(v_1), ...,$$
$$24\phi(23v_k) - 300\phi(22v_k) + 2024\phi(21v_k) - 10350\phi(20v_k) + 42504\phi(19v_k)$$
$$-136620\phi(18v_k) + 346104\phi(17v_k) - 724845\phi(16v_k) + 1307504\phi(15v_k)$$
$$-2003760\phi(14v_k) + 2496144\phi(13v_k) - 2569560\phi(12v_k) + 2496144\phi(11v_k)$$
$$-2307360\phi(10v_k) + 1307504\phi(9v_k) + 346104\phi(7v_k) - 1442100\phi(6v_k)$$
$$+42504\phi(5v_k) + 1950630\phi(4v_k) \quad +2024\phi(3v_k) - \frac{24!}{2}\phi(2v_k) + 24!\phi(v_k)\Big)\Big\| \le \frac{3}{2}\delta \tag{29}$$

forall $v_1, ..., v_k \in \mathcal{F}$. Switching $v_1, ..., v_k$ into $11v_1, ..., 11v_k$ and Replacing $w_1, w_2, ..., w_k$ by $v_1, ..., v_k$ in (27) and using evenness of ϕ, one gets

$$\sup_{k\in\mathbb{N}} \|(\phi(23v_1) - 24\phi(22v_1) + 276\phi(21v_1) - 2024\phi(20v_1) + 10626\phi(19v_1)$$
$$-42504\phi(18v_1) + 134596\phi(17v_1) - 346104\phi(16v_1) + 735471\phi(15v_1)$$
$$-1307504\phi(14v_1) + 1961256\phi(13v_1) - 2496144\phi(12v_1) + 2704156\phi(11v_1)$$
$$-2496144\phi(10v_1) + 1961256\phi(9v_1) - 1307504\phi(8v_1) + 735471\phi(7v_1) - 346104\phi(6v_1)$$
$$+134596\phi(5v_1) - 42504\phi(4v_1) + 10626\phi(3v_1) - 2024\phi(2v_1) - 24!\phi(v_1), ...,$$
$$\phi(23v_k) - 24\phi(22v_k) + 276\phi(21v_k) - 2024\phi(20v_k) + 10626\phi(19v_k)$$
$$-42504\phi(18v_k) + 134596\phi(17v_k) - 346104\phi(16v_k) + 735471\phi(15v_k)$$
$$-1307504\phi(14v_k) + 1961256\phi(13v_k) - 2496144\phi(12v_k) + 2704156\phi(11v_k)$$
$$-2496144\phi(10v_k) + 1961256\phi(9v_k) - 1307504\phi(8v_k) + 735471\phi(7v_k) - 346104\phi(6v_k)$$
$$+134596\phi(5v_k) - 42504\phi(4v_k) + 10626\phi(3v_k) - 2024\phi(2v_k) - 24!\phi(v_k))\| \le \delta \tag{30}$$

forall $v_1, ..., v_k \in \mathcal{F}$. Multiplying by 24 on both sides of (30),then it follows from (29) and the resulting equation one gets

$$\sup_{k\in\mathbb{N}} \|(276\phi(22v_1) - 4600\phi(21v_1) + 38226\phi(20v_1) - 212520\phi(19v_1) + 883476\phi(18v_1)$$
$$-2884200\phi(17v_1) + 7581651\phi(16v_1) - 16343800\phi(15v_1) + 29376336\phi(14v_1)$$
$$-44574000\phi(13v_1) + 57337896\phi(12v_1) - 62403600\phi(11v_1) + 57600096\phi(10v_1)$$
$$-45762640\phi(9v_1) - 31380096\phi(8v_1) - 17305200\phi(7v_1) + 6864396\phi(6v_1)$$
$$-3187800\phi(5v_1) + 2970726\phi(4v_1) - 253000\phi(3v_1) + \frac{24!}{2}\phi(2v_1) + 24!(25)\phi(v_1), ...,$$

$$276\phi(22v_k) - 4600\phi(21v_k) + 38226\phi(20v_k) - 212520\phi(19v_k) + 883476\phi(18v_k)$$
$$-2884200\phi(17v_k) + 7581651\phi(16v_k) - 16343800\phi(15v_k) + 29376336\phi(14v_k)$$
$$-44574000\phi(13v_k) + 57337896\phi(12v_k) - 62403600\phi(11v_k) + 57600096\phi(10v_k)$$
$$-45762640\phi(9v_k) - 31380096\phi(8v_k) - 17305200\phi(7v_k) + 6864396\phi(6v_k)$$
$$-3187800\phi(5v_k) + 2970726\phi(4v_k) - 253000\phi(3v_k) + \frac{24!}{2}\phi(2v_k) + 24!(25)\phi(v_k)\Big)\Big\| \leq \frac{51}{2}\delta \tag{31}$$

On Simplification by Using (6), (8), (10), (12), (14), (16), (18), (20), (22), (24) we arrive at

$$\sup_{k\in\mathbb{N}}\left\|\left(\frac{1}{16777216}\phi(2v_1) - \phi(v_1), ..., \frac{1}{16777216}\phi(2v_k) - \phi(v_k)\right)\right\| \leq \frac{16777218}{24!(16777217)}\delta \tag{32}$$

forall $v_1, ..., v_k \in \mathcal{F}$. Let $\Psi = \{l : \mathcal{F} \to \mathcal{G} | l(0) = 0\}$ and introduce the generalized metric d defined on Ψ by

$$d(l, m) = \inf\left\{\Psi \in [0,\infty] | \sup_{k\in\mathbb{N}} \|l(v_1) - m(v_1), ..., l(v_k) - m(v_k)\|_k \leq \Psi \quad \forall \quad v_1, ..., v_k \in \mathcal{F}\right\}$$

Then it is easy to show that Ψ, d is a generalized complete metric space, See [11]. We define an operator $\mathcal{J} : \Psi \to \Psi$ by

$$\mathcal{J}l(v) = \frac{1}{2^{24}}l(2v) \qquad v \in \mathcal{F}$$

We assert that $\mathcal{J}$ is a strictly contractive operator. Given $l, m \in \Psi$, let $\Psi \in [0,\infty]$ be an arbitary constant with $d(l, m) \leq \Psi$. From the definition if follows that

$$\sup_{k\in\mathbb{N}} \|l(v_1) - m(v_1), ..., l(v_k) - m(v_k)\|_k \leq \Psi \quad v_1, ..., v_k \in \mathcal{F}.$$

Therefore, $\sup_{k\in\mathbb{N}} \|(\mathcal{J}l(v_1) - \mathcal{J}m(v_1), ..., \mathcal{J}l(v_k) - \mathcal{J}m(x_k))\|_k \leq \frac{1}{2^{24}}\Psi$, $v_1, ..., v_k \in \mathcal{F}$. Hence, it holds that

$$d(\mathcal{J}l, \mathcal{J}m) \leq \frac{1}{2^{24}}\Psi$$
$$d(\mathcal{J}l, \mathcal{J}m) \leq \frac{1}{2^{24}}d(l, m), \ \ \forall \ l, m \in \Psi.$$

This Means that $\mathcal{J}$ is strictly contractive operator on Ψ with the Lipschitz constant $L = \frac{1}{2^{24}}$. By (32), we have $d(\mathcal{J}\phi, \phi) \leq \frac{16777218}{24!(16777217)}\delta$. Applying the Theorem 2.2 in [14], we deduce the existence of a fixed point of $\mathcal{J}$ that is the existence of mapping $\mathcal{Q}_{24} : \mathcal{F} \to \mathcal{G}$ such that

$$\mathcal{Q}_{24}(2v) = 2^{24}\mathcal{Q}_{24}(v) \qquad \forall v \in \mathcal{F}.$$

Moreover, we have $d(\mathcal{J}^n\phi, \mathcal{Q}_{24}) \to 0$, which implies

$$\mathcal{Q}_{24}(v) = \lim_{n\to\infty} \mathcal{J}^n\phi(v) = \lim_{n\to\infty} \frac{\phi(2^n v)}{2^{24n}}$$

for all $v \in \mathcal{F}$. Also, $d(\phi, \mathcal{Q}_{24}) \leq \frac{1}{1-\mathcal{L}}d(\mathcal{J}\phi, \phi)$ implies the inequality

$$\leq \frac{16777218}{24!(16777216)}\delta.$$

Doing $v_1 =, ..., = v_k = 2^n v$, and $w_1 =, ..., = w_k = 2^n w$ in (27) and dividing by 2^{24n}. Now, applying the property (a) of multi-norms, we have

$$\begin{aligned}\|\mathcal{D}\mathcal{Q}_{24}(v,w)\| &= \lim_{n\to\infty} \frac{1}{2^{24n}} \|\mathcal{D}\phi\left(2^n v, 2^n w\right)\| \\ &\leq \lim_{n\to\infty} \frac{1}{2^{24n}} = 0\end{aligned}$$

for all $v, w \in \mathcal{F}$. The uniqueness of $\mathcal{Q}_{24}$ follows from the fact that $\mathcal{Q}_{24}$ is the unique fixed point of $\mathcal{J}$ with the property that there exists $\ell \in (0,\infty)$ such that

$$\sup_{k\in\mathbb{N}} \|(\phi(v_1) - \mathcal{Q}_{24}(v_1), ..., \phi(v_k) - \mathcal{Q}_{24}(v_k))\|_k \leq \ell$$

for all $v_1, ..., v_k \in \mathcal{F}$. □

Corollary 3.3. *Let $\mathcal{F}$ be a linear space, and let $(\mathcal{G}^n, \|.\|_n)$ be a multi-Banach space. Let $\theta > 0, 0 < p < 24$ and $\phi : \mathcal{F} \to \mathcal{G}$ be a mapping satisfying $f(0) = 0$*

$$\sup_{k\in\mathbb{N}} \|\mathcal{D}\phi(v_1, w_1), ..., \mathcal{D}\phi(v_k, w_k)\|_k \leq \theta\left(\|v_1\|^p + \|w_1\|^p, .., \|v_k\|^p + \|w_k\|^p\right) \tag{33}$$

for all $v_1, ..., v_k, w_1 ..., w_k \in \mathcal{F}$. Then there exists a unique mapping $\mathcal{Q}_{24} : \mathcal{F} \to \mathcal{G}$ such that

$$\sup_{k\in\mathbb{N}} \|(\phi(v_1) - \mathcal{Q}_{24}(v_1), ..., \phi(v_k) - \mathcal{Q}_{24}(v_k))\| \leq \frac{1}{2^{24} - 2^p}\Delta_A(\|v_1\|^p, ..., \|v_k\|^p) \tag{34}$$

where

$$\begin{aligned}\Delta_A = \frac{2}{24!}\theta &\left[\frac{1}{2}2^p + 12^p + 1 + 24(11^p + 1)276(10^p + 1) + 2024(9^p + 1)\right. \\ &+10626(8^p + 1) + 42504(7^p + 1) + 134596(6^p + 1) + 346104(5^p + 1) \\ &\left.+7354710(4^p + 1) + 1307504(3^p + 1) + 1961256(2^p + 1) + 51274958\right]\end{aligned}$$

Proof. Proof is similar to that of Theorem 3.2 replacing δ by $\theta\left(\|v_1\|^p + \|w_1\|^p, .., \|v_k\|^p + \|w_k\|^p\right)$ we arrive the result. □

Corollary 3.4. *Let $\mathcal{F}$ be a linear space, and let $(\mathcal{G}^n, \|.\|_n)$ be a multi-Banach space. Let $\theta > 0, 0 < r + s = p < 24$ and $\phi : \mathcal{F} \to \mathcal{G}$ be a mapping satisfying $f(0) = 0$*

$$\sup_{k\in\mathbb{N}} \|\mathcal{D}\phi(v_1, w_1), ..., \mathcal{D}\phi(v_k, w_k)\|_k \leq \theta\left(\|v_1\|^r . \|w_1\|^s, .., \|v_k\|^r . \|w_k\|^s\right) \tag{35}$$

for all $v_1, ..., v_k, w_1 ..., w_k \in \mathcal{F}$. Then there exists a unique mapping $\mathcal{Q}_{24} : \mathcal{F} \to \mathcal{G}$ such that

$$\sup_{k\in\mathbb{N}} \|(\phi(v_1) - \mathcal{Q}_{24}(v_1), ..., \phi(v_k) - \mathcal{Q}_{24}(v_k))\| \leq \frac{1}{2^{24} - 2^p}\Delta_{AA}(\|v_1\|^{r+s}, ..., \|v_k\|^{r+s}) \tag{36}$$

where

$$\begin{aligned}\Delta_{AA} = \frac{2}{24!}\theta &\left[12^r + 24(11)^r + 276(10)^r + 2024(9)^r + 10626(8)^r + 42504(7)^r + 134596(6)^r + 346104(5)^r\right. \\ &\left.+7354710(4)^r + 1307504(3)^r + 1961256(2)^r + 24961440\right]\end{aligned}$$

Proof. Proof is similar to that of Theorem 3.2 replacing δ by $\theta\left(\|v_1\|^r . \|w_1\|^s, .., \|v_k\|^r . \|w_k\|^s\right)$ we arrive the result. □

Corollary 3.5. *Let $\mathcal{F}$ be a linear space, and let $(\mathcal{G}^n, \|.\|_n)$ be a multi-Banach space. Let $\theta > 0, 0 < r + s = p < 24$ and $\phi : \mathcal{F} \to \mathcal{G}$ be a mapping satisfying $f(0) = 0$*

$$\sup_{k\in\mathbb{N}} \|\mathcal{D}\phi(v_1, w_1), ..., \mathcal{D}\phi(v_k, w_k)\|_k \leq \theta \left(\|v_1\|^r . \|w_1\|^s + (\|v_1\|^{r+s} + \|w_1\|^{r+s}), .., \ \|v_k\|^r . \|w_k\|^s + (\|v_k\|^{r+s} + \|w_k\|^{r+s})\right) \tag{37}$$

for all $v_1, ..., v_k, w_1..., w_k \in \mathcal{F}$. Then there exists a unique mapping $\mathcal{Q}_{24} : \mathcal{F} \to \mathcal{G}$ such that

$$\sup_{k\in\mathbb{N}} \|(\phi(v_1) - \mathcal{Q}_{24}(v_1), ..., \phi(v_k) - \mathcal{Q}_{24}(v_k))\| \leq \frac{1}{2^{24} - 2^p} \Delta_{AAA}(\|v_1\|^{r+s}, ..., \|v_k\|^{r+s}) \tag{38}$$

where

$$\begin{aligned}\Delta_{AAA} = \frac{2}{24!}\theta \big[&2^{r+s} + 12^r + 12^{r+s} + 24(11^r + (11)^{r+s}) + 276(10^r + (10)^{r+s}) + 2024(9^r + (9)^{r+s}) \\ &+10626(8^r + (8)^{r+s}) + 42504(7^r + (7)^{r+s}) + 134596(6^r + (6)^{r+s}) + 346104(5^r + (5)^{r+s}) \\ &+7354710(4^r + 4^{r+s}) + 1307504(3^r + 3^{r+s}) + 1961256(2^r + 2^{r+s}) + 1159630\big]\end{aligned}$$

Proof. Proof is similar to that of Theorem 3.2 replacing δ by $\theta \left(\|v_1\|^r . \|w_1\|^s + (\|v_1\|^{r+s} + \|w_1\|^{r+s}), ..., \|v_k\|^r . \|w_k\|^s \right.$ $\left. +(\|v_k\|^{r+s} + \|w_k\|^{r+s})\right)$ we arrive the result. □

4. Counter Examples

Example 4.1. *Let $\beta : \mathbb{R} \to \mathbb{R}$ be a function defined by*

$$\beta(v) = \begin{cases} \epsilon v^{24}, & |v| < 1 \\ \epsilon, & otherwise \end{cases}$$

where $\epsilon > 0$ is a constant, and define a function $\phi : \mathbb{R} \to \mathbb{R}$ by

$$\phi(v) = \sum_{n=0}^{\infty} \frac{\beta(2^n)v}{2^{24n}}$$

for all $v \in \mathbb{R}$. Then ϕ satisfies the inequality

$$\|\mathcal{D}\phi(v, w)\| \leq \frac{6.204484017 \times 10^{23}}{16777215}(16777216)^2\epsilon \left(|v|^{24} + |w|^{24}\right) \tag{39}$$

for all $v, w \in \mathbb{R}$. Then there does not exists a Quattuorvigintic mapping $\mathcal{Q}_{24} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ such that

$$|\phi(v) - \mathcal{Q}_{24}(v)| \leq \lambda |v|^{24} \quad \forall v \in \mathbb{R}. \tag{40}$$

Proof. It is easy to see that ϕ is bounded by $\dfrac{16777216}{16777215}\epsilon$ on $\mathbb{R}$.
If $|v|^{24} + |w|^{24} = 0$, then (39) is trivial. If $|v|^{24} + |w|^{24} \geq \dfrac{1}{2^{24}}$, then there exists a non-negative integer k such that

$$\frac{1}{2^{24(k+1)}} \leq |v|^{24} + |w|^{24} \leq \frac{1}{2^{24k}} \tag{41}$$

Hence From definition of ϕ and (41), we arrive that

$$|\mathcal{D}\phi(v, w)| \leq \sum_{n=k}^{\infty} \frac{(6.204484017 \times 10^{23})\epsilon}{2^{24n}}$$

$$\leq \frac{(16777216)(6.204484017 \times 10^{23})\epsilon}{16777215(2^{24k})}$$
$$\leq \frac{(6.204484017 \times 10^{23})}{16777215}(16777216)^2\epsilon\left(|v|^{24} + |w|^{24}\right)$$

Therefore, ϕ satisfies (39) for all $v, w \in \mathbb{R}$. Now, we claim that functional equation (1) is not stable for $\kappa = 24$ in Corollary (3.3). Suppose on the contrary that there exists a Quattuorvigintic mapping $\mathcal{Q}_{24} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ satisfying (46). Then there exists a constant $c \in \mathbb{R}$ such that $\mathcal{Q}_{24}(v) = cv^{24}$ for any $v \in \mathbb{R}$. Thus we obtain the following inequality

$$|\phi(v)| \leq (\lambda + |c|)\,|v|^{24} \tag{42}$$

Let $m \in \mathbb{N}$ with $m\epsilon > \lambda + |c|$. If $v \in (0, \frac{1}{2^{m-1}})$, then $2^n v \in (0,1)$ for all $n = 0, 1, 2...m-1$ and for this case we get

$$\phi(v) = \sum_{n=0}^{\infty} \frac{\beta(2^n)v}{2^{24n}} \geq \sum_{n=0}^{m-1} \frac{\epsilon(2^n v)^{24}}{2^{24n}} = m\epsilon v^{24} > (\lambda + |c|)\,|v|^{24}$$

which is a contradiction to (42). Therefore the Quattuorvigintic functional equation (1) is not stable for $\kappa = 24$. □

Example 4.2. *Let $\beta : \mathbb{R} \to \mathbb{R}$ be a function defined by*

$$\beta(v) = \begin{cases} \epsilon v^{24}, & |v| < 1 \\ \epsilon, & otherwise \end{cases}$$

where $\epsilon > 0$ is a constant, and define a function $\phi : \mathbb{R} \to \mathbb{R}$ by

$$\phi(v) = \sum_{n=0}^{\infty} \frac{\beta(2^n)v}{2^{24n}}$$

for all $v \in \mathbb{R}$. Then ϕ satisfies the inequality

$$\|\mathcal{D}\phi(v,w)\| \leq \frac{6.204484017 \times 10^{23}}{16777215}(16777216)^2\epsilon\left(|v|^{12}\,.\,|w|^{12}\right) \tag{43}$$

for all $v, w \in \mathbb{R}$. Then there does not exists a Quattuorvigintic mapping $\mathcal{Q}_{24} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ such that

$$|\phi(v) - \mathcal{Q}_{24}(v)| \leq \lambda\,|v|^{24} \quad \forall v \in \mathbb{R}. \tag{44}$$

Proof. The proof is analogous to the proof of Example 4.1. □

Example 4.3. *Let $\beta : \mathbb{R} \to \mathbb{R}$ be a function defined by*

$$\beta(v) = \begin{cases} \epsilon v^{24}, & |v| < 1 \\ \epsilon, & otherwise \end{cases}$$

where $\epsilon > 0$ is a constant, and define a function $\phi : \mathbb{R} \to \mathbb{R}$ by

$$\phi(v) = \sum_{n=0}^{\infty} \frac{\beta(2^n)v}{2^{24n}}$$

for all $v \in \mathbb{R}$. Then ϕ satisfies the inequality

$$\|\mathcal{D}\phi(v,w)\| \leq \frac{6.204484017 \times 10^{23}}{16777215}(16777216)^2\epsilon\left(|v|^{12}\,.\,|w|^{12} + (|v|^{24} + |w|^{24})\right) \tag{45}$$

for all $v, w \in \mathbb{R}$. Then there does not exists a Quattuorvigintic mapping $\mathcal{Q}_{24} : \mathbb{R} \to \mathbb{R}$ and a constant $\lambda > 0$ such that

$$|\phi(v) - \mathcal{Q}_{24}(v)| \leq \lambda\,|v|^{24} \quad \forall v \in \mathbb{R}. \tag{46}$$

Proof. The proof is analogous to the proof of Example 4.1. □

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