International Journal of Mathematics And its Applications
Volume 5, Issue 2–B (2017), 203–210.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Decomposition of M-rg-Continuous Maps

Research Article

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Abstract: In this paper, we introduce new types of sets like m-gpr-closed sets and m-gα**-closed sets and new classes of maps namely M-C$_r$-continuity and M-C$_r$*-continuity in minimal spaces and study some of their properties. We obtain some decompositions of M-rg-continuity in minimal spaces.

MSC: 54A05, 54D15, 54D30.

Keywords: m-gα**-closed set, m-C$^{\#}$-set, m-C*-set, m-Cr-set, m-Cr*-set.

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1. Introduction

In 1961, Levine [3] obtained a decomposition of continuity in topological spaces. Further Rose [11, 12] improved Levine's decomposition results. In 1986, Tong [14] obtained a decomposition of continuity and proved that his decomposition is independent of Levine's. In 1989, Tong [15] improved upon his earlier decomposition and obtained yet another decomposition of continuity. In 1990, Ganster and Reilly [2] obtained a decomposition of continuity improving the first result of Tong [15]. A year later, Ganster and Reilly [2] improved Tong's second decomposition. The concept of minimal structures (briefly, m-structure) were widely studied by Popa and Noiri [10] in 2000. In this paper, we introduce new types of sets like m-gpr-closed sets and m-gα**-closed sets and new classes of maps namely M-C$_r$-continuity and M-C$_r$*-continuity in minimal spaces and study some of their properties. We obtain some decompositions of M-rg-continuity in minimal spaces.

2. Some Basic Results

Definition 2.1. *Let S be a subset of X. Then S is said to be*

(1) m-α-open [9] if $S \subseteq m-Int(m-Cl(m-Int(S)))$,

(2) m-preopen [7] if $S \subseteq m-Int(m-Cl(S))$.

The complements of their open sets are called their corresponding closed sets.

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Remark 2.2. *The m-preInterior, m-pInt(S) [resp. m-α-Interior, m-α-Int(S)], of S is the union of all m-preopen sets [resp. m-α-open sets] contained in S. The m-preclosure, m-pCl(S) [resp. m-α-closure, m-α-Cl(S)], of S is the intersection of all m-preclosed sets [resp. m-α-closed sets] that contains S.*

Definition 2.3. *Let S be a subset of X. Then S is said to be*

(1) m-g-open if $F \subseteq m - Int(S)$ where F is m-closed and $F \subseteq S$,

(2) m-rg-open if $F \subseteq m - Int(S)$ where F is regular m-closed and $F \subseteq S$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.4 ([1])**.** *A minimal structure m_x on a nonempty set X is said to have property B if the union of any family of subsets belonging to m_x belongs to m_x.*

Lemma 2.5 ([1])**.** *The following are equivalent for the minimal space (X, m_x).*

(1) m_x have property B.

(2) If $m_x - Int(E) = E$, then $E \in m_x$,

(3) If $m_x - Cl(F) = F$, then $F^c \in m_x$.

Theorem 2.6. *Let (X, m_x) have property B and S be a subset of X. Then*

(1) $m - pInt(S) = S \cap m - Int(m - Cl(S))$,

(2) $m - \alpha - Int(S) = S \cap m - Int(m - Cl(m - Int(S)))$.

Definition 2.7. *A subset S of X is said to be*

(1) regular m-open [1] if $= m - Int(m - Cl(S))$,

(2) regular m-closed [12] if $S = m - Cl(m - Int(S))$.

The family of all regular m-closed sets of X is denoted m-RC(X).

Theorem 2.8 ([6])**.** *Let (X, m_x) have property B. Then every regular m-open set is m-open. However the converse is not true.*

Theorem 2.9 ([6])**.** *Let (X, m_x) have property B. Then every m-g-open set is m-rg-open. However the converse is not true.*

Definition 2.10 ([6])**.** *A subset S of X is said to be*

(1) a m-t set if $m - Int(m - Cl(S)) = m - Int(S)$,

(2) a m-h set if $m - Int(m - Cl(m - Int(S))) = m - Int(S)$.

Theorem 2.11 ([6])**.** *Any m-t set is m-h set.*

Definition 2.12 ([6])**.** *A minimal space (X, m_x) has the property I if the any finite intersection of m-open sets is m-open.*

Remark 2.13 ([6])**.** *For subsets A and B of a minimal space (X, m_x) satisfying property I, the following holds: $m - Int(A \cap B) = m - Int(A) \cap m - Int(B)$.*

3. M-C$_r$-Continuity and M-C$_r$*-Continuity

Here we introduce new types of sets as follows:

Definition 3.1. *Let S be a subset of X. Then S is said to be*

(1) a generalized pre-regular m-closed (briefly, m-gpr-closed) if $m - pCl(S) \subseteq G$ *whenever* $S \subseteq G$ *and G is regular m-open.*

(2) a m-gα***-closed if* $m - \alpha - Cl(S) \subseteq m - Int(m - Cl(U))$ *whenever* $S \subseteq U$ *and U is* $m - \alpha - open$.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 3.2. *Let S be a subset of X. Then S is said to be a*

(1) m-$C^{\#}$*-set if* $S = A \cap B$ *where A is m-g-open and B is a m-t set in X,*

(2) m-C-set if* $S = A \cap B$ *where A is m-g-open and B is a m-h set in X,*

(3) m-Cr-set if $S = A \cap B$ *where A is m-rg-open and B is a m-t set in X,*

(4) m-Cr-set if* $S = A \cap B$ *where A is m-rg-open and B is a m-h set in X.*

Proposition 3.3. *A m-t set is a m-Cr-set.*

Proof. $S = X \cap S$ where X is m-rg-open and S is a m-t set. □

Proposition 3.4. *A m-rg-open set is a m-*C_r*-set.*

Proof. $S = X \cap S$ where X is a m-t set and S is m-rg-open. □

Proposition 3.5. *A m-h set is a m-*C_r**-set.*

Proof. $S = X \cap S$ where X is m-rg-open and S is a m-h set. □

Proposition 3.6. *A m-rg-open set is a m-*C_r**-set.*

Proof. $S = X \cap S$ where X is m-h set and S is m-rg-open set. □

Proposition 3.7. *A m-t set is a m-*C_r**-set.*

Proof. $S = X \cap S$ where X is m-rg-open and S is a m-t set. By Theorem 2.4, S is a m-h set. □

Proposition 3.8. *Let* (X, m_x) *have property B. Then every m-*$C^{\#}$*-set is a m-*C_r*-set.*

Proof. $S = A \cap B$ where A is m-g-open and B is a m-t set. By Theorem 2.4, A is m-rg-open and B is a m-h set. □

Proposition 3.9. *Let* (X, m_x) *have property B. Then every m-*$C^{\#}$*-set is a m-*C_r**-set.*

Proof. $S = A \cap B$ where A is m-g-open and B is a m-t set. By Theorem 2.3 and Theorem 2.4, A is m-rg-open and B is a m-h set. □

Proposition 3.10. *A m-*C_r*-set is a m-*C_r**-set.*

Proof. $S = A \cap B$ where A is m-rg-open and B is a m-t set. By Theorem 2.4, B is a m-h set. □

Proposition 3.11. *A m-*$C^{\#}$*-set is a m-C*-set.*

Proposition 3.12. *Let (X, m_x) have property B. Then every m-C*-set is a m-C_r*-set.*

By means of simple examples on finite spaces, it can be seen that the converses of all the above Propositions need not necessarily hold. We introduce new classes of mappings as follows.

Proposition 3.13. *A mapping $f : X \to Y$ is said to be*

(1) M-$C^{\#}$-continuous if $f^{-1}(V)$ is a m-$C^{\#}$-set in X for every m_y-open set V in Y.

(2) M-C-continuous if $f^{-1}(V)$ is a m-C*-set in X for every m_y-open set V in Y.*

(3) M-C_r-continuous if $f^{-1}(V)$ is a m-C_r-set in X for every m_y-open set V in Y.

(4) M-C_r-continuous if $f^{-1}(V)$ is a m-C_r*-set in X for every m_y-open set V in Y.*

Proposition 3.14. *Let (X, m_x) have property $\mathcal{B}$ and $f : X \to Y$ be a mapping. Then the following results are easily obtained.*

(1) A M-$C^{\#}$-continuous mapping is M-C_r-continuous.

(2) A M-C_r-continuous mapping is M-C_r-continuous.*

(3) A M-C-continuous mapping is M-C_r*-continuous.*

(4) A M-$C^{\#}$-continuous mapping is M-C_r-continuous.*

(5) A M-$C^{\#}$-continuous mapping is M-C-continuous.*

The converses of all the above results need not necessarily hold as seen from the following examples.

Example 3.15. *Let $X = Y = \{a, b, c\}$, $m_x = \{\phi, X, \{a, b\}\}$ and $m_y = \{\phi, Y, \{b, c\}\}$. Let $f : X \to Y$ be the identity mapping. Then f is both M-C*-continuous and M-C_r-continuous but it is not M-$C^{\#}$-continuous.*

Example 3.16. *Let $X = \{a, b, c, d\}$, $Y = \{x, y\}$, $m_x = \{\phi, X, \{c, d\}, \{b, c, d\}\}$ and $m_y = \{\phi, Y, \{x\}\}$. Define $f : X \to Y$ as $f(a) = f(b) = f(c) = x$ and $f(d) = y$. Then f is M-C_r*-continuous but it is not M-C_r-continuous.*

Example 3.17. *Let $X = Y = \{a, b, c\}$, $m_x = \{\phi, X, \{a, b\}\}$ and $m_y = \{\phi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Then the identity mapping f: $X \to Y$ is M-C_r*-continuous but it is not M-C*-continuous.*

Example 3.18. *In Example 3.1, f is M-C_r*-continuous but it is not M-$C^{\#}$-continuous.*

Remark 3.19. *M-C*-continuity and M-C_r-continuity are independent of each other. In Example 3.3, f is M-C_r-continuous but it is not M-C*-continuous. In Example 3.2, f is M-C*-continuous but it is not M-C_r-continuous.*

Remark 3.20. *From the above results, we have the following implications:*

$$\begin{array}{ccc} M-C^{\#}-continuity & \to & M-Cr-continuity \\ \downarrow & & \downarrow \\ M-C*-continuity & \to & M-Cr*-continuity \end{array}$$

The above examples and remark enable us to realize that none of the above implications is reversible.

4. m-gpr-Open Sets and m-gα**-Open Sets

In this section, we give the relation between m-rg-closed sets, m-gα**-closed sets and m-gpr-closed sets and their relations with m-C_r-sets and m-C_r*-sets.

Lemma 4.1. *Let (X, m_x) have property B. Then a subset A of X is m-gα**-closed in X if and only if $m-\alpha-Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a regular m-open set in X.*

Proof. Necessity is trivial. Assume that $A \subseteq U$ and U is m-α-open in X. Then $A \subseteq U \subseteq m-Int(m-Cl(m-Int(U))) \subseteq m-Int(m-Cl(U))$. Let $B = U \cup m-Int(m-Cl(U)) = m-Int(m-Cl(U))$. Then $m-Int(m-Cl(B)) = m-Int(m-Cl(U)) = B$. Therefore B is regular m-open. Since $A \subseteq B$, $m-\alpha-Cl(A) \subseteq B = m-Int(m-Cl(U))$. Therefore A is m-g$\alpha$**-closed. Hence the proof. □

Lemma 4.2. *For an $x \in X$, its complement $X - \{x\}$ is m-gpr-closed or regular m-open.*

Proof. Suppose X−{x} is not regular m-open. Then X is the only regular m-open set containing $X - \{x\}$. This implies $m-pCl(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is m-gpr-closed. The class of all m-gpr-closed [resp. m-preopen, m-preclosed] sets of X is denoted by m-GPRC(X) [resp. m-PO(X), m-PC(X)]. □

Proposition 4.3. *Let (X, m_x) have property B. Then if $m-PO(X) = m-PC(X)$, then $m-GPRC(X) = \wp(X)$.*

Proof. Suppose $A \subseteq O$, where O is regular m-open in X. Since O is m-preopen, it is m-preclosed by hypothesis. Hence $m-pCl(A) \subseteq O$ and so A is m-gpr-closed. Thus $m-GPRC(X) = \wp(X)$. □

Proposition 4.4. *Let (X, m_x) have property B. Then every m-rg-closed set is m-gα**-closed.*

Proof. Since A is m-rg-closed, we have, $m-\alpha-Cl(A) \subseteq m-Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular m-closed. By Lemma 4.1., A is m-gα**-closed. □

Proposition 4.5. *Let (X, m_x) have property B. Then every m-gα**-closed set is m-gpr-closed.*

Proof. Let A be a m-gα**-closed set. Then, by Lemma 4.1, $m-\alpha-Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular m-open. Since $m-pCl(A) \subseteq m-\alpha-Cl(A)$, we have $m-pCl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular m-open. Therefore A is m-gpr-closed. □

Example 4.6. *The converse of Proposition 4.3 need not be true in general. Let $X = \{a, b, c, d\}$ and $m_x = \{\phi, X, \{c, d\}, \{b, c, d\}\}$. Clearly $\{c\}$ is m-gpr-closed but it is not m-gα**-closed.*

Example 4.7. *The converse of Proposition 4.2 need not be true in general. Let $X = \{a, b, c, d, e\}$ and $m_x = \{\phi, X, \{b\}, \{b, c\}, \{a, b, c, d\}\}$. Clearly $\{c\}$ is m-gα**-closed but it is not m-rg-closed.*

Remark 4.8. *We have the following implications:*

$$m-rg-closed \rightarrow m-g\alpha**-closed \rightarrow m-gpr-closed.$$

The above examples enable us to realize that none of the above implications is reversible.

Remark 4.9. *m-gpr-open sets and m-C_r-sets are independent of each other. In Example 4.1, $\{a, b\}$ is m-C_r-set but it is not m-gpr-open. Moreover $\{a, b, c\}$ is m-gpr-open set but it is not m-C_r-set.*

Remark 4.10. *m-gα**-open sets and m-C_r-sets are independent of each other. In Example 4.2, $\{a, d, e\}$ is m-C_r-set but it is not m-gα**-open. Moreover $\{a, b, d, e\}$ is m- gα**-open set but it is not m -C_r-set.*

Remark 4.11. *m-gα**-open sets and m-C_r*-sets are independent of each other. In Example 4.2, $\{a, d, e\}$ is m-C_r*-set but it is not m-gα**-open. Moreover $\{a, b, d, e\}$ is m-gα**-open set but it is not m-C_r*-set.*

Theorem 4.12. *Let (X, m_x) have property B and property I. Then a subset S of X is*

(1) m-rg-open in X if and only if it is both m-gpr-open and a m-C_r-set in X.

*(2) m-rg-open in X if and only if it is both m-gα**-open and a m-C_r-set in X.*

*(3) m-rg-open in X if and only if it is both m-gα**-open and a m-C_r*-set in X.*

Proof.

(1) Necessity is trivial. Assume that S is m-gpr-open and a m-C_r-set in X. Then $S = A \cap B$ where A is m-rg-open and B is a m-t set in X. Let $F \subseteq S$, where F is regular m-closed in X. Since S is m-gpr-open in X,

$$\begin{aligned} F \subseteq m - pInt(S) &= S \cap m - Int(m - Cl(S)) \\ &= (A \cap B) \cap m - Int(m - Cl(A \cap B)) \\ &\subseteq (A \cap B) \cap m - Int(m - Cl(A)) \cap m - Int(m - Cl(B)) \\ &= (A \cap B) \cap m - Int(m - Cl(A)) \cap m - Int(B). \end{aligned}$$

This implies F $\subseteq$ m-Int(B). Note that A is m-rg-open and that $F \subseteq A$. So $F \subseteq m - Int(A)$. Therefore, $F \subseteq m - Int(A) \cap m - Int(B) = m - Int(S)$ by property I. Thus S is m-rg-open. Hence the proof of (1).

(2) Necessity is trivial. Assume that S is m-gα**-open and a m-C_r-set in X. Then $S = A \cap B$ where A is m-rg-open and B is a m-t set in X. Let $F \subseteq S$, where F is regular m-closed in X. Since S is m-gα**-open, using Lemma 4.1, we have,

$$\begin{aligned} F \subseteq m - \alpha - Int(S) &= S \cap m - Int(m - Cl(m - Int(S))) \\ &\subset A \cap B \cap m - Int(m - Cl(m - Int(A))) \cap m - Int(m - Cl(m - Int(B))) \\ &= A \cap B \cap m - Int(m - Cl(m - Int(A))) \cap m - Int(B) \end{aligned}$$

because a m-t set is a m-h set. This implies, $F \subseteq m - Int(B)$. Since A is m-rg-open and F is regular m-closed in X, $F \subseteq m - Int(A)$. So $F \subseteq m - Int(A) \cap m - Int(B) = m - Int(S)$ by property I. Therefore F is m-rg-open. Hence the proof of (2).

(3) The proof of it is similar to the proof of (2).

□

Remark 4.13. *In the above theorem, both properties are used and so, the theorem is nothing but topological results.*

5. Decompositions of M-rg-Continuity

Definition 5.1. *A mapping $f : X \to Y$ is called M-rg-continuous if $f^{-1}(V)$ is m_x-rg-open in X for every m_y-open set V in Y.*

We introduce new classes of mappings as follows.

Definition 5.2. *A mapping $f : X \to Y$ is called*

(1) generalized pre-regular M-continuous (briefly, M-gpr-continuous) if $f^{-1}(V)$ is m_x-gpr-open in X for every m_y-open set V in Y.

*(2) M-gα**-continuous if $f^{-1}(V)$ is m_x-gα**-open in X for every m_y-open set V in Y.*

Theorem 5.3. *Let (X, m_x) have property B and property I and $f : X \to Y$ be a mapping. Then f is M-rg-continuous if and only if it is both*

(1) M-gpr-continuous and M-C_r-continuous.

*(2) M-gα**-continuous and M-C_r-continuous.*

*(3) M-gα**-continuous and M-C_r*-continuous.*

Remark 5.4. *In the above theorem, both properties are used and so, the theorem is nothing but topological result.*

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