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On Quasi Soft sg-open and Quasi Soft sg-closed Functions in Soft Topological Spaces

Research Article

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Abstract: Soft set theory was firstly introduced by Molodtsov in 1999 as a general Mathematical tool for dealing with uncertainty. He has shown several applications of this theory in solving many practical problems in Economics, Engineering,Social science, Medical science, etc. The purpose of this paper is to give a new type of soft open and soft closed function is called quasi Soft sg-open function and quasi Soft sg-closed function in Soft Topological spaces . Also, we obtain its characterizations and its basic properties.

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Keywords: Soft sg-open set, Soft sg-closed set, Soft sg-interior, Soft sg-closure, quasi Soft sg-open function. quasi Soft sg-closed function.

1. Introduction

It is known that Topology is an important area of Mathematics with many applications in the domains of Computer science and Physical sciences. Soft topology isa relatively new and promising domain which can lead to the development of new Mathematical models and innovative approaches that will significantly contribute to the solution of complex problems in natural sciences. In 1999, D. Molodtsov [7] introduced the notion of soft set. He applied the soft theory in several fields such as smoothness of functions, Game theory, Probability, Perron integration, Riemann integration, theory of measurement. The concept of soft set is used to solve complicated problems in other sciences such as, Engineering, Economics etc.. Maji et al.[6] described an application of soft set theory to a decision-making problem. As a generalization of closed sets, the notion of sg-closed sets were introduced and studied by Bhattacharyya and Lahiri [2]. In this chapter, we will continue the study of related functions by involving soft sg-open sets. We introduce and characterize the concept of Quasi Soft sg-open and Quasi Soft sg-closed Functions in soft topological spaces.

1.1. Preliminaries

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let $A\widetilde{\subseteq}E$.

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Definition 1.1 ([6, 7]). *A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular $e \in A$. F(e) may be considered the set of e-approximate elements of the soft set (F, A).*

Definition 1.2 ([6, 7]). *For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if*

(1). $A\widetilde{\subseteq}B$, and

(2). $\forall e\widetilde{\in}A$, $F(e)\widetilde{\subseteq}G(e)$.

We write $(F, A)\widetilde{\subseteq}(G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A)\widetilde{\supseteq}(G, B)$.

Definition 1.3 ([6, 7]). *A soft set (F, A) over U is said to be*

(1). null soft set denoted by $\widetilde{\emptyset}$ if $\forall e\widetilde{\in}A$, $F(e) = \widetilde{\phi}$.

(2). absolute soft set denoted by A, if $\forall e\widetilde{\in}A$,F(e) = U.,

Definition 1.4 ([6, 7]). *A soft set (F, A) over U is said to beFor two soft sets (F, A) and (G, B) over a common universe U,union of two soft sets of (F, A) and (G, B) is the soft set (H, C), where $C = A\widetilde{\cup}B$, and if $\forall$ e $\widetilde{\in}C$, $H(e) = F(e)$ if $e\widetilde{\in}A - B$, $G(e)$ if $e\widetilde{\in}B - A$, $F(e)\widetilde{\cup}G(e)$ if $e\widetilde{\in}A\widetilde{\cap}B$. We write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.*

Definition 1.5 ([6, 7]). *The Intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U denoted $(F, A)\widetilde{\cap}(G, B)$ is defined as $C = A\widetilde{\cap}B$ and $H(e) = F(e)\widetilde{\cap}G(e)$ for all $\forall$ e $\widetilde{\in}C$.*

Definition 1.6 ([6, 7]). *Let Y be a non-empty subset of X, then denotes the soft set (Y, E) over X for which $Y(e) = Y$, $\forall$ $e\widetilde{\in}E$. In particular, (X, E), will be denoted by $\widetilde{X}$*

Definition 1.7 ([6, 7]). *For a soft set (F, A) over the universe U, the relative complement of (F, A) is denoted by (F, A)' and is defined by $(F, A)' = (F', A)$, where $F' : A \to P(U)$ is a mapping defined by $F'(e) = U - F(e)$ for all $e\widetilde{\in}A$.*

Definition 1.8 ([6, 7]). *Let $\widetilde{\tau}$ be the collection of soft sets over X, then τ is called a soft topology on X if $\widetilde{\tau}$ satisfies the following axioms:*

(1). $\widetilde{\phi}$, $\widetilde{X}$ belongs to $\widetilde{\tau}$.

(2). The union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

(3). The intersection of any two soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The triplet (X, $\widetilde{\tau}$, E) is called a soft topological space over X.

Definition 1.9 ([9]). *Let U be an initial universe and E be a set of parameters. Let $F_A\widetilde{\in}S(U)$. A soft topology on F_A, denoted by $\widetilde{\tau}$, is a collection of soft subsets of F_A having the following properties:*

(1). F_ϕ, F_A belong to $\widetilde{\tau}$.

(2). $\{F_{A_i} \widetilde{\subseteq} F_A : i \widetilde{\in} I\}\widetilde{\subseteq} \widetilde{\tau} \Rightarrow \widetilde{\cup}_i \widetilde{\in} I\ F_{A_i} \widetilde{\in} \widetilde{\tau}$.

(3). $\{F_{A_i} \widetilde{\subseteq} F_A : 1 \leq i \leq n, \widetilde{\in} N\} \widetilde{\in} \widetilde{\tau} \Rightarrow \widetilde{\cap}_{i=0}^{n} \{F_{A_i}\widetilde{\in} \widetilde{\tau}\}$ .The pair (F_A ,$\widetilde{\tau}$) is called a soft topological spaces.The members of $\widetilde{\tau}$ are called Soft Open sets in X and complements of them are called Soft Closed sets in X.

Example 1.10. *Suppose that there are Three type of persons like hot drinks in a coffee shop $U = \{P_1, P_2, P_3\}$, under consideration and that $E = \{x_1, x_2, x_3\}$ is a set of decision parameters. The x_i (i =1, 2, 3,) stand for the parameters of like 'Coffee', 'Milk', 'Tea', respectively. Consider the mapping f_A given by 'persons (.)', where (.) is to be filled in by one of the parameters $x_i \widetilde{\in} E$. For instance, $f_A(x_1)$ means 'person 1 (like 'Coffee) and its functional value is the set $\{p_1 \widetilde{\in} U$: person 1 like Coffee$\}$. Let $U = \{P_1, P_2, P_3\}$, $E = \{x_1, x_2, x_3\}$, and $A = \{x_1, x_2\}$. Then, we can view the soft set F_A as consisting of the following collection of approximations Let $U = \{p_1, p_2, p_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \widetilde{\subseteq} E$, and $F_A = \{(x_1, \{p_1, p_2\}), (x_2, \{p_2, p_3\})\}$. Then, $F_{A_1} = \{(x_1, \{p_1\})\}$, $F_{A_2} = \{(x_1, \{p_2\})\}$, $F_{A_3} = \{(x_1, \{p_1, p_2\})\}$, $F_{A_4} = \{(x_2, \{p_2\})\}$, $F_{A_5} = \{(x_2, \{p_3\})\}$, $F_{A_6} = \{(x_2, \{p_2, p_3\})\}$, $F_{A_7} = \{(x_1, \{p_1\}), (x_2, \{p_2\})\}$, $F_{A_8} = \{(x_1, \{p_1\}), (x_2, \{p_3\})\}$, $F_{A_9} = \{(x_1, \{p_1\}), (x_2, \{p_2, p_3\})\}$, $F_{A_{10}} = \{(x_1, \{p_2\}), (x_2, \{p_2\})\}$, $F_{A_{11}} = \{(x_1, \{p_2\}), (x_2, \{p_3\})\}$ $F_{A_{12}} = \{(x_1, \{p_2\}), (x_2, \{p_2, p_3\})\}$, $F_{A_{13}} = \{(x_1, \{p_1, p_2\}), (x_2, p_2\})\}$, $F_{A_{14}} = \{(x_1, \{p_1, p_2\}), (x_2, \{p_3\})\}$, $F_{A_{15}} = F_A$, $F_{A_{16}} = F_\phi$. Then the soft topology $\widetilde{\tau} = \{F_A, F_\phi, F_{A_2}, F_{A_3}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}\}$.*

Definition 1.11 ([6, 7])**.** *Let (X, $\widetilde{\tau}$, E) be soft space over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement $(F, E)'$ belongs to τ. The relative complement is a mapping $F' : E \to P(X)$ defined by $F'(e) = X - F(e)$ for all $e \widetilde{\in} A$.*

Definition 1.12. *Let (X, $\widetilde{\tau}$, E) be soft space over X. A soft set (F, E) over X is said to be soft closed in X, if its relative complement $(F, E)'$ belongs to τ. The relative complement is a mapping $F' : E \to P(X)$ defined by $F'(e) = X - F(e)$ for all $e \widetilde{\in} A$.*

Definition 1.13 ([6, 7])**.** *Let X be an initial universe set, E be the set of parameters and $\widetilde{\tau} = \{\widetilde{\phi}, \widetilde{X}\}$. Then $\widetilde{\tau}$ is called the soft indiscrete topology on X and (X, $\widetilde{\tau}$, E) is said to be a soft indiscrete space over X. If $\widetilde{\tau}$ is the collection of all soft sets which can be defined over X, then $\widetilde{\tau}$ is called the soft discrete topology on X and (X, $\widetilde{\tau}$, E) is said to be a soft discrete space over X.*

Definition 1.14 ([6, 7])**.** *Let (X,$\widetilde{\tau}$, E) be a soft topological space over X and the soft interior of (F, E) denoted by Int(F, E) is the union of all soft open subsets of (F, E). Clearly, (F, E) is the largest soft open set over X which is contained in (F, E). The soft closure of (F, E) denoted by soft Cl(F, E) is the intersection of all soft closed sets containing (F, E). Clearly, (F, E) is smallest soft closed set containing (F, E). soft Int $(F, E) = \widetilde{\cup} E(O, E) : (O, E)$ is soft open and $(O, E) \widetilde{\subseteq} (F, E)$. soft $Cl(F, E) = \widetilde{\cap}(O, E) : (O, E)$ is soft closed and $(F, E) \widetilde{\subseteq} (O, E)$.*

Definition 1.15 ([1])**.** *A Soft subset (F, A) of a space (X, $\widetilde{\tau}$, E) is called semi-open if $(F, A) \widetilde{\subseteq}$ Soft Cl(Soft Int((F, A))). The complement of a Soft semi-open set is called Soft semi-closed.*

Definition 1.16 ([2])**.** *A Soft subset (F, A) of a space X is called:*

(1). Soft sg-closed [1] if Soft $sCl((F, A)) \widetilde{\subseteq} (U, E)$ whenever $(F, A) \widetilde{\subseteq} (U, E)$ and (U, E) is Soft semi-open in X. The complement of Soft sg-closed set is called Soft sg-open.

(2). The union of all Soft sg-open sets, each contained in a set (F, A) in a space X is called the Soft sg-interior of (F, A) and is denoted by Soft sg-Int((F, A)) .

(3). The intersection of all Soft sg-closed sets containing a set (F, A) in a space X is Called the Soft sg-closure of (F, A) and is denoted by Soft sg-Cl((F, A))

Definition 1.17. *A function $f : X \to Y$ is said to be*

(1). soft sg-continuous if for every soft closed set in Y, its inverse image is soft sg- closed in X.

(2). soft sg-irresolute if for every soft sg-closed set in Y, its inverse image is soft sg- closed in X.

Definition 1.18. *A function $f : X \to Y$ is said to be soft sg*-closed [6], if for every soft sg-closed set (A, E) in X, f(A, E) is a soft sg-closed set in Y.*

Definition 1.19. *A function $f : X \to Y$ is said to be soft sg*-closed [6], if for every soft sg-closed set (A,E) in X, f(A, E) is a soft sg-closed set in Y.*

Definition 1.20. *A soft topological space X is said to be soft sg-normal if for any pair of disjoint soft sg-closed subsets (V_1, E) and (V_2, E) of X, there exist disjoint soft open sets (A, E) and (B, E) such that $V_1 \tilde{\subseteq} A$ and $V_2 \tilde{\subseteq} B$.*

2. Quasi Soft sg-Open Functions

We introduce the following definition.

Definition 2.1. *We introduce a new definition as follows: A function $f : X \to Y$ is said to be quasi Soft sg-open if the image of every Soft sg-open set in X is open in Y. It is evident that, the concepts quasi Soft sg-openness and Soft sg-continuity coincide if the function is a bijection.*

Theorem 2.2. *A function $f : X \to Y$ is quasi Soft sg-open if and only if for every Soft subset (U, E) of X, f (Soft sg-Int((U, E))) $\tilde{\subseteq}$ Int(f((U, E))).*

Proof. Let f be a quasi Soft sg-open function. Now, we have Soft $Int((U, E)) \tilde{\subseteq} (U, E)$ and Soft $sg - Int((U, E))$ is a Soft sg-open set. Hence, we obtain that $f(Soft \ \ sg - Int((U, E))) \tilde{\subseteq} f((U, E))$. As $f(Soft \ sg - Int((U, E)))$ is open, $f(Soft \ sg - Int((U, E))) \tilde{\subseteq} Int(f((U, E)))$. Conversely, assume that (U, E) is a Soft sg-open set in X. Then, $f((U, E)) = f(Soft \ sg - Int((U, E))) \tilde{\subseteq} \ SoftInt(f((U, E)))$ but $SoftInt(f((U, E))) \tilde{\subseteq} f((U, E))$. Consequently, $f((U, E)) = Int(f((U, E)))$ and hence f is quasi Soft sg-open. □

Lemma 2.3. *If a function $f : \ X \to \ Y$ is quasi Soft sg-open, then Soft sg-Int(f^{-1}((V,K))) $\tilde{\subseteq}$ f^{-1}(Soft int((V,K))) for every Soft subset (V,K) of Y .*

Proof. Let (V,K) be any arbitrary Soft subset of Y . Then, Soft sg-Int(f^{-1}((V,K))) is a Soft sg-open set in X and since f is quasi Soft sg-open, then f (Soft sg-Int(f-1((V,K)))) $\tilde{\subseteq}$ Soft int(f (f $^{-1}$((V,K)))) $\tilde{\subseteq}$Int((V,K)). Thus, Soft sg-Int(f $^{-1}$((V,K))) $\tilde{\subseteq}$ f $^{-1}$(Int((V,K))). □

Definition 2.4. *Soft subset S is called a Soft sg-neighbourhood of a point x of X if there exists a Soft sg-open set (F, A) such that $x \tilde{\in}$(F, A) $\tilde{\subseteq}$(U, E). A soft subset (U,E) of X is called a soft sg-neighbourhood of a point x in X, if there exists a soft sg-open set (F,A) such that $x \tilde{\in}$(F,A) $\tilde{\subseteq}$ (U,E).*

Theorem 2.5. *For a function $f : \ X \to Y$, the following conditions are equivalent:*

(1). f is quasi soft sg-open

(2). For every soft subset (U,E) of X, f(soft sg-int(U,E)) $\tilde{\subseteq}$ soft int(f(U,E))

(3). For each $x \tilde{\in}$ X and each soft sg-neighbourhood (U,E) of x in X, there exists a soft neighbourhood (V,K) of f(x) in Y such that (V,K) $\tilde{\subseteq}$ f(U,E).

.

Proof.

(1) ⇒(2) It follows from Theorem 3.2.

(2) ⇒(3) Let x $\widetilde{\in}$X and (U,E) be an arbitrary soft sg-neighbourhood of x in X. Then there exists a soft sg-open set (F,A) in X such that x $\widetilde{\in}$(F,A)$\widetilde{\in}$(U,E). From (2),we have f(F,A) = f(soft sg-int(F,A)) $\widetilde{\in}$soft int(f(F,A)), but soft int(f(F,A)) $\widetilde{\in}$f(F,A). Hence f(F,A) = soft int(f(F,A)) ⇒ f(F,A) is soft open in Y such that f(x) $\widetilde{\in}$(V,K) $\widetilde{\in}$ (U,E), where (V,K) = f(F,A).

(3)⇒ (1) Let (U,E) be an arbitrary soft sg-open set in X. By (3), for each y $\widetilde{\in}$f(U,E), there exists a soft neighbourhood (Vy,K) of y in Y such that (V_y,K) $\widetilde{\in}$f(U,E). Thus there exists a soft open set (W_y,K) in Y such that y $\widetilde{\in}$(Wy,K) $\widetilde{\in}$(V_y,K). Hence f(U,E) = (W_y,K) : y $\widetilde{\in}$f(U,E) which is soft open in Y. Thus f is quasi soft sg-open functions. □

Theorem 2.6. *A function f : X → Y is quasi soft sg-open if and only if for any soft subset (A,K) of Y and for any sgclosed set (M,E) of X containing f $^{-1}$(A,K), there exists a soft closed set (N,K) of Y containing (A,K) such that f $^{-1}$(N,K) $\widetilde{\subseteq}$ (M,E).*

Proof. Let f be quasi soft sg-open and (A,K) $\widetilde{\subseteq}$Y. Also let (M,E) be a soft sg-closed set of X containing f $^{-1}$(A,K). Let (N,K) = Y \ f(X \ (M,E)). Then f $^{-1}$(A,K) $\widetilde{\subseteq}$(M,E) ⇒ (A,K) $\widetilde{\subseteq}$(N,K). Since f is quasi soft sg-open, f(X\ (M,E)) is soft open. Hence (N,K) is a soft closed set of Y and f^{-1}(N,K) $\widetilde{\subseteq}$(M,E). Conversely, suppose that (U,E) be sg-open in X and let (A,K) = Y \ f(U,E). Then X \ (U,E) is soft sg-closed in X containing f $^{-1}$(A,K). By hypothesis, there exists a soft closed set (M,K) of Y such that (A,K) $\widetilde{\subseteq}$ (M,K) and f $^{-1}$(M,K) $\widetilde{\subseteq}$ X \ U ⇒f(U,E) $\widetilde{\subseteq}$Y \ (M,K). Also A $\widetilde{\subseteq}$ (M,K) ⇒ Y\ (M,K)$\widetilde{\subseteq}$ Y\ (A,K) = f(U,E). Thus f(U,E) = Y \ (M,K) is soft open and hence f is a quasi soft sg-open function. □

Theorem 2.7. *A function f : X → Y is quasi soft sg-open if and only if f^{-1}(soft cl(A,K)) $\widetilde{\subseteq}$ softsg-cl(f^{-1}(A,K)) for every soft subset (A,K) of Y.*

Proof. Let f be quasi soft sg-open. For any soft subset (A,K) of Y, f^{-1}(A,K) $\widetilde{\subseteq}$soft sg-cl(f^{-1}(A,K)). By theorem 3.6, there exists a soft closed set (N,K) in Y such that (A,K) $\widetilde{\subseteq}$(N,K) and f^{-1}(N,K) $\widetilde{\subseteq}$ soft sg-cl(f^{-1}(A,K)), which implies soft cl(A,K)$\widetilde{\subseteq}$ soft cl(N,K) = (N,K). So f^{-1}(soft cl(A,K)) $\widetilde{\subseteq}$ f^{-1}(N,K) $\widetilde{\subseteq}$soft sg-cl(f^{-1}(A,K)).

Conversely, let (A,K) $\widetilde{\subseteq}$Y and (M,E) be a soft sg-closed set in X such that f^{-1}(A,K) $\widetilde{\subseteq}$(M,E), which implies soft sg-cl(f^{-1}(A,K)) $\widetilde{\subseteq}$soft sg-cl(f^{-1}(M,E)) = (M,E). By hypothesis f^{-1}(soft cl(A,K)) $\widetilde{\subseteq}$ soft sg-cl(f^{-1} (A,K)) $\widetilde{\subseteq}$(M,E), f is quasi soft sg-open (by Theorem 3.6). □

Theorem 2.8. *Let f : X → Y and g : Y → Z be two functions such that g ∘ f : X → Z is quasi soft sg-open. If g is injective and continuous, then f is quasi soft sg-open.*

Proof. Let (U,E) be soft sg-open in X. Since g ∘ f is quasi soft sg-open, (g ∘ f)(U;E) is soft open in Z. Moreover since g is an injective continuous function, f(U,E) = g^{-1}(g ∘ f(U,E)) is soft open in Y. Hence f is quasi soft sg-open. □

3. Quasi Soft sg-Closed Functions

Definition 3.1. *A function f : X → Y is said to be quasi soft sg-closed if the image of every soft sg-closed set in X is soft closed in Y.*

Example 3.2. *Let X = {a, b, c}, Y = {y_1, y_2, y_3} and E = {e_1, e_2}. Then Let X = {a, b, c}, Y ={y_1, y_2, y_3} and E = {e_1, e_2}. Then $\widetilde{\tau}$ =($\widetilde{\phi}$), (X), (F_1 ,E) (F_2, E) (F_3, E)} is a soft topology over X . $\widetilde{\tau'}$ ={($\widetilde{\phi}$), (Y) ,(G_1 ,E) (G_2,E) (G_3,E)} is a soft topology over Y . where (F_1, E), (F_2, E) and (F_3, E) are soft sets over X and (G_1, E), (G_2, E) and (G_3, E) are soft sets over Y, which are defined as follows: [$F_1(e_1)$ = {a}, $F_1(e_2)$ ={b}] ,[$F_2(e_1)$ = X, $F_2(e_2)$ = {b}] [$F_3(e_1)$ = {a, b},*

$F_3(e_2) = \{a,\ b\}$] and[$G_1(e_1) = Y$, $G_1(e_2) = \{\ y_2,\ y_1\}$], [$G_2(e_1) = \{y_1\}$, $G_2(e_2) = \{y_2\}$], [$G_3(e_1) = \{y_1,\ y_2\}$, $G_3(e_2) = \{y_2,\ y_1\}$] Then (X ,τ,E) and (Y ,τ ' ,E) are soft topological spaces. Soft closed sets over Y are $\widetilde{(\phi)}$, (Y) ,(G_4,E), (G_5,E) and (G_6,E), where [$G_4(e_1) = \widetilde{(\phi)}$, $G_4(e_2)$ =$\{y_3\}$], [$G_5(e_1)$ =$\{y_2,\ y_3\}$ $G_5(e_2) = \{y_1,\ y_3\}$], [$G_6(e_1) = \{y_3\}$, $G_6(e_2) = \{y_3\}$] If we define the mapping f :(X ,$\widetilde{\tau}$,E) $\to$ (Y,$\widetilde{\tau'}$,E) as f(a) = y_2, f(b) = y_3 and f(c) = y_1. Then clearly f is Soft sg-closed as well as closed but not quasi Soft sg-closed

Theorem 3.3. *A function f : X $\to$ Y is quasi soft sg-closed if and only if for any soft subset (B,K) of Y and for any soft sg-open set (A,E) of X containing f^{-1}(B,K), there exists a soft open set (N,K) of Y containing (B,K) such that f^{-1}(N,K) $\widetilde{\subseteq}$ (A,E).*

Theorem 3.4. *If f : X $\to$ Y and g : Y $\to$ Z are two quasi soft sg-closed functions, then g $\circ$ f : X $\to$ Z is a quasi soft sg-closed function.*

Definition 3.5. *A function f : X $\to$ Y is called sg*-closed if the image of every sg-closed subset of X is sg-closed in Y .*

Theorem 3.6. *Let f : X $\to$ Y and g : Y $\to$ Z be two functions.*

(1). If f is soft sg-closed and g is quasi soft sg-closed, then g $\circ$ f is soft closed.

(2). If f is quasi soft sg-closed and g is soft sg-closed, then g $\circ$ f is soft sg-closed.*

(3). If f is soft sg-closed and g is quasi soft sg-closed, then g $\circ$ f is quasi soft sg-closed.*

Proof.

(1). Let f be soft sg-closed and g be quasi soft sg-closed. Let (U,E) be a soft closed set in X. Then f(U,E) is soft sg-closed in Y. Since g is quasi soft sg-closed, g(f(U,E)) is soft closed in Z. Hence g $\circ$ f is closed.

(2). Let f be a quasi soft sg-closed and g be soft sg-closed. Let (V,E) be a soft closed set in X. Then f(V,E) is soft closed in Y and g(f(V,E)) is soft sg-closed in Z. Since everysoft closed set is soft sg-closed f(V,E) is soft sg-closed in Y. Thus g $\circ$ f is soft sg*-closed.

(3). Let f be soft sg*-closed and g be quasi soft sg-closed. Let (V,E) be a soft sg-closed set in X. Then f(V,E) is soft sg-closed in Y and g(f(V,E)) is soft closed in Z. So g$\circ$ f is quasi soft sg-closed.

□

Theorem 3.7. *Every soft sg-closed function is soft sg*-closed.*

Proof. Let f : X $\to$ Y be soft sg-closed function, then for every soft closed set (U,E) in X, its image f(U,E) is soft sg-closed in Y. Since every soft closed set is soft sg-closed, (U,E) is soft sg-closed in X. Hence f is soft sg*-closed. □

Theorem 3.8. *Let f : X $\to$ Y and g : Y $\to$ Z be two functions such that g $\circ$ f : X $\to$Z is quasi soft sg-closed.*

(1). If g is injective and soft continuous, then f is quasi soft sg-closed.

(2). If f is soft sg-irresolute surjective, then g is soft closed.

(3). If g is soft sg-continuous injective, then f is soft sg-closed.*

Proof.

(1). It is similar to the proof of theorem 3.8.

(2). Let (M,E) be an arbitrary soft closed set in Y. Since f is soft sg-irresolute, f^{-1}(M,E) is soft sg-closed in X. Moreover g ∘ f is quasi soft sg-closed and f is surjective, we have (g ∘ f)(f^{-1}(M,E)) = g(M,E), which is soft closed in Z. Hence g is a soft closed unction.

(3). Let (V,E) be any soft sg-closed in X. Since g ∘ f is quasi soft sg-closed, (g ∘ f)(V,E) is soft closed in Z. If g is a soft sg-continuous injective function, we have g^{-1}(g ∘ f(V,E)) = f(V,E), which is soft sg-closed in Y. Thus f is soft sg*-closed.

□

Theorem 3.9. *A function f : X → Y is quasi soft sg-closed if and only if f(X,E) is soft closed in Y and f(U,E) \f((X,E)\(U,E)) is soft open in f(X,E) for every soft sg-open set (U,E) in X.*

Proof. Let f : X → Y be a quasi soft sg-closed function. Then (X,E) is soft sg-closed and f(X,E) is soft closed in Y. For an arbitrary soft sg-open set (U,E) in X, f(U,E) \f((X,E) \ (U,E)) = f(U,E) $\widetilde{\cap}$ f(X,E)\ f((X,E)\(U,E)) is soft open in f(X,E). Conversely, suppose that f(X,E) is soft closed in Y and for every soft sg-open set (U,E) in X, f(U,E) $\widetilde{\cap}$ f((X,E) $\widetilde{\cap}$ (U,E)) is soft open in f(X,E). Let (A,E) be soft closed in X. Then f(A,E) = f(X,E) \ (f(X,E) \ f(A,E)) is soft closed in f(X,E) and hence it is soft closed in Y. Thus f is quasi soft sg-closed. □

Theorem 3.10. *A surjective function f : X → Y is quasi soft sg-closed if and only if f(U,E) \f((X,E) \ (U,E)) is soft open in Y, for every soft sg-open set (U,E) in X.*

Theorem 3.11. *If f : X→ Y is soft sg-continuous surjective and quasi soft sg-closed function, then for every soft sg-open set (U,E) in X, f(U,E) \ f((X,E) \ (U,E)) is the soft topology on Y.*

Let (V,K) be a soft open set in Y. Since f is soft sg-continuous, f^{-1}(V,K) is soft sg-open in X. Moreover f(f^{-1}(V,K)) \ f((X,E) \ f^{-1}(V,K)) = (V,K). Hence all soft open sets in Y are of the form f(U,E) \ f((X,E) \ (U,E)), where (U,E) is soft sg-open in X. Also from theorem 4.8, we have all the soft sets of the form f(U,E) \ f((X,E) \ (U,E)), for every soft sg-open set (U,E) in X are soft open in Y.

Theorem 3.12. *If X is soft sg-normal and f : X → Y is soft sg-continuous surjective and quasi soft sg- closed function, then Y is soft normal*

Proof. Let (R_1,K) and (R_2,K) be disjoint soft closed subsets of Y. Since f is soft sg-continuous, f^{-1}(R_1,K) and f^{-1}(R_2,K) are disjoint soft sg-closed subsets of X. Also since X is soft sg-normal, there exist disjoint soft open sets (A,E) and (B,E) such that f^{-1}(R_1,K) $\widetilde{\subseteq}$ (A,E) and f^{-1}(R_2,K) $\widetilde{\subseteq}$ (B,E) ⇒ (R_1,K) $\widetilde{\subseteq}$ f(A,E) and (R_2,K) $\widetilde{\subseteq}$ f(B,E). i.e., (R_1,K)$\widetilde{\subseteq}$ f(A,E) f((X,E) (A,E)) and (R_2,K) $\widetilde{\subseteq}$ f(B,E) f((X,E) (B,E)) which are soft open sets in Y (by theorem 4.8), we have (f(A,E) \ f((X,E) \(A,E))) $\widetilde{\cap}$f(B,E) \ f((X,E)\ (B,E))) = $\widetilde{\phi}$ □

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