International Journal of Mathematics And its Applications
Volume 5, Issue 2–C (2017), 315–319.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Electrons Drifting Effect on Dust Ion Acoustic Solitary Waves Using Relativistic Effect

Research Article

Riju Kumar[1]*

1 Department of Mathematics, Pandu College, Guwahati, Assam, India.

Abstract: In this paper we study dust ion acoustic solitary waves (DIASW) based on the dust charge $Z_d = \frac{n_{d_0}}{n_{i_0}} = \frac{equilibrium\ density\ of\ dust\ ions}{equilibrium\ density\ of\ ions}$. Compressive solitons are found to exist in presence of electron's drift velocity (v_e'). It is observed that compressive solitons exist for smaller values of dust charge Z_d and greater values ofv_e'.

Keywords: Dust ion acoustic solitary waves, electron's drift velocity, dust charge.

1. Introduction

In the study of plasma dust ion acoustic (DIA) waves have created immense impact on research workers. Formation of solitary waves in plasma is done both in laboratory and space plasma. Any way the presence of dust particle in plasmas of space environment has created increasing interest in the domain of plasma research and influence the plasma properties leading to new significant result. By means of reductive perturbation method, Rao et.al (1990) have reported the existence of dust acoustic waves. The role and theoretical applications [3] of dust particles observed in earth's magnetosphere, cometary tail, planetary rings, lower ionosphere of the earth [4–9] have contributed a lot towards study of dusty plasma. On the otherhand, dust ion acoustic waves (DIAW) are studied by [10–12]. Duan et.al (2003) have investigated dusty plasma with variable dust charge. EL-Labany and El-Taibany (2003) have investigated the effects of variable dust charge, dust temperature and an arbitrary streaming ion beam on small amplitude dust acoustic waves.

In their investigation they found both compressive and rarefactive solitons exists. Zhang and Xue (2005) have studied the effects of dust charge variation and non thermal ions on dust acoustic solitary waves. Das and Chatterjee (2009) have also investigated the formation of large amplitude double layers in dusty plasma. Recently Tiwari et.al (2011) have investigated the characteristics of ion acoustic soliton in dusty plasma.

In this paper, we have investigated the drifting effect of electrons on dust-ion acoustic solitary waves. Various substantial and characteristic changes on solitons amplitude and growth are observed in this investigation due to the presence of the drifting motion of the electrons.

* *E-mail: kumarriju1@yahoo.com*

2. Dynamics of Motion

In the model of plasma under consideration, we assume it to be composed of negatively charged dust particles in the plasma, together with ions and electrons. The governing equations in one-dimension are given by

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0 \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = Z_d \frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}\right)(\gamma_i v_i) = -Z_d \frac{\partial \phi}{\partial x} \tag{4}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right)(\gamma_e v_e) = \frac{Z_d}{Q}\left(\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial n_e}{\partial x}\right) \tag{6}$$

where Z_d stands for dust charges, 'i' for positive ions and 'e' for electrons respectively, $Q = \frac{m_e}{m_i} = \frac{mass\ of\ the\ electrons}{mass\ of\ the\ positive\ ions}$ and $Z_d = \frac{n_{d_0}}{n_{i_0}} = \frac{equilibrium\ density\ of\ dust\ ions}{equilibrium\ density\ of\ ions}$. In these equations, densities are normalized by the unperturbed densities n_{d0}, n_{i0} and n_{e0} respectively; space by electron Debye length $\lambda_{De} = \left(\frac{Z_d T_e}{4\,\pi\, n_{e0} e^2}\right)^{\frac{1}{2}}$; time by ion plasma period $\omega_{p_i}^{-1} = \left(\frac{m_i}{4\,\pi\, n_{e0} e^2}\right)^{\frac{1}{2}}$; velocities by acoustic velocity $C_S = \left(\frac{Z_d T_e}{m_i}\right)^{\frac{1}{2}}$ and the potential ϕ by $\frac{T_e}{e}$. Again for charge imbalances these equations are to be combined by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_e + Z_d n_d - n_i \tag{7}$$

3. Derivation of the KdV Equation

To derive the KdV equation from the basic equations (1)-(6), we use the stretched variables

$$\xi = \varepsilon^{\frac{1}{2}}(x - Ut), \quad \tau = \varepsilon^{\frac{3}{2}} t \tag{8}$$

With phase velocity U and expand the variables asymptotically about the equilibrium state in terms of smallness parameter ε as follows:

$$\begin{aligned}
n_d &= n_{do} + \varepsilon n_{d1} + \varepsilon^2 n_{d2} + \ldots \\
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \ldots \\
n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \ldots \\
u_d &= \varepsilon u_{d1} + \varepsilon^2 u_{d2} + \ldots \\
v_i &= \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \ldots \\
v_e &= v_e' + \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \ldots \\
\phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \ldots
\end{aligned} \tag{9}$$

For unidirectional transformation related to solitary waves in our plasma model, we need to use Equation (9) and the stretched variables in the set of Equations (1)-(7). Equating the coefficients of ε^1, we get from the first order perturbed quantities, the equation for the phase velocity U namely,

$$\frac{1}{Z_d - Q(U - v_e')^2\left(1 + \frac{3v'^2_e}{2c^2}\right)} - \frac{Z_d}{U^2} - \frac{1}{U^2} = 0 \tag{10}$$

Using $n_{i1} = n_{e1} = n_{d1} = 0$, $v_{i1} = v_{e1} = v_{d1} = 0$, $\phi_1 = 0$ at $|\xi| \to \infty$. This can be solved for the phase velocity U in terms of the equilibrium quantities in it and dust charge Z_d and C for relativistic effects. Again equating the coefficients of ε^2 from (1)-(7) and eliminating $\frac{\partial u_{d2}}{\partial \xi}$, $\frac{\partial v_{i2}}{\partial \xi}$, $\frac{\partial v_{e2}}{\partial \xi}$ we get

$$\frac{\partial \phi_1}{\partial \tau} + p\phi_1 \frac{\partial \phi_1}{\partial \xi} + q\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{11}$$

Where $p = \frac{B}{A}$ and $q = \frac{1}{A}$ with A and B given by

$$A = \left[\frac{(Z_d - 1)}{(v'_e - U)\left\{z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}} - \frac{(Z_d - 1)}{(v'_e - U)\left\{z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^2} - \frac{QZ_d(v'_e - U)\left(1 + \frac{3v'_e{}^2}{2c^2}\right)}{\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^2} + \frac{4Z_d}{U^3}\right] \tag{12}$$

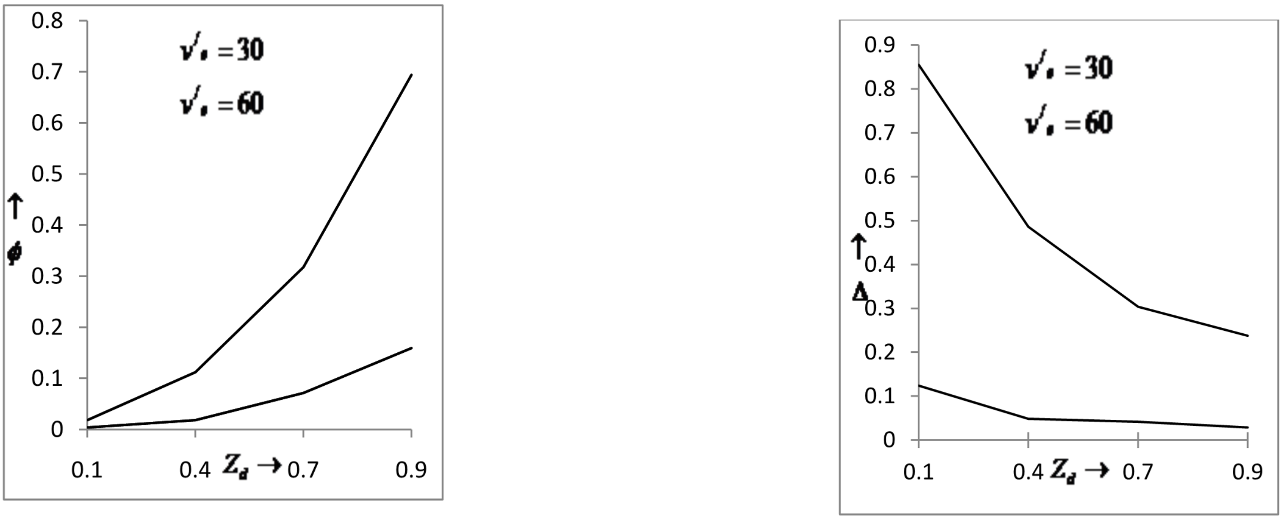
$$\begin{aligned} B = \Bigg[& \frac{\left(Z_d{}^4 - Z_d{}^3\right)}{2Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^3} \\ & + \frac{\left(Z_d{}^2 - Z_d\right)}{2Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}} \\ & + \frac{3v'_e\left(Z_d{}^4 - Z_d{}^3\right)}{2Qc^2(v'_e - U)\left(1 + \frac{3v'_e{}^2}{2c^2}\right)^2\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^3} \\ & + \frac{3v'_e\left(Z_d{}^2 - Z_d\right)}{2Qc^2(v'_e - U)\left(1 + \frac{3v'_e{}^2}{2c^2}\right)^2\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}} \\ & + \frac{\left(Z_d{}^3 - Z_d{}^2\right)}{\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^3} - \frac{\left(Z_d{}^2 - Z_d\right)}{\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^2} \\ & - \frac{3v'_e\left(Z_d{}^3 - Z_d{}^2\right)}{Qc^2(v'_e - U)\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^2} \\ & - \frac{\left(Z_d{}^3 - Z_d{}^2\right)}{Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^2} - \frac{\left(Z_d{}^3 - Z_d{}^2\right)}{2\left\{Z_d - Q(v'_e - U)^2\left(1 + \frac{3v'_e{}^2}{2c^2}\right)\right\}^3}\Bigg] \end{aligned} \tag{13}$$

The KdV equation (11) will give the soliton solution for real suitable values of p and q based on the parameters involved in (12) and (13). For the desired soliton solution,we consider the transformation $\eta = \xi - V\tau$. Using the transformation $\eta = \xi - V\tau$, for solitary wave solution, we integrate once and use the boundary conditions $\phi_1 = 0$, $\frac{d^2\phi_1}{d\eta^2} = 0$ as $\eta \to \infty$ to get $\frac{d^2\phi_1}{d\eta^2} = \left(\frac{V}{q}\right)\phi_1 - \left(\frac{p}{2q}\right)\phi_1^2$.

Multiplying both sides by $2\frac{d\phi_1}{d\eta}$ and again integrating we get $\frac{d\phi_1}{d\eta} = \pm\sqrt{\left(\frac{V}{q}\right)}\phi_1\sqrt{\left\{\left(1 - \frac{p}{3V}\phi_1\right)\right\}}$. Integrating again we get $\phi_1 = \left(\frac{3V}{p}\right)\sec h^2\left(\frac{1}{2}\sqrt{\frac{V}{q}}\eta\right)$. Where V is the velocity with which the solitary waves travel to the right. The corresponding amplitudes (ϕ_0) and width (Δ) of the soliton are respectively $\phi_0 = \left(\frac{3V}{p}\right)$ and $\Delta = 2\sqrt{\frac{q}{V}}$.

4. Discussion

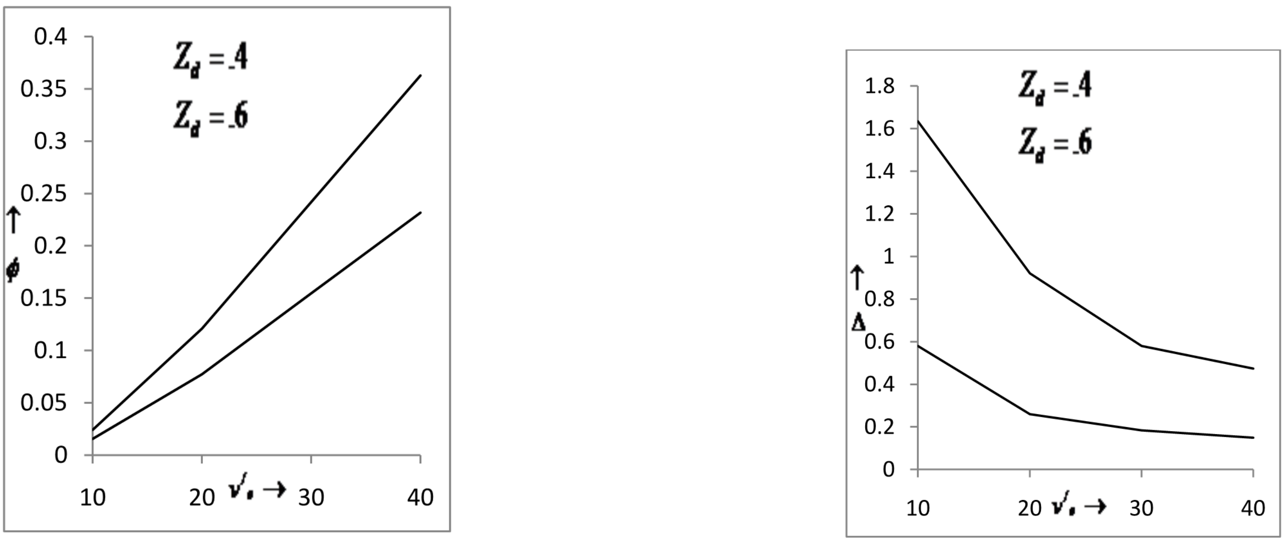
Using relativistic effects in electrons, only compressive solitons are established depending on the dust charge Z_d and drift velocity v_e' with the real acceptable Values of U. Here we have considered $V = 0.2$, $Q = 0.0054$, $C = 300$ (normalized).



(a) Amplitude of dust acoustic compressive solitons versus dust charge Z_d

(b) Width Δ of dust acoustic compressive solitons versus dust charge Z_d

Figure 1:



(a) Amplitude ϕ of dust acoustic compressive solitons versus drift velocity v_e'.

(b) Width Δ of dust acoustic compressive solitons versus drift velocity v_e'.

Figure 2:

References

[1] N.N.Rao, P.K.Shukla and M.Y.Yu, *Dust Acoustic Waves in Dusty Plasma*, Planet Space Science, 38(1990), 543-546.

[2] P.K.Shukla and V.P.Silin, *Dust Ion Acoustic Waves*, Phys. Scr., 45(1992).

[3] N.D' Angelo, *Coulomb solids and low-frequency fluctuations in RF dusty plasmas*, J. Phys., D28(1995).

[4] A.Barkan, R.L.Merlino and N.D'Angelo, *Labortary observation of the dust ion acoustic wave mode*, Phys. Plasmas., 2(1995), 3563-3565.

[5] P.K.Shukla and A.A.Mamun, *Introduction to Dusty Plasma physics*, Institute of Physics, Bristol, (2002).

[6] P.K.Shukla, D.A.Mendis and T.Desai, *Advances in Dusty Plasmas*, World Scientific, Singapore, (1997).

[7] Y.Nakamura, H.Bailung and P.K.Shukla, *Observation of Ion-Acoustic shocks in a Dusty Plasma*, Phy. Rev. Lett., 83(1999), 1602-1605.

[8] W.S.Duan, K.P.Lu and J.B.Zhao, *Hot Dust Acoustic Solitary Waves in Dust plasma with variable Dust charge*, Chain. Phys. Lett., 18(2001), 1088-1089.

[9] A.A.Mamun and P.K.Shukla, *Introduction to dusty plasma physics*, Physics IOP, London, (2002).

[10] F.Verheest, *Waves in Dusty Plasma*, Kluwar Academic, Dordrecht, (2000).

[11] S.Ghosh, *Dust Acoustic Solitary waves with variable dust charge: Role of negative ions*, Phys. Plasmas., 12(2005), 94504-94507.

[12] H.R.Pakzad, *Rarefactive and compressive Soliton Waves in unmagnetized dusty plasma with non-thermal electron and ion distribution*, Astrophys Space Science, 324(2009), 41-45.

[13] B.Das and P.Chatterjee, *Large amplitude double layers in dusty plasma with non-thermal electrons and two temperature isothermal ions*, Phy. Lett., A373(2009), 1144-1147.

[14] B.C.Kalita and R.Kumar, *Dust Acoustic Solitary Waves in a weakly relativistic unmagnetized plasma*, Int. J.of Math.Sci & Engg. Appls., 5(IV)(2011), 213-224.