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Fuzzy Mappings Via Fuzzy $\widehat{\Omega}$-closed Sets

Research Article

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Abstract: This paper aims to extend two kinds of mappings on the family of all fuzzy $\widehat{\Omega}$-closed sets in a fuzzy topological space. They are nothing but fuzzy contra $\widehat{\Omega}$-continuous and fuzzy almost contra $\widehat{\Omega}$-continuous mappings. Moreover, fuzzy $\widehat{\Omega}$-interior and fuzzy $\widehat{\Omega}$-closure have been defined and their basic properties have been derived. In classical topological space, the family of all $\widehat{\Omega}$-closed sets forms a topology. It has been shown that this concept can not be fuzzified by giving suitable example.

MSC: AMS Classification

Keywords: $f\widehat{\Omega}$-int, $f\widehat{\Omega}$-cl, fuzzy contra $\widehat{\Omega}$-continuity, fuzzy almost contra $\widehat{\Omega}$-continuity.

1. Introduction

In 1965, Zadeh [19] the concept of fuzzy sets had been investigated to meet the the challenges due to lack of solving the problems involved in our day to day life. It plays an important role in the study of fuzzy topological spaces which had been introduced by Chang [6] in 1968. After that many authors have applied various basic concepts of general topology to fuzzy sets and developed theories of fuzzy topological spaces. In this paper, we introduce and study the new classes of mappings called fuzzy contra $\widehat{\Omega}$-continuous and fuzzy almost contra $\widehat{\Omega}$-continuous in fuzzy topological spaces.

1.1. Preliminaries

Now, we recall some of the basic definitions and results that are used in subsequent sections.

Definition 1.1 ([19])**.** *Let X be a non-empty set. A **fuzzy set** A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of elements x in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(x, \mu_A(x)) : x \in X\}$.*

Definition 1.2 ([15])**.** *A fuzzy set in X is called a **fuzzy singleton** if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is ϵ $(0 < \epsilon \leq 1)$ we denote this fuzzy singleton by x_ϵ, where the point x is called its support. For any fuzzy singleton x_ϵ and any fuzzy set μ, we write $x_\epsilon \in \mu$ if and only if $\epsilon \leq \mu(x)$.*

Definition 1.3 ([15])**.** *A fuzzy singleton x_ϵ is called **quasi-coincident** with a fuzzy set ρ, denoted $x_\epsilon q \rho$ if and only if $\epsilon + \rho(x) > 1$. A fuzzy set μ is called quasi-coincident with a fuzzy set ρ, denoted by $\mu q \rho$ if and only if there exists a $x \in X$ such that $\mu(x) + \rho(x) > 1$.*

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Definition 1.4 ([6]). *Let (X,τ) and (Y,σ) be any two fuzzy topological spaces. Let f be a function from the fuzzy topological space (X,τ) to the fuzzy topological space (Y,σ). Let λ be a fuzzy set in (Y,σ). The inverse image of λ under f written as $f^{-1}(\lambda)$ is the fuzzy sets in (X,τ) defined by $f^{-1}(\lambda)(x)=\lambda(f(x))$, $\forall x\in X$. Also the image of λ in (X,τ) under f written as f(λ)is the fuzzy set in (Y,σ) defined by*

$$f(\lambda)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)}\lambda(x), & \text{if } f^{-1}(y) \text{ is non-empty each } y\in Y\\ 0, & \text{if otherwise.}\end{cases}$$

Definition 1.5 ([6]). *A function $f:(X,\tau)\rightarrow(Y,\sigma)$ be a mapping. For fuzzy sets λ and μ of (X,τ) and (Y,σ) respectively. Then the following statements hold:*

(1) $ff^{-1}(\mu)\subseteq\mu$.

(2) $f^{-1}f(\lambda)\supseteq\lambda$.

(3) $f(1-\lambda)\supseteq 1-f(\lambda)$.

(4) $f^{-1}(1-\mu)=1-f^{-1}(\mu)$.

(5) If f is injective, then $f^{-1}f(\lambda)=\lambda$.

(6) If f is surjective, then $ff^{-1}(\mu)=\mu$.

(7) If f is bijective, then $f(1-\lambda)=1-f(\lambda)$.

Definition 1.6 ([14]). *A family $\tau\subseteq I^X$ of fuzzy sets is called a **fuzzy topology** for X, if it satisfies the following three axioms:*

(1) $\overline{0},\overline{1}\in\tau$.

(2) $A,B\in\tau\Rightarrow\ A\wedge B\in\tau$.

(3) $\forall(A_j)_{j\in J}\in\tau\Rightarrow\bigvee\limits_{j\in J}A_j\in\tau$.

*The pair (X,τ) is called a **fuzzy topological space** (in short, fts).The elements of τ are called fuzzy open sets. A fuzzy set K is called fuzzy closed, if $K^c\in\tau$.*

Definition 1.7 ([14]). *The closure $\overline{A}$ and the interior A^0 of a fuzzy set A of X are defined as $\overline{A}=inf\{K:A\leq K,K^c\in\tau\}$ and $A^0=sup\{0:0\leq A,0\in\tau\}$ respectively. Also, $cl(A)$ and $int(A)$ denotes the fuzzy interior and fuzzy closure respectively.*

Definition 1.8 ([16]). *A fuzzy set A of a fuzzy topological space (X,τ) is said to be **fuzzy δ-closed set**, if $A=\delta cl(A)$, where $\delta cl(A)=inf\{U\subseteq I^X:A\leq U,U=cl(int(A))\}$.*

Definition 1.9 ([2]). *Let A be a fuzzy semi-open subset of fuzzy topological space(X,τ). Then there exists a fuzzy open set μ such that $\mu\subseteq A\subseteq cl(\mu)$ if and only if $A\subseteq cl(int(A))$.*

Definition 1.10. *A fuzzy set A of fuzzy topological space (X,τ) is called*

(1) ***fuzzy semi-open*** *[2], if $A\leq cl(int(A))$.*

(2) ***fuzzy pre closed*** *[5], if $cl(int(A))\leq\ A$.*

(3) ***fuzzy α-closed*** *[5], if $cl(int(cl(A))) \leq A$.*

Definition 1.11. *A fuzzy set A of a fuzzy topological space (X, τ) is said to be*

(1) ***fuzzy generalized closed set****(in short, fg-closed)[4], if $cl(A) \leq U$, whenever $A \leq U$ and U is fuzzy open set in X*

(2) ***fuzzy $\widehat{\Omega}$-closed set****(in short, f$\widehat{\Omega}$-closed)[1], if $\delta cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open set in (X, τ).*

Definition 1.12. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1) ***fuzzy contra continuous*** *[9], if the pre image of every fuzzy open set in Y is fuzzy closed in X.*

(2) ***fuzzy contra semi-continuous*** *[7], if the pre image of every fuzzy open set in Y is fuzzy semi-closed in X.*

(3) ***fuzzy contra pre-continuous*** *[11], if the pre image of every fuzzy open set in Y is fuzzy pre-closed in X.*

(4) ***fuzzy contra α-continuous*** *[10], if the pre image of every fuzzy open set in Y is fuzzy α-closed in X.*

(5) ***fuzzy contra g-continuous*** *[17], if the pre image of every fuzzy open set in Y is fuzzy g-closed in X.*

(6) ***fuzzy almost contra continuous*** *[8], if the pre image of every fuzzy regular open set in Y is fuzzy closed in X.*

(7) ***fuzzy almost contra pre-continuous*** *[8], if the pre image of every fuzzy regular open set in Y is fuzzy pre-closed in X.*

(8) ***fuzzy almost contra semi-continuous*** *[8], if the pre image of every fuzzy regular open set in Y is fuzzy semi-closed in X.*

(9) ***fuzzy almost contra α-continuous*** *[8], if the pre image of every fuzzy regular open set in Y is fuzzy α-closed in X.*

Definition 1.13 ([12]). *A fuzzy topological space (X, τ) is said to be* ***fuzzy semi-$T_{\frac{1}{2}}$****, if every fuzzy semi-generalised closed set in X is fuzzy semi-closed.*

Theorem 1.14 ([18]). *A fuzzy topological space (X, τ) is said to be fuzzy semi-$T_{\frac{1}{2}}$, if and only if every fuzzy singleton is either fuzzy semi-open or fuzzy semi-closed set.*

Definition 1.15 ([2]). *Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$, defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ is called the* ***fuzzy graph function*** *f.*

Throughout this paper, $\mathcal{F}(X)$ denotes the set of all fuzzy sets on X and $\mathcal{F}\widehat{\Omega}O(X)$ (resp. $\mathcal{F}\widehat{\Omega}$C(X)) denotes the set of all fuzzy $\widehat{\Omega}$-open sets (resp.fuzzy $\widehat{\Omega}$-closed sets) in X.

2. Fuzzy $\widehat{\Omega}$-interior and Fuzzy $\widehat{\Omega}$-closure

Definition 2.1. *Let μ be a fuzzy $\widehat{\Omega}$-open set in a fuzzy topological space (X, τ). Then* ***fuzzy $\widehat{\Omega}$-interior*** *is denoted by $f\widehat{\Omega}$-$int(\mu)$ and defined by $f\widehat{\Omega}$-$int(\mu) = sup\{\eta \in f\widehat{\Omega}O(X) : \eta \subseteq \mu\}$.*

Proposition 2.2. *In a fuzzy topological space (X, τ), the following holds for any $\mu \in f\widehat{\Omega}O(X)$*

(1) $f\widehat{\Omega}$-$int(0) = 0$.

(2) $f\widehat{\Omega}$-$int(1) = 1$.

(3) $f\widehat{\Omega}\text{-}int(\mu) \subseteq \mu$.

(4) $\mu \subseteq \eta \Rightarrow f\widehat{\Omega}\text{-}int(\mu) \subseteq f\widehat{\Omega}\text{-}int(\eta)$.

(5) $f\widehat{\Omega}\text{-}int(f\widehat{\Omega}\text{-}int(\mu)) = f\widehat{\Omega}\text{-}int(\mu)$.

(6) $[f\widehat{\Omega}\text{-}int(\mu)]^c = f\widehat{\Omega}\text{-}cl((\mu)^c)$.

Definition 2.3. *Let μ be a fuzzy $\widehat{\Omega}$-open set in a fuzzy topological space (X, τ). Then **fuzzy $\widehat{\Omega}$-closure** is denoted by $f\widehat{\Omega}\text{-}cl(\mu)$ and defined by $f\widehat{\Omega}\text{-}cl(\mu) = inf\{\sigma \in f\widehat{\Omega}C(X) : \mu \subseteq \sigma\}$.*

Proposition 2.4. *In a fuzzy topological space (X, τ) the following holds for any $\mu \in f\widehat{\Omega}C(X)$*

(1) $f\widehat{\Omega}\text{-}cl(0) = 0$.

(2) $f\widehat{\Omega}\text{-}cl(1) = 1$.

(3) $\mu \subseteq f\widehat{\Omega}\text{-}cl(\mu)$.

(4) $\mu \subseteq \eta \Rightarrow f\widehat{\Omega}\text{-}cl(\mu) \subseteq f\widehat{\Omega}\text{-}cl(\eta)$.

(5) $f\widehat{\Omega}\text{-}cl(f\widehat{\Omega}\text{-}cl(\mu)) = f\widehat{\Omega}\text{-}cl(\mu)$.

(6) $[f\widehat{\Omega}\text{-}cl(\mu)]^c = f\widehat{\Omega}\text{-}int(\mu)^c$.

Proposition 2.5. *The union of two fuzzy $\widehat{\Omega}$-closed sets in a fuzzy topological space (X, τ) is a fuzzy $\widehat{\Omega}$-closed set in (X, τ).*

Proof. Suppose that μ and σ are any two fuzzy $\widehat{\Omega}$-closed sets in X. Let η be any fuzzy semi-open set in X such that $(\mu \cup \sigma) \subseteq \eta$. Then, $\mu \subseteq (\mu \cup \sigma) \subseteq \eta$ and $\sigma \subseteq (\mu \cup \sigma) \subseteq \eta$. Since μ and σ are fuzzy $\widehat{\Omega}$-closed sets, we have $\delta cl(\mu) \subseteq \eta$ and $\delta cl(\sigma) \subseteq \eta$. By[16], $\delta cl(\mu \cup \sigma) = \delta cl(\mu) \cup \delta cl(\sigma) \subseteq \eta$. So, $\mu \cup \sigma$ is a fuzzy $\widehat{\Omega}$-closed set in X. □

Remark 2.6. *The intersection of two fuzzy $\widehat{\Omega}$-closed sets is not necessarily a fuzzy $\widehat{\Omega}$-closed set.*

Example 2.7. *Let $X = \{a, b, c\}$ be a set. The fuzzy sets A, B and C and fuzzy sets μ and σ are given in a tabular form.*

fs(X)	*a*	*b*	*c*
A	*0.2*	*0*	*0*
B	*0*	*0*	*0.5*
C	*0.2*	*0*	*0.5*

fs(X)	*a*	*b*	*c*
μ	*0.9*	*0.2*	*0.4*
σ	*0.7*	*0.5*	*0.8*
$\mu \cap \sigma$	*0.7*	*0.2*	*0.4*

Let $\tau = \{0, A, B, C, 1\}$ be a fuzzy topological space on X. The family of fuzzy semi-open sets

$$FSO(X,\tau) = \begin{cases} & 0.2 \le \alpha \le 0.8, \qquad 0 \le \alpha \le 0.8, \\ 0, 1, \eta(a) = \alpha, \eta(b) = \beta, \eta(c) = \gamma \text{ either } & 0 \le \beta \le 1, \qquad \text{or} \qquad 0 \le \beta \le 1, \\ & 0 \le \gamma \le 0.5 \qquad\qquad \gamma = 0.5 \end{cases}$$

The fuzzy set 1 is the only fuzzy semi-open set such that $\mu \subseteq 1$ and $\sigma \subseteq 1$. Therefore, μ and σ are fuzzy $\widehat{\Omega}$-closed sets. Here, $\mu \cap \sigma$ is fuzzy semi-open set in X , but $\delta cl(\mu \cap \sigma) = c^c \nsubseteq \mu \cap \sigma$. Hence $\mu \cap \sigma$ is not a fuzzy $\widehat{\Omega}$-closed set.

Corollary 2.8. *$f\widehat{\Omega}\text{-}cl(\mu)$ is not necessarily a fuzzy $\widehat{\Omega}$-closed set.*

3. Behaviour of fuzzy $\widehat{\Omega}$-closed Sets

Theorem 3.1. *Let μ be any fuzzy $\widehat{\Omega}$-closed set in a fuzzy topological space (X,τ). If there exists a fuzzy set σ of X such that $\mu \subseteq \sigma \subseteq \delta cl(\mu)$, then σ is a fuzzy $\widehat{\Omega}$-closed set.*

Proof. Suppose that, μ is any fuzzy $\widehat{\Omega}$-closed set in (X,τ) and σ be any fuzzy set in X such that $\mu \subseteq \sigma \subseteq \delta cl(\mu)$. Let η be any fuzzy semi-open set in X such that $\sigma \subseteq \eta$. Since μ is a fuzzy $\widehat{\Omega}$-closed set, we have $\delta cl(\mu) \subseteq \eta$. By hypothesis, $\sigma \subseteq \delta cl(\mu)$. By[[16],Corollary 2.12], $\delta cl(\sigma) \subseteq \delta cl(\delta cl(\mu)) = \delta cl(\mu)$. Then, $\delta cl(\sigma) \subseteq \delta cl(\mu) \subseteq \eta$. Then, σ is fuzzy $\widehat{\Omega}$-closed set. □

Theorem 3.2. *Let μ be any fuzzy $\widehat{\Omega}$-closed set in X. Then $\delta cl(\mu) \setminus \mu$ does not contain a nonzero fuzzy semi-closed set of X.*

Proof. Let μ be any fuzzy $\widehat{\Omega}$-closed set in X. Assume that there exists a non-zero fuzzy semi-closed subset η of X such that $\eta \subseteq \delta cl(\mu) \setminus \mu$. Then, $\mu \subseteq \eta^c$. Since η is fuzzy semi-closed set, η^c is a fuzzy semi-open set such that $\mu \subseteq \eta^c$. By hypothesis, $\delta cl(\mu) \subseteq \eta^c$. Then, $\eta \subseteq [\delta cl(\mu)]^c$. Now, we have $\eta \subseteq \delta cl(\mu)$ and $\eta \subseteq [\delta cl(\mu)]^c$. So, $\eta \subseteq \delta cl(\mu) \cap [\delta cl(\mu)]^c = 0$ a, contradiction to our assumption. Therefore, $\delta cl(\mu) \setminus \mu$ does not contain a non-zero fuzzy semi-closed set of X. □

Theorem 3.3. *In a fuzzy topological space (X,τ), μ is fuzzy $\widehat{\Omega}$-open if and only if $\sigma \subseteq \delta$-$int(\mu)$ whenever σ is fuzzy semi-closed set in X such that $\sigma \subseteq \mu$.*

Proof. Let μ be any fuzzy $\widehat{\Omega}$-open set in X and $\sigma \subseteq \mu$ where σ is fuzzy semi-closed set in X. Then $\mu^c \subseteq \sigma^c$, where σ^c is fuzzy semi-open set in X. Since μ^c is fuzzy $\widehat{\Omega}$-closed set in X, we have $\delta cl(\mu^c) \subseteq \sigma^c$. By[16], $[\delta int(\mu)]^c \subseteq \sigma^c$. Hence $\sigma \subseteq \delta int(\mu)$.

Conversely, suppose $\mu^c \subseteq \sigma$ is any fuzzy semi-open set of X. Then $\sigma^c \subseteq \mu$ where σ^c is fuzzy semi-closed set in X. By hypothesis, $\sigma^c \subseteq \delta$- $int(\mu)$. By [16], $\delta - cl(\mu^c) = (\delta - int(\mu)^c \subseteq \sigma$. Then, μ^c is fuzzy $\widehat{\Omega}$-closed set in X and hence μ is fuzzy $\widehat{\Omega}$-open set in X. □

Theorem 3.4. *If μ is a fuzzy $\widehat{\Omega}$-closed subset of X, then $\delta cl(\mu) \setminus \mu$ is a fuzzy $\widehat{\Omega}$-open set of X.*

Proof. Let μ be a fuzzy $\widehat{\Omega}$-closed subset in X and σ be any fuzzy semi-closed set such that $\sigma \subseteq \delta cl(\mu) \setminus \mu$. Since μ is fuzzy $\widehat{\Omega}$-closed set, $\delta cl(\mu) \setminus \mu$ does not contain any non zero fuzzy semi-closed set. It follows that $\sigma = 0$. So, $\sigma = 0 \subseteq \delta int[\delta cl(\mu) \setminus \mu]$. Then, $\delta cl(\mu) \setminus \mu$ is fuzzy $\widehat{\Omega}$-open set. □

Corollary 3.5. *If μ is any fuzzy $\widehat{\Omega}$-open set in X such that $\delta int(\mu) \subseteq \sigma \subseteq \mu$, then σ is fuzzy $\widehat{\Omega}$-open set in X.*

Proof. Let μ be any fuzzy $\widehat{\Omega}$-open set in X such that $\delta int(\mu) \subseteq \sigma \subseteq \mu$. Then μ^c is fuzzy $\widehat{\Omega}$-closed set in X and $\mu^c \subseteq \sigma^c \subseteq \delta cl(\mu^c)$. By theorem-4.1, σ^c is fuzzy $\widehat{\Omega}$-closed set. Then, σ is fuzzy $\widehat{\Omega}$-open subset of X. □

Theorem 3.6. *A fuzzy topological space (X,τ) is fuzzy semi-$T_{\frac{1}{2}}$, if and only if every fuzzy $\widehat{\Omega}$-closed set is fuzzy semi-closed.*

Proof. Let μ be any fuzzy $\widehat{\Omega}$-closed set in (X,τ). By ([1] and ([13],Remark-2.4)), μ is fuzzy semi-generalised closed set. Since (X,τ) is fuzzy semi-$T_{\frac{1}{2}}$, μ is fuzzy semi-closed subset of X.

Conversely, Let x_α be a fuzzy singleton semi-open in X. If x_α is not a fuzzy semi-closed, then $(1 - x_\alpha)$ is not a fuzzy semi-open subset of X. Now, the fuzzy set 1 is the only fuzzy semi-open set such that $(1 - x_\alpha) \subseteq 1$. Then, $\delta cl(1 - x_\alpha) \subseteq 1$. So, $(1 - x_\alpha)$ is a fuzzy $\widehat{\Omega}$-closed subset of X. By hypothesis, $(1 - x_\alpha)$ is a fuzzy semi-closed subset of X. It follows that every fuzzy singleton (x_α) is fuzzy semi-open subset of X. Therefore, (X,τ) is fuzzy semi-$T_{\frac{1}{2}}$. □

4. Fuzzy Contra $\widehat{\Omega}$-continuous

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be* ***fuzzy contra $\widehat{\Omega}$-continuous****, if the pre image of every fuzzy open set in Y is fuzzy $\widehat{\Omega}$-closed set in X.*

Example 4.2. *Let $X = \{a, b, c\}$ and $Y = \{r, s, t\}$ be two nonempty sets.*

The fuzzy sets A,B and C of X and fuzzy sets P and Q of Y are given in a tabular form as follows.

fs(X)	a	b	c
A	0.2	0.1	0
B	0.3	0.2	0.4
C	0.5	0.3	0.6

fs(Y)	r	s	t
P	0.9	0.8	1
Q	0.8	0.7	0.6

Let $\tau = \{0, A, B, C, 1\}$ and $\sigma = \{0, P, Q, 1\}$ be two fuzzy topologies on X and Y respectively. Define $f : (X, \tau) \to (Y, \sigma)$ by f(a)=s, f(b)=r, f(c)=t. It is fuzzy contra $\widehat{\Omega}$-continuous.

Remark 4.3. *From the following two examples it is known that fuzzy contra $\widehat{\Omega}$-continuous is independent of fuzzy contra continuous.*

Example 4.4. *Define the fuzzy sets A,B and C of X and fuzzy sets P and Q of Y as follows.*

fs(X)	a	b	c
A	0	0.4	0.1
B	0.1	0.5	0.2
C	0.2	0.6	0.3

fs(Y)	r	s	t
P	0.5	0.8	0.9
Q	0.6	0.3	0.2

Let $\tau = \{0, A, B, 1\}$ and $\sigma = \{0, P, Q, 1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a)=t, f(b)=r, f(c)=s is fuzzy contr $\widehat{\Omega}$-continuous but not fuzzy contra continuous.

Example 4.5. *Let $X = \{a, b, c\}$ and $Y = \{r, s, t\}$ be two nonempty sets and the fuzzy sets A,B and C of X and fuzzy set P of Y be defined as follows.*

fs(X)	a	b	c
A	0.2	0.4	0.5
B	0.4	0.5	0.6
C	0.5	0.7	0.8

fs(Y)	r	s	t
P	0.3	0.5	0.2

Let $\tau = \{0, A, B, C, 1\}$ and $\sigma = \{0, P, 1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a)=s, f(b)=r, f(c)=t is fuzzy contra continuous but not fuzzy contra $\widehat{\Omega}$-continuous.

Theorem 4.6. *Let (X, τ) and (Y, σ) be two fuzzy topological spaces and $f : (X, \tau) \to (Y, \sigma)$ be any function. Then, f is fuzzy contra $\widehat{\Omega}$-continuous if and only if $f^{-1}(\lambda)$ is fuzzy $\widehat{\Omega}$-open set in X for each fuzzy closed set λ in Y.*

Proof. It follows from the definition of fuzzy contra $\widehat{\Omega}$-continuity and the concepts of the complement of fuzzy $\widehat{\Omega}$-closed sets. □

Theorem 4.7. *If f is a fuzzy contra $\widehat{\Omega}$-continuous, then for each $x \in X$ and each fuzzy closed set λ containing the point $f(x)$ in Y, there exists a fuzzy $\widehat{\Omega}$-open set η containing the point x in X such that $f(\eta) \subseteq \lambda$.*

Proof. Let λ be any fuzzy closed set containing the point $f(x)$ in Y. By hypothesis, $f^{-1}(\lambda)$ is fuzzy $\widehat{\Omega}$-open set in X containing the point x. Take $\eta = f^{-1}(\lambda)$. Then η is a fuzzy $\widehat{\Omega}$-open set containing the point x in X such that $f(\eta) \subseteq \lambda$. □

Theorem 4.8. *Let (X,τ) and (Y,σ) be any fuzzy topological spaces and $f:(X,\tau)\to(Y,\sigma)$ be any function. Then the following statements are equivalent:*

(i) for each $x \in X$ and each fuzzy closed set λ containing the point $f(x)$ in Y, there exists a fuzzy $\widehat{\Omega}$-open set η containing the point x in X such that $f(\eta) \subseteq \lambda$.

(ii) for each $x \in X$ and each fuzzy open set μ not containing the point $f(x)$ in Y, there exists a fuzzy $\widehat{\Omega}$-closed set ζ not containing the point x in X such that $f^{-1}(\mu) \subseteq \zeta$.

Proof. (i)⇒(ii) Let μ be any fuzzy open subset of Y not containing the point $f(x)$ in Y. Then $(1-\mu)$ is a fuzzy closed set containing the point $f(x)$. By hypothesis, there exists a fuzzy $\widehat{\Omega}$-open subset η containing the point x in X such that $f(\eta) \subseteq (1-\mu)$. Then $\eta \subseteq f^{-1}(f(\eta)) \subseteq f^{-1}(1-\mu) \subseteq 1-f^{-1}(\mu)$. Moreover, $f^{-1}(\mu) \subseteq (1-\eta)$. Take $\zeta = 1-\eta$. Then ζ is fuzzy $\widehat{\Omega}$-closed subset of X not containing the point x in X such that $f^{-1}(\mu) \subseteq \zeta$.
(ii)⇒(i) Let λ be any fuzzy closed subset of Y containing the point $f(x)$ in Y. Then $(1-\lambda)$ is fuzzy open subset of Y not containing the point $f(x)$ in Y. By hypothesis, there exists a fuzzy $\widehat{\Omega}$-closed subset ζ in X not containing the point x in X such that $f^{-1}(1-\lambda) \subseteq \zeta$. Therefore, $(1-\zeta) \subseteq f^{-1}(\lambda)$. Then, $(1-\zeta)$ is a fuzzy $\widehat{\Omega}$-open subset of X containing the point x such that $f(1-\zeta) \subseteq \lambda$. By taking η as $1-\zeta$, the requirement is satisfied. □

Proposition 4.9. *Every fuzzy contra $\widehat{\Omega}$-continuous is fuzzy contra g-continuous.*

Proof. Assume that $f:(X,\tau)\to(Y,\sigma)$ is fuzzy contra $\widehat{\Omega}$-continuous function. Let A be any fuzzy open set in Y. By the hypothesis, $f^{-1}(A)$ is fuzzy $\widehat{\Omega}$-closed set in X. Since every fuzzy $\widehat{\Omega}$-closed set is fuzzy g-closed set, $f^{-1}(A)$ is fuzzy g-closed set in X. So, f is fuzzy contra g-continuous. □

Remark 4.10. *The converse of the above statement is not necessarily true.*

Example 4.11. *Let $X=\{a,b,c\}$ and $Y=\{r,s,t\}$ be two nonempty sets.*
The fuzzy sets A,B and C of X and fuzzy sets P and Q of Y are given in a tabular form as follows.

fs(X)	a	b	c
A	0.3	0.1	0.4
B	0.4	0.2	0.5
C	0.6	0.7	0.8

fs(Y)	r	s	t
P	0.5	0.6	0.8
Q	0.2	0.4	0.3

Let $\tau=\{0,A,B,C,1\}$ and $\sigma=\{0,P,Q,1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f:(X,\tau)\to(Y,\sigma)$ defined by f(a)=s, f(b)=t, f(c)=r is fuzzy contra g-continuous but not fuzzy contra $\widehat{\Omega}$-continuous.

5. Fuzzy Almost Contra $\widehat{\Omega}$-continuous

Definition 5.1. *A fuzzy function $f:(X,\tau)\to(Y,\sigma)$ is said to be* ***fuzzy almost contra $\widehat{\Omega}$-continuous****, if the pre image of each fuzzy regular open set in Y is fuzzy $\widehat{\Omega}$-closed subset in X*

Example 5.2. *Let $X=\{a,b,c\}$ and $Y=\{r,s,t\}$ be two nonempty sets.*
The fuzzy sets A,B and C of X and fuzzy set P of Y are given in a tabular form as follows.

fs(X)	a	b	c
A	0.2	0.4	0.3
B	0.3	0.5	0.4
C	0.5	0.6	0.7

fs(Y)	r	s	t
P	0.5	0.3	0.4

Let $\tau = \{0, A, B, C, 1\}$ and $\sigma = \{0, P, 1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a)=r, f(b)=t, f(c)=s is fuzzy almost contra $\widehat{\Omega}$-continuous.

Remark 5.3. *The following two examples show that fuzzy almost contra $\widehat{\Omega}$-continuous is independent of fuzzy almost contra continuous.*

Example 5.4. *Let $X = \{a, b, c\}$ and $Y = \{r, s, t\}$ be two nonempty sets.*

The fuzzy sets A,B and C of X and fuzzy set P of Y are given in a tabular form as follows.

fs(X)	a	b	c
A	0.4	0.5	0.3
B	0.5	0.6	0.4
C	0.6	0.7	0.5

fs(Y)	r	s	t
P	0.5	0.4	0.3

Let $\tau = \{0, A, B, 1\}$ and $\sigma = \{0, P, 1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a)=s, f(b)=t, f(c)=r is fuzzy almost contra $\widehat{\Omega}$-continuous but not fuzzy almost contra continuous.

Example 5.5. *Let $X = \{a, b, c\}$ and $Y = \{r, s, t\}$ be two nonempty sets.*

The fuzzy sets A,B and C of X and fuzzy set P of Y are given in a tabular form.

fs(X)	a	b	c
A	0.1	0.3	0.4
B	0.2	0.4	0.5
C	0.5	0.6	0.7

fs(Y)	r	s	t
P	0.4	0.3	0.5

Let $\tau = \{0, A, B, C, 1\}$ and $\sigma = \{0, P, 1\}$ be two fuzzy topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by f(a)=t, f(b)=r, f(c)=s is fuzzy almost contra continuous but not fuzzy almost contra $\widehat{\Omega}$-continuous.

Theorem 5.6. *If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy almost contra $\widehat{\Omega}$-continuous if and only if pre image of every fuzzy regular closed subset of Y is fuzzy $\widehat{\Omega}$-open set in X.*

Proof. It follows from the definition of fuzzy almost contra $\widehat{\Omega}$ -continuous. □

Theorem 5.7. *For any fuzzy almost contra-continuous function $f : (X, \tau) \to (Y, \sigma)$, if μ is any fuzzy regular closed subset of Y and $x_\epsilon \in X$ is any point such that $f(x_\epsilon)q\mu$, then there exists a fuzzy $\widehat{\Omega}$-open subset η of X such that such that $(x_\epsilon)q\eta$ and f(η)$\subseteq$ μ.*

Proof. Let μ be any fuzzy regular closed subset of Y and $x_\epsilon \in X$ be any point such that $f(x_\epsilon)q\mu$. Then, $\mu(f(x_\epsilon)) + \epsilon > 1 \Leftrightarrow (f^{-1}(\mu))(x_\epsilon) + \epsilon > 1$. By the definition of quasi coincident, $(x_\epsilon)qf^{-1}(\mu)$. By hypothesis, $f^{-1}(\mu) = \eta$ (say) is fuzzy $\widehat{\Omega}$-open set in X such that $(x_\epsilon)q\eta$. Since $f\left(f^{-}1(\mu)\right) \subseteq \mu$, we have $f(\eta) \subseteq \mu$. □

Theorem 5.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be any fuzzy function. Then f is fuzzy almost contra $\widehat{\Omega}$-continuous if and only if $f^{-1}(int(cl(\mu)))$ (resp. $f^{-1}(cl(int(\mu)))$) is fuzzy $\widehat{\Omega}$-closed (resp.fuzzy $\widehat{\Omega}$-open)for any subset μ of Y.*

Proof. Let μ be any fuzzy subset of Y. By [[2],Theorem-3.6] $int(cl(\mu))$ is fuzzy regular open. By hypothesis, $f^{-1}(int(cl(\mu)))$ is fuzzy $\widehat{\Omega}$-closed subset of X.

Conversely, Let λ be any fuzzy regular open set in Y. By hypothesis, $f^{-1}(int(cl(\lambda)))$ is fuzzy $\widehat{\Omega}$-closed set in X. Then $f^{-1}(\lambda)$ is fuzzy $\widehat{\Omega}$-closed set in X. Hence, f is fuzzy almost contra $\widehat{\Omega}$-continuous. □

Theorem 5.9. *Let $f : X \to Y$ be any fuzzy function. If graph of f is fuzzy almost contra $\widehat{\Omega}$ continuous, then f is fuzzy almost contra $\widehat{\Omega}$ continuous.*

Proof. Let $g : X \to X \times Y$ be the fuzzy graph of f, defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ for every fuzzy point x_ϵ of X. Let η be any fuzzy regular closed set in Y. Then, $X \times \eta$ is fuzzy regular closed set in the product fuzzy topological space $X \times Y$. Since g is fuzzy almost contra $\widehat{\Omega}$ continuous, $g^{-1}(X \times \eta) = f^{-1}(\eta)$ is fuzzy $\widehat{\Omega}$-open subset of X. Thus, f is fuzzy almost contra $\widehat{\Omega}$ continuous. □

6. Conclusion

In this paper, an attempt has been made to fuzzify the concept of contra $\widehat{\Omega}$-continuity and almost contra $\widehat{\Omega}$-continuity on fuzzy topological spaces. It has been shown that intersection of fuzzy $\widehat{\Omega}$-closed sets not necessarily a fuzzy $\widehat{\Omega}$-closed set.

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