International Journal of Mathematics And its Applications
Volume 5, Issue 2–B (2017), 231–238.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Complementary Tree Nil Domination Number of Splitting Graphs

Research Article

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Abstract: A set D of a graph $G = (V, E)$ is a dominating set, if every vertex in $V(G) - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set. A dominating set D is called a complementary tree nil dominating set, if $V(G) - D$ is not a dominating set and also the induced subgraph $\langle V(G) - D\rangle$ is a tree. The minimum cardinality of a complementary tree nil dominating set is called the complementary tree nil domination number of G and is denoted by $\gamma_{ctnd}(G)$. In this paper, some results regarding the complementary tree nil domination number of splitting graphs of connected graphs are found.

Keywords: Complementary tree domination, Complementary tree nil domination, Splitting graphs.

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1. Introduction

Graphs discussed in this paper are finite, undirected and simple graphs. For a graph G, let $V(G)$ and $E(G)$ denote its vertex set and edge set respectively. A graph G with p vertices and q edges is denoted by $G(p, q)$. The concept of domination in graphs was introduced by Ore [5]. A set $D \subseteq V(G)$ is said to be a dominating set of G, if every vertex in $V(G) - D$ is adjacent to some vertex in D. The cardinality of a minimum dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. Muthammai, Bhanumathi and Vidhya [4] introduced the concept of complementary tree dominating set. A dominating set $D \subseteq V(G)$ is said to be a complementary tree dominating set (ctd-set) if the induced subgraph $\langle V(G) - D\rangle$ is a tree. The minimum cardinality of a ctd-set is called the complementary tree domination number of G and is denoted by $\gamma_{ctd}(G)$. Any undefined terms in this paper may be found in Harary [1]. Splitting graphs were first studied by Sampathkumar and Walikar [7]. For a graph G, let $V'(G) = \{v' : v \in V(G)\}$ be a copy of $V(G)$. The splitting graph $Sp(G)$ of G is the graph with vertex set $V(G) \cup V'(G)$ and edge set $\{uv, u'v, uv' : uv \in E(G)\}$. A graph G and its splitting graph are given in Figure 1. The concept of complementary tree nil dominating set is introduced in [3]. A dominating set $D \subseteq V(G)$ is said to be a complementary tree nil dominating set (ctnd-set) if the induced subgraph $\langle V(G) - D\rangle$ is a tree and $V(G) - D$ is not a dominating set. The minimum cardinality of a ctnd-set is called the complementary tree nil domination number of G and is denoted by $\gamma_{ctnd}(G)$. In this paper, some results regarding the complementary tree nil domination number of splitting graphs of graphs are found.

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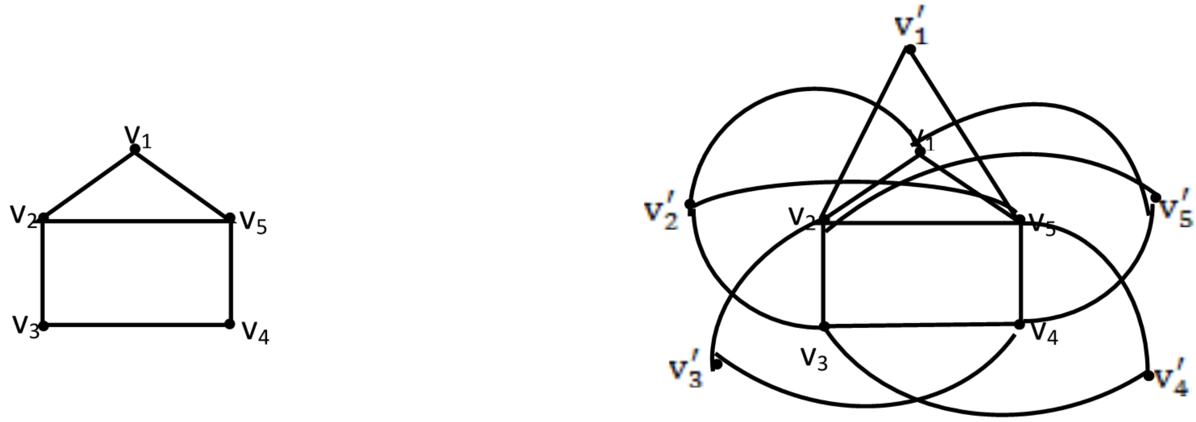

Figure 1:

2. Prior Results

Theorem 2.1 ([2])**.** *Radius of* $Sp(G) = \max\{2, radius\ of\ G\}$*.*

Theorem 2.2 ([3])**.** *For any connected graph G,* $\delta(G) + 1 \leq \gamma_{ctnd}(G)$*.*

Theorem 2.3 ([3])**.** *For any connected graph G with p vertices,* $2 \leq \gamma_{ctnd}(G) \leq p$*, where* $p \geq 2$*.*

Theorem 2.4 ([3])**.** *Let G be a connected graph with p vertices. Then* $\gamma_{ctnd}(G) = 2$ *if and only if G is a graph obtained by attaching a pendant edge at a vertex of degree* $p - 2$ *in* $T + K_1$*, where T is a tree on* $(p - 2)$ *vertices.*

Theorem 2.5 ([3])**.** *For any connected graph G,* $\gamma_{ctnd}(G) = p$ *if and only if* $G \cong K_p$*, where* $p \geq 2$*.*

Theorem 2.6 ([3])**.** *Let G be a connected graph with* $p \geq 3$ *and* $\delta(G) = 1$*. Then* $\gamma_{ctnd}(G) = p - 1$ *if and only if the subgraph of G induced by vertices of degree atleast 2 is* K_2 *or* K_1*.*

That is, G is one of the graphs $K_{1,p-1}$ or $S_{m,n}$ $(m + n = p, m, n \geq 1)$, where $S_{m,n}$ is a bistar which is obtained by attaching $m - 1$ pendant edges at one vertex of K_2 and $n - 1$ pendant edges at other vertex of K_2.

Theorem 2.7 ([3])**.** *Let G be a connected noncomplete graph such that* $\delta(G) \geq 2$*. Then* $\gamma_{ctnd}(G) = p - 1$ *if and only if each edge of G is a dominating edge.*

Theorem 2.8 ([3])**.** *Let T be a tree on p vertices such that* $\gamma_{ctnd}(T) \leq p - 2$*. Then* $\gamma_{ctnd}(T) = p - 2$ *if and only if T is one of the following graphs.*

1. *T is obtained from a path* P_n $(n \geq 4$ *and* $n < p)$ *by attaching pendant edges at atleast one of the end vertices of* P_n*.*

2. *T is obtained from* P_3 *by attaching pendant edges at either both the end vertices or all the vertices of* P_3*.*

Notation 2.9 ([3])**.** *Let G be the class of connected graphs G with* $\delta(G) = 1$ *having one of the following properties.*

1. *There exist two adjacent vertices u, v in G such that* $deg_G(u) = 1$ *and* $\langle V(G) - \{u, v\} \rangle$ *contains* P_3 *as an induced subgraph such that end vertices of* P_3 *have degree atleast 2 and the central vertex of* P_3 *has degree atleast 3.*

2. *Let P be the set of all pendant vertices in G and let there exist a vertex* $v \in V(G) - P$ *having minimum degree in* $V(G) - P$ *and is not a support of G such that* $V(G) - (N_{\langle V-P \rangle}[v] - P)$ *contains* P_3 *as an induced subgraph such that the end vertices of* P_3 *have degree atleast 2 and the central vertex of* P_3 *has degree atleast 3.*

Theorem 2.10 ([3]). *Let G be a connected graph with $\delta(G) = 1$ and $\gamma_{ctnd}(G) \neq p-1$. Then $\gamma_{ctnd}(G) = p-2$ if and only if G does not belong to the class G of graphs.*

Theorem 2.11 ([3]). *Let G be a connected, noncomplete graph with p vertices ($p \geq 4$) and $\delta(G) \geq 2$. Then $\gamma_{ctnd}(G) = p-2$ if and only if G is one of the following graphs.*

1. *A cycle on atleast five vertices.*
2. *A wheel on six vertices.*
3. *G is the one point union of complete graphs.*
4. *G is obtained by joining two complete graphs by an edges.*
5. *G is a connected noncomplete graph such that there exists a vertex $\in V(G)$ such that $G - v$ is a complete graph on $(p-1)$ vertices.*
6. *G is a graph such that there exists a vertex $v \in V(G)$ such that $G - v$ is $K_{p-1} - e$, ($e \in E(K_{p-1})$) and $N(v)$ contains atleast one vertex of degree $(p-3)$ in $K_{p-1} - e$.*

Theorem 2.12 ([7]). *If G is a (p, q) graph, then $Sp(G)$ is a (2p, 3q) graph.*

Theorem 2.13 ([7]). *For every vertex $v_i \in G$. $deg(v_i) = deg(v'_i)$ for every v'_i in $Sp(G)$.*

3. Main Results

Observation 3.1. *For any connected graph G, $\gamma_{ctnd}(G) \leq \gamma_{ctnd}(Sp(G))$.*

This is illustrated by the following example.

Example 3.2. *For the graph G given in Figure 2 a, $\{v_3, v_4, v_5, v_6\}$ is a γ_{ctnd}-set of G and hence $\gamma_{ctnd}(G) = 4$. For the graph $Sp(G)$, given in Figure 2b, $\{v_3, v_4, v'_5, v'_6\}$ is a γ_{ctnd}-set of $Sp(G)$ and $\gamma_{ctnd}(Sp(G)) = 4$. Therefore $\gamma_{ctnd}(G) = \gamma_{ctnd}(Sp(G))$.*

(a) (b)

Figure 2:

For the graph G given in Figure 3a, $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ is a γ_{ctnd}-set of G and hence $\gamma_{ctnd}(G) = 6$. For the graph $Sp(G)$ given in Figure 3b, $\{v_1, v_2, v_8, v'_1, v'_4, v'_5, v'_6\}$ is a γ_{ctnd}-set of Sp(G) and $\gamma_{ctnd}(Sp(G)) = 7$. Therefore $\gamma_{ctnd}(G) < \gamma_{ctnd}(Sp(G))$.

(a)

(b)

Figure 3:

Observation 3.3.

1. *For any path P_p on p vertices, $\gamma_{ctnd}(Sp(P_p)) = p$, $p \geq 5$. $\gamma_{ctnd}(Sp(P_2)) = 3$, $\gamma_{ctnd}(Sp(P_3)) = 4$, $\gamma_{ctnd}(Sp(P_4)) = 5$.*
2. *For any cycle C_p on p vertices, $\gamma_{ctnd}(Sp(C_p)) = p-1$, $p \geq 7$. $\gamma_{ctnd}(Sp(C_3)) = 3$, $\gamma_{ctnd}(Sp(C_4)) = 5$, $\gamma_{ctnd}(Sp(C_5)) = 4$, $\gamma_{ctnd}(Sp(C_6)) = 6$.*
3. *For any star $K_{1,p-1}$, $\gamma_{ctnd}(Sp(K_{1,p-1})) = p+1$, $p \geq 2$.*
4. *For any complete bipartite graph $K_{m,n}\gamma_{ctnd}(Sp(K_{m,n})) = 2m+n-1$, $m, n \geq 2$.*
5. *$\gamma_{ctnd}(Sp(\overline{mK_2})) = 2m+1$, $m \geq 2$.*
6. *For the graph $K_p - e$, $\gamma_{ctnd}(Sp(K_p - e)) = p$, $p \geq 4$, where e is an edge in K_p.*
7. *For the graph $K_{m,n} - e$, $\gamma_{ctnd}(Sp(K_{m,n} - e)) = 2m+n-3$, $m, n \geq 3$, where e is an edge in $K_{m,n}$.*
8. *For the graph $\overline{K_{m,n} - e}$, $\gamma_{ctnd}(Sp(\overline{K_{m,n} - e})) = m+n$, $m, n \geq 2$.*
9. *$\gamma_{ctnd}(Sp(P_n \circ K_1)) = 2n+1$, $n \geq 3$.*
10. *$\gamma_{ctnd}(Sp(C_n \circ K_1)) = 2n+2$, $n \geq 3$.*

Theorem 3.4. *For any connected graph G with atleast three vertices and $\delta(G) \geq 2$, $3 \leq \gamma_{ctnd}(Sp(G)) \leq 2p-1$.*

Proof. Since radius of $Sp(G)$ is atleast 2, $\gamma_{ctnd}(Sp(G)) \geq 2$. By Theorem 2.4., if $\gamma_{ctnd}(G) = 2$, then $Sp(G)$ is a graph obtained by attaching a pendant edge at a vertex of degree $2p-2$ in $T+K_1$, where T is a tree on $2p-2$ vertices. But, no such connected graph $Sp(G)$ exists. Therefore $\gamma_{ctnd}(Sp(G)) \geq 3$. Also, since $Sp(G)$ is not complete, $\gamma_{ctnd}(Sp(G)) \leq 2p-1$. □

Remark 3.5. *By Theorem 2.2, $\delta(Sp(G))+1 \leq \gamma_{ctnd}(Sp(G))$, for any connected graph G. But, $\delta(Sp(G)) = \delta(G)$. Therefore, $\delta(G)+1 \leq \gamma_{ctnd}(Sp(G))$. $\gamma_{ctnd}(Sp(G)) \geq \delta(G)+1$. Equality holds, if $G \cong C_3$.*

Theorem 3.6. *Let G be a connected graph, which is not a star. Then $\gamma_{ctnd}(Sp(G)) = 3$ if and only if $G \cong K_2$, C_3 or G is the graph obtained by attaching a pendant edge at exactly one vertex of C_3 or C_4.*

Proof. Let D be a ctnd-set of Sp(G) such that $|D| = 3$. By Remark 3.5, $\delta(G)+1 \leq \gamma_{ctnd}(Sp(G))$. Therefore $\delta(G)+1 \leq 3$ and this implies $\delta(G) \leq 2$.

Case 1: $\delta(G) = 1$. Then D contains all the pendant vertices and atleast one support of Sp(G). Since $|D| = 3$, D contains atmost two pendant vertices. Also, no vertex of G is a pendant vertex of Sp(G), since $deg^v_{Sp(G)} = 2deg^v_G$, for every $v \in V(G)$.

Subcase 1: D contains exactly two pendant vertices. Then that vertices belong to $V'(G)$ and the remaining vertex of D is a vertex of G. Let $v \in D$, where $v \in V(G)$. Since no two vertices of $V'(G)$ in Sp(G) are adjacent, vertices of $V'(G)$ other than v' in $V(Sp(G)) - D$ is adjacent to v. But the vertex v' is not adjacent to any of vertices in D. Therefore v' must belong to D. If $V(Sp(G)) - D$ contains a P_3 induced by any three vertices of G, then the vertex in $V'(G)$ corresponding to the central vertex of P_3 must belong to D. Otherwise $V(Sp(G)) - D$ contains C_4. Therefore $G \cong K_2$ or P_3. But $\gamma_{ctnd}(Sp(P_3)) = 4$. Therefore $G \cong K_2$.

Subcase 2: D contains one pendant vertex of Sp(G). Since any ctnd-set of a graph contains all the pendant vertices, both G and Sp(G) contain one pendant vertex. Let $v \in V(G)$ be the pendant vertex and u be the support in G, adjacent to v. Then $v' \in V(Sp(G))$ is a pendant vertex and $v' \in D$. Also, since there exists a vertex $w \in D$ such that $N(w) \subseteq D$, and u is the only vertex adjacent to v' in Sp(G), both $u, v' \in D$. Let $v \in D$. If $V(G) - (D - \{v'\})$ contains a P_3, $V(Sp(G)) - D$ contains a C_4. Therefore, $V(G) - (D - \{v\}) \cong K_2$. Let $V(K_2) = \{w, x\}$. Since v is a pendant vertex in G, both w, x are adjacent to u. Then G is a graph obtained from C_3 by attaching a pendant edge at a vertex of C_3. Let $v \in D$. Let $w \in D$, $w \in V(G)$ and $w \in v$. Then $v \in V(Sp(G) - D) \cap \{v\})$. That is, $v \in (V(Sp(G)) \cap V(G)$. If u is not adjacent to w, then $u' \in V(Sp(G) - D$ is not adjacent to any of the vertices in D. Therefore $uw \in E(G)$.

Also $(V(Sp(G) - D) \cap V(G)$ contains atleast 2 vertices of G, other than v, since otherwise, $\langle V(Sp(G)) - D\rangle$ will not be a tree. Let $x, y \in (V(Sp(G) - D) \cap V(G)$. If atleast one of x and y is adjacent to both u and w, then $\langle\{x, y, u'\}\rangle \in C_3$ in $V(Sp(G)) - D$. Each of x and y is adjacent to exactly one of u and w. Therefore G is a graph obtained from C_4 by attaching a pendant edge at a vertex of C_4. If $(V(Sp(G)) - D) \cap V(G)$ contains atleast 3 vertices then $\langle V(Sp(G) - D\rangle$ contains a cycle.

Case 2: $\delta(G) = 2$. Therefore $\langle D\rangle$ is isomorphic to one of the graphs: $3K_1, K_2 \cup K_1, P_3, C_3$. Also, $(V(Sp(G)) - D) \bigcap V(G)$ contains atmost 2 vertices. If $\langle D\rangle$ is one of the graphs as above, then either $\delta(G) = 1$ or there exist no vertex $u \in D$, $N(u) \in D$. Therefore D contains atleast one vertex of $V'(G)$. Let $D = \{u, v, x'\}$. Since there is a vertex $w \in D$ such that $N(w) \subseteq Dx' \neq u'$ and v'. Also x' is adjacent to both u and v. That is, x is adjacent to both u and v in G. Also u and v are adjacent in G, otherwise D is not a dominating set of Sp(G). Further $(V(Sp(G)) - D) \cap V(G)$ contains no vertex other than x, otherwise, $\langle V(Sp(G)) - D\rangle$ contains a cycle. Therefore $G \cong C_3$. Hence $G \cong K_2$, C_3 or G is a graph obtained by attaching a pendant edge at exactly one vertex of C_3 or C_4.

Conversely, if $G \cong K_2$, C_3 or G is a graph obtained by attaching a pendant edge at exactly one vertex of C_3 or C_4, then $\gamma_{ctnd}(Sp(G)) = 3$. □

Theorem 3.7. *For any nontrivial connected graph G, $\gamma_{ctnd}(Sp(G)) = 2p - 1$ if and only if $G \cong K_2$.*

Proof. Assume $\gamma_{ctnd}(Sp(G)) = 2p - 1$. Let D be a γ_{ctnd}-set of Sp(G). Assume $p \geq 3$.

Case 1: $\delta(G) = 1$. By Theorem 2.6., if $\delta(Sp(G)) = 1$, then $\gamma_{ctnd}(Sp(G)) = 2p - 1$ if and only if the subgraph of Sp(G) induced by vertices of degree atleast 2 is K_2 or K_1. But there is no graph G with Sp(G) satisfying above condition. That is, the subgraph of Sp(G) induced by vertices of degree atleast 2 in Sp(G) is neither K_2 nor K_1.

Case 2: $\delta(G) \geq 2$. By Theorem 2.7, if $\delta(Sp(G)) \geq 2$, then $\gamma_{ctnd}(Sp(G)) = 2p - 1$ if and only if each edge of Sp(G) is a dominating edge. But in Sp(G), there exists atleast one edge that is not a dominating edge. Therefore $p = 2$. Then $G \cong K_2$.

Conversely, assume $G \cong K_2$, Since G is connected and $Sp(K_2) = P_4$ and $\gamma_{ctnd}(P_4) = 3 = 2p-1$. Then $\gamma_{ctnd}(Sp(G)) = 2p-1$. □

Observation 3.8. *If G is a connected graph with atleast three vertices, then Sp(G) is not a tree, since $Sp(P_3)$ contains a C_4 as an induced subgraph.*

Theorem 3.9. *Let G be a connected graph with atleast three vertices. Then $\gamma_{ctnd}(Sp(G)) \leq 2p-2$. Equality holds, if and only if $G \cong P_3$.*

Proof. Assume $\gamma_{ctnd}(Sp(G)) = 2p-2$. Let D be a γ_{ctnd}-set of Sp(G).

Case 1: δ(G) = 1. By Theorem 2.8., if $\delta(Sp(G)) = 1$, if G is a tree, then $Sp(G) \cong T$. By Observation 3.8, if G is a connected graph with atleast three vertices, then Sp(G) is not a tree. Therefore, there is no graph G with Sp(G) to be a tree and hence Sp(G) is a graph satisfying one of the following

1. There exist two adjacent vertices u, v in Sp(G) such that $deg_{Sp(G)}(u) = 1$ and $\langle V(Sp(G)) - \{u, v\}\rangle$ contains P_3 as an induced subgraph such that end vertices of P_3 have degree atleast 2 and the central vertex of P_3 has degree atleast 3.

2. Let P be the set of all pendant vertices in Sp(G) and let there exist a vertex $v \in V(Sp(G)) - P$ having minimum degree in $V(Sp(G)) - P$ and is not a support of Sp(G) such that $V(Sp(G)) - (N_{\langle V-P\rangle}[v] - P)$ contains P_3 as an induced subgraph such that the end vertices of P_3 have degree atleast 2 and the central vertex of P_3 has degree atleast 3. But the only case possible is $G \cong P_3$.

Case 2: $\delta(G) \geq 2$. There exists no graph G with Sp(G) to be one of the graphs mentioned in Theorem 2.11. Therefore $G \cong P_3$.

Conversely, if $G \cong P_3$, then $\gamma_{ctnd}(Sp(G)) = 2p-2$. □

In the following, upper bounds of $\gamma_{ctnd}(Sp(G))$ are found.

Remark 3.10. *If G is a connected graph with atleast 4 vertices and is not a bistar, then $\gamma_{ctnd}(Sp(G)) \leq 2p-3$. Equality holds, if $G \cong C_3$.*

Theorem 3.11. *Let G be a connected noncomplete graph and $\delta(G) \geq 3$. Then $\gamma_{ctnd}(Sp(G)) \leq 2p-4$.*

Proof. Since G is not complete, G contains P_3 as an induced subgraph. Let the vertices of P_3 be u, v, w, where v is the central vertex of P_3. Since $deg_G v \geq 3$, there exists a vertex x in G, $x \in N(v)$ and $x \neq u$ or w. Let $D = \{v, u', w', x'\}$ and $D' = V(Sp(G)) - D$. Since $(G) \geq 3$, each vertex $V(Sp(G)) - D'$ is adjacent to atleast one vertex in D' and $\langle V(Sp(G)) - D'\rangle \cong K_{1,3}$ with v as the central vertex and $N(v') \subseteq D'$. Therefore, D' is a ctnd-set of Sp(G). Hence, $\gamma_{ctnd}(Sp(G)) \leq 2p-4$. □

Theorem 3.12. *Let G be a connected graph with $\delta(G) \geq 2$ and $diam(G) \geq 3$. Then $\gamma_{ctnd}(Sp(G)) \leq 2p-4$.*

Proof. Since $diam(G) \geq 3$, there exists a vertex say $u \in V(G)$ with eccentricity 3. Let $e = (u, v) \in E(G)$, $v \in V(G)$. Let $D = \{u, v, u', v'\} \subseteq V(Sp(G))$ and let $D' = V(Sp(G)) - D$. Since $(G) \geq 2$, each vertex in $V(Sp(G)) - D'$ is adjacent to atleast one vertex in D' and $\langle V(Sp(G)) - D'\rangle \cong P_4$ in Sp(G). Let w be a vertex in G such that $deg_G(w) = 3$. Then w is adjacent to neither u nor v and $N_{Sp(G)}(w) \subseteq D'$ in Sp(G). Therefore D' is a ctnd-set of G and hence $\gamma_{ctnd}(Sp(G)) \leq |D'| = 2p-4$. □

Theorem 3.13. *Let G be a connected noncomplete graph with $\delta(G) \geq 3$. If G contains a P_3 as an induced subgraph and if there exists a vertex $x \in V(G) - V(P_3)$ such that $x \notin N(V(P_3))$, then $\gamma_{ctnd}(Sp(G)) \leq 2p-5$.*

Proof. Let G be a connected noncomplete graph with $\delta(G) \geq 3$ and G contains a P_3 as an induced subgraph. Let $V(P_3) = \{u, v, w\}$, where $u, v, w \in V(G)$ where v is the central vertex of P_3. Let $D = \{u, v, w, u', w'\}$ and $D' = V(Sp(G)) - D$.

Since $\delta(G) \geq 3$, each vertex in $V(Sp(G)) - D'$ is adjacent to atleast one vertex in D' and $\langle V(Sp(G)) - D' \rangle \cong K_{1,4}$ with v as the central vertex.

It is given that , there exists a vertex $x \in V(G) - V(P_3)$ not adjacent to any of the vertices of P_3. Hence, $N(x) \subseteq D'$ in Sp(G). Therefore, D' is ctnd-set of Sp(G) and $\gamma_{ctnd}(Sp(G)) \leq |D'| = 2p - 5$. □

Remark 3.14. *Let G be a connected graph with $\delta(G) \geq 2$. If G contains a P_3 as an induced subgraph such that central vertex of P_3 is of degree atleast 3 and the other two vertices in P_3 are of degree atleast two and if there exists a vertex $x \in V(G) - V(P_3)$ such that x is not adjacent to any of the vertices of P_3, then $\gamma_{ctnd}(Sp(G)) \leq 2p - 5$.*

Theorem 3.15. *Let G be a connected graph with $\delta(G) \geq 2$. If $diam(G) \geq 2$, then $\gamma_{ctnd}(Sp(G)) \leq 2p - \delta(G) - 1$.*

Proof. Since $diam(G) \geq 2$, there exists a vertex $v \in V(G)$ such that eccentricity of v is atleast 2. Let $D = \{u' \in V(Sp(G)) : u \in N(v)\}$ and $|D| = deg_G u$. Let $D' = V(Sp(G)) - D - \{v\} \subseteq V(Sp(G))$. Then $V(Sp(G)) - D' = D \cup \{v\}$ and $\langle V(Sp(G)) - D' \rangle \cong K_{1,deg}(v)$. Let $u \in N(v)$. Since $\delta(G) \geq 2$, degree of u in G is atleast 2. Therefore, $N(u) - \{v\}$ is nonempty. Let $w \in N(u) - \{v\}$, where $w \in V(G)$. Then $w' \in V(Sp(G)) - D'$ is adjacent to a vertex in D'. Also $v \in V(Sp(G)) - D'$ is adjacent to a vertex in D'. Since eccentricity of v in G is atleast 2, there exists a vertex, say x in G such that $d_G(v,x) \geq 2$. Therefore, $N(x) \subseteq V(Sp(G))$. Since $d_G(v,x) \geq 2$, no vertex in $N(x)$ is adjacent to a vertex in $V(Sp(G)) - D'$ and hence $N(x) \subseteq D'$. Therefore D' is a ctnd-set of G and $\gamma_{ctnd}(Sp(G)) \leq |D'| = |V(Sp(G)) - D - \{v\}|$, which implies $\gamma_{ctnd}(Sp(G)) \leq 2p - deg_G u - 1 = 2p - (G) - 1$. □

Remark 3.16. *Let G be a connected graph with $\delta(G) \geq 2$ and $diam(G) \geq 2$. Let v be a vertex of maximum degree in G. If eccentricity of v is atleast 2, then $\gamma_{ctnd}(Sp(G)) \leq \gamma_{ctnd}(G) + \Delta(G) - 1$.*

Remark 3.17. *Let D be a ctnd-set of Sp(G). Then D contain vertices from both V(G) and $V'(G)$.*

Theorem 3.18. *For any connected graph G with p vertices, $\gamma_{ctnd}(Sp(G)) \leq \gamma_{ctnd}(G) + p - 1$.*

Proof. Let D and D' be minimum ctnd-sets of G and Sp(G) respectively. Therefore, $\gamma_{ctnd}(G) = |D|$ and $\gamma_{ctnd}(Sp(G)) = |D'|$. By Remark 3.17, atleast one of the vertices of G, say $v \in D$ must be in D'. Therefore, $\gamma_{ctnd}(Sp(G)) \leq \gamma_{ctnd}(G) + |V'(G)| - 1$. Hence $\gamma_{ctnd}(Sp(G)) \leq \gamma_{ctnd}(G) + p - 1$. In the following, upper bounds of $\gamma_{ctnd}(Sp(G))$ are found. □

Theorem 3.19. *Let G be a connected noncomplete graph and graph such that $\delta(G) \geq 2$, then $\gamma_{ctnd}(Sp(G)) \geq \gamma_{ctnd}(G) + 1$.*

Proof. Let D be a γ_{ctnd}-set of G. Then $\langle V(G) - D \rangle$ is a tree and there exists a vertex $u \in D$ such that $N(u) \subseteq D$. Also the vertex u' in Sp(G) corresponding to $u \in D$ is isolated in $\langle V(Sp(G) - D \rangle$. Therefore, atleast one vertex, say u in Sp(G) is to be added with D such that $D \cup \{u\}$ will be a ctnd-set of Sp(G) and hence $\gamma_{ctnd}(Sp(G)) \geq \gamma_{ctnd}(G) + 1$. Equality holds if $G \cong C_p$, $p \geq 7$. □

In the following, the connected splitting graphs for which $\gamma_{ctnd}(Sp(G)) = \gamma_{ctnd}(G)$ is characterized.

Theorem 3.20. *Let G be a connected graph such that $\delta(G) = 1$ and let S and T be the set of supports and pendant vertices of G respectively such that $S \cup T$ is a minimum ctnd-set of G. Then $\gamma_{ctnd}(Sp(G)) = \gamma_{ctnd}(G)$ if and only if G is a graph obtained from C_4 by attaching pendant vertices at atmost two adjacent vertices of C_4.*

Proof. Let $D = S \cup T$. Assume $S \cup T$ is a minimum ctnd-set of G and $\gamma_{ctnd}(Sp(G)) = \gamma_{ctnd}(G) = |D|$. Let $T' = \{v' \in V'(G)/v \in T\}$. Since T' is an independent set in Sp(G), and since $\gamma_{ctnd}(Sp(G)) = |D|$, the set $D' = S \cup T'$ is a minimum ctnd-set of Sp(G). If $\langle V(G) - D \rangle$ contains P_3, then $\langle V(Sp(G) - D' \rangle$ contains C_4 since $Sp(P_3)$ contains C_4. Therefore,

$\langle V(G) - D\rangle \cong K_2$. If D contains atleast three vertices of G, then atleast one vertex in V(G)-D is adjacent to atleast two vertices in D and $\langle V(G) - D\rangle$ contains P_3. Therefore D contains 1 or 2 vertices.

If D contains 1 vertex, then G is a graph obtained from C_3 by attaching pendant vertices at a vertex of C_3. For this graph G, $\gamma_{ctnd}(G) = 2$, $\gamma_{ctd}(G) = 2$. D contains exactly 2 vertices. Let $D = \{v_1, v_2\}$. If v_1 and v_2 are adjacent in G, then D is not a dominating set of G. Therefore, v_1 and v_2 are not adjacent in G. If a vertex in $\langle V(G) - D\rangle$ is adjacent to 2 vertices of D, then also $\langle V(G) - D\rangle$ contains P_3. Therefore each vertex in $V(G) - D$ is adjacent to exactly one vertex in D. Therefore, G is a graph obtained from C_4 by attaching pendant vertices at atmost two adjacent vertices of C_4.

Conversely, if G is a graph obtained from C_4 by attaching pendant vertices at atmost two adjacent vertices of C_4, then $S \cup T$ is a minimum ctnd-set of G where S and T be the set of supports and pendant vertices of G. □

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