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Soft $g\zeta^*$-Closed Sets in Soft Topological Spaces

Research Article

V.Kokilavani[1] and M.Myvizhi[2]*

1 Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, India.

2 Department of Mathematics, Asian College of Engineering and Technology, Coimbatore, India.

Abstract: In this paper we consider a new class of soft set called soft generalized ζ^*-closed sets in soft topological spaces. Also, we discuss some basic properties of soft $g\zeta^*$-closed sets.

Keywords: $g\zeta^*$-closed set, soft $g\zeta^*$-closed set, soft $g\zeta^*$-continuous function, soft $g\zeta^*$-irresolute function and soft $g\zeta^*$-spaces.

1. Introduction

The concept of soft sets was first introduced by Molodtsov [12] in 1999 who began to develop the basics of corresponding theory as a new approach to modeling uncertainties. In [12, 13], Molodtsov successfully applied the soft theory in several directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. In recent years, an increasing number of papers have been written about soft sets theory and its applications in various fields [10, 16]. Shabir and Naz [15] introduced the notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameters. In addition, Maji et al. [11] proposed several operations on soft sets, and some basic properties of these operations have been revealed so far. I.Arockiarany and A.ArokiaLancy [4] introduced the notion of soft gs-closed sets in soft topological spaces in 2013.

Many researchers extended the results of generalization of various soft closed sets in many directions. V.R.Karuppayal et.al, [8] introduced the notion of soft $g^{\#}\alpha$-closed sets and V.Kokilavani et.al, [9] introduced the notion of soft ${}^{\#}g\alpha$-closed sets in soft topological spaces. Our motivation in this paper is to define soft $g\zeta^*$-closed sets and also we introduce soft $g\zeta^*$-spaces and investigate their properties which are important for further research on soft topology. Furthermore, we will study soft $g\zeta^*$-continuous and soft $g\zeta^*$-irresolute functions and obtain some characterizations of such functions.

2. Preliminaries

Let U be an initial universe and E be a set of parameters. Also let A be a non-empty subset of E and $P(U)$ denote the power set of U. If F is a mapping given by $F : A \to P(U)$, then the pair (F, A) is called a soft set over U.

* E-mail: myvizhimaths@gmail.com

Definition 2.1 ([5]). *Let X be an initial universe set and E be a universe set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set X and $A, B \subseteq E$. Then (F, A) is a subset of (G, B) denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if: (i) $A \subseteq B$; (ii) for all $e\widetilde{\in}A$, $F(e) \subseteq G(e)$. (F, A) equals (G, B) denoted by $(F, A) = (G, B)$, if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$.*

Definition 2.2 ([5]). *A soft set (F, A) over X is called a null soft set which is denoted by $\emptyset$, if $\forall\ e \in \widetilde{A}$, $F(e) = \emptyset$.*

Definition 2.3 ([5]). *A soft set (F, A) over X is called an absolute soft set which is denoted by $\widetilde{A}$, if $\forall\ e\widetilde{\in}A$, $F(e) = X$. If $A = E$, then the absolute soft set (F, A) over X is denoted by $\widetilde{X}$.*

Definition 2.4 ([5]). *The union of two soft sets (F, A) and (G, B) over a common universe X is the soft set (H, C), where $C = A \cup B$ and $\forall\ e\widetilde{\in}C$,*

$$H(e) = \begin{cases} F(e), & \text{if } e\widetilde{\in}A - B \\ G(e), & e\widetilde{\in}B - A \\ F(e)\widetilde{\cup}G(e), & e\widetilde{\in}B\widetilde{\cap}A. \end{cases}$$

We write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.5 ([5]). *The intersection of two soft sets of (F, A) and (G, B) over a common universe X is the soft set (H, C), where $C = A\widetilde{\cap}B$ and $\forall\ e\widetilde{\in}C$, $H(e) = F(e)\widetilde{\cap}G(e)$. We write $(F, A)\widetilde{\cap}(G, B) = (H, C)$.*

Definition 2.6 ([5]). *The Difference of two soft sets (F, A) and (G, A) over a common universe X is the soft set (H, A) which is denoted by $(F, A)\backslash(G, A)$ and is defined as $H(e) = F(e)\backslash G(e)$ for all $e\widetilde{\in}A$.*

Definition 2.7 ([5]). *Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if it satisfies the following axioms:*

1. *$\Phi, \tilde{X}$ belongs to τ,*
2. *The union of any number of soft sets in τ belongs to τ,*
3. *The intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called a soft topological space over X. Let (X, τ, E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. A soft set (F, A) over X is said to be soft closed set in X, if its relative complement $(F, A)^c$ belongs to τ.

Definition 2.8 ([2]). *For a soft set (F, A) over X, the relative complement of (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = F^c(A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e\widetilde{\in}A$.*

Definition 2.9 ([6]). *Let (X, τ, E) be a soft topological space and let (F, A) be a soft set over X. The soft closure of (F, A) is the soft set defined by $\widetilde{s}cl(F, A) = \widetilde{\cap}\{(S, A) : (S, A) \quad \text{is soft closed and } (F, A)\widetilde{\subseteq}(S, A)\}$.*

Note that, $\widetilde{s}cl(F, A)$ is the smallest closed set containing (F, A).

Definition 2.10 ([3]). *A soft set (F, A) of a soft topological space (X, τ, E) is said to be*

1. *soft open if its complement is soft closed.*
2. *soft semi open if $(F, A)\widetilde{\subseteq}\tilde{s}cl(\tilde{s}int(F, A))$. soft semi closed if $\tilde{s}int(\tilde{s}cl(F, A))\widetilde{\subseteq}(F, A)$.*
3. *soft α-open if $(F, A)\widetilde{\subseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int(F, A)))$, soft α-closed if $\tilde{s}cl(\tilde{s}int(\tilde{s}cl(F, A)))\widetilde{\subseteq}(F, A)$.*

Definition 2.11 ([7]). *A soft set (A, E) is called a soft generalized closed (soft g-closed) set in a soft topological space (X, τ, E), if $\tilde{s}cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft open in X.*

Definition 2.12 ([4]). *A subset (A, E) of a topological space X is called soft generalized-semi closed (soft gs-closed) if $\tilde{s}scl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft open in X.*

Definition 2.13 ([8]). *A soft set (A, E) is called a soft generalized $^{\#}\alpha$-closed (soft $g^{\#}\alpha$-closed) set in a soft topological space (X, τ, E), if $\tilde{s}\alpha cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft g-open in X.*

Definition 2.14 ([9]). *A soft set (A, E) is called a soft $^{\#}$generalized α-closed (soft $^{\#}g\alpha$-closed) set in a soft topological space (X, τ, E), if $\tilde{s}\alpha cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft $g^{\#}\alpha$-open in X.*

Definition 2.15 ([1]). *A soft mapping $f : X \to Y$ is called soft α-continuous if the inverse image of each soft open set in Y is soft α-open set in X.*

3. Soft $g\zeta^*$-closed Sets

Definition 3.1. *A soft set (A, E) is called a soft generalized ζ^*-closed (soft $g\zeta^*$-closed) set in a soft topological space (X, τ, E) if $\tilde{s}\alpha cl(A, E)\widetilde{\subseteq}(U, E)$ whenever $(A, E)\widetilde{\subseteq}(U, E)$ and (U, E) is soft $^{\#}g\alpha$-open in X.*

Theorem 3.2. *Every soft closed set is soft $g\zeta^*$-closed but not conversely.*

Proof. Let (A, E) be a soft closed set in (X, τ, E) and (U, E) be soft $^{\#}g\alpha$-open in X such that $(A, E)\widetilde{\subseteq}\ (U, E)$. Hence $\tilde{s}cl(A,\ E)\ =\ (A, E)\ \widetilde{\subseteq}(U,\ E)$. Since $\tilde{s}\alpha cl\,(A,\ E)\,\widetilde{\subseteq}\tilde{s}cl(A,\ E)\ \widetilde{\subseteq}(U,\ E)$. Therefore (A, E) is soft $g\zeta^*$-closed. □

Converse of the above theorem need not be true by the following example.

Example 3.3. $U = \{a, b\}$, $A = \{e_1, e_2\}$. *Define* $(F, A)_1 = \{(e_1, \phi), (e_2, \phi)\}$, $(F, A)_2 = \{(e_1, \phi), (e_2, \{a\})\}$, $(F, A)_3 = \{(e_1, \phi), (e_2, \{b\})\}$, $(F, A)_4 = \{(e_1, \phi), (e_2, \{a, b\})\}$, $(F, A)_5 = \{(e_1, \{a\}), (e_2, \phi)\}$, $(F, A)_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$, $(F, A)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F, A)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, $(F, A)_9 = \{(e_1, \{b\}), (e_2, \phi)\}$, $(F, A)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$, $(F, A)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F, A)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$, $(F, A)_{13} = \{(e_1, \{a, b\}), (e_2, \phi)\}$, $(F, A)_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$, $(F, A)_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$, $(F, A)_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$ *are all soft sets on universal set U under the parameter set A.*
$\widetilde{\tau} = \{(F, A)_1, (F, A)_5, (F, A)_7, (F, A)_8, (F, A)_{16}\}$; $\widetilde{\tau}^c = \{(F, A)_1, (F, A)_9, (F, A)_{10}, (F, A)_{12}, (F, A)_{16}\}$. *Soft closed sets are* $(F, A)_1, (F, A)_9, (F, A)_{10}, (F, A)_{12}, (F, A)_{16}$. *Soft $g\zeta^*$-closed sets are* $(F, A)_1$, $(F, A)_2$, $(F, A)_3$, $(F, A)_4$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{11}$, $(F, A)_{12}$, $(F, A)_{16}$. *Here* $(F, A)_2$, $(F, A)_3$, $(F, A)_4$ *and* $(F, A)_{11}$ *are soft $g\zeta^*$-closed sets but not soft closed sets.*

Theorem 3.4. *Every soft α-closed set is soft $g\zeta^*$-closed set but not conversely.*

Proof. Let (A, E) be a soft α-closed set in (X, τ, E) and (U, E) be soft $^{\#}g\alpha$-open in X such that $(A, E)\ \widetilde{\subseteq}\ (U, E)$. Since $\tilde{s}\alpha cl(A,\ E)\ =\ (A,\ E)\widetilde{\subseteq}(U,\ E)$. Therefore $\tilde{s}\alpha cl(A,\ E)\ \widetilde{\subseteq}(U,\ E)$ and hence $(A,\ E)$ is soft $g\zeta^*$-closed. □

Converse of the above theorem need not be true by the following example.

Example 3.5. $U = \{a, b\}$, $A = \{e_1, e_2\}$. *Define* $(F, A)_1 = \{(e_1, \phi), (e_2, \phi)\}$, $(F, A)_2 = \{(e_1, \phi), (e_2, \{a\})\}$, $(F, A)_3 = \{(e_1, \phi), (e_2, \{b\})\}$, $(F, A)_4 = \{(e_1, \phi), (e_2, \{a, b\})\}$, $(F, A)_5 = \{(e_1, \{a\}), (e_2, \phi)\}$, $(F, A)_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$, $(F, A)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F, A)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, $(F, A)_9 = \{(e_1, \{b\}), (e_2, \phi)\}$, $(F, A)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$, $(F, A)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F, A)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$, $(F, A)_{13} = \{(e_1, \{a, b\}), (e_2, \phi)\}$, $(F, A)_{14} =$

$\{(e_1,\{a,b\}),(e_2,\{a\})\}$, $(F,A)_{15} = \{(e_1,\{a,b\}),(e_2,\{b\})\}$, $(F,A)_{16} = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ are all soft sets on universal set U under the parameter set A. $\widetilde{\tau} = \{(F,A)_1,(F,A)_6,(F,A)_{16}\}$; $\widetilde{\tau}^c = \{(F,A)_1,(F,A)_{11},(F,A)_{16}\}$

Soft α-closed sets are $(F,A)_1$, $(F,A)_3$, $(F,A)_9$, $(F,A)_{11}$, $(F,A)_{16}$. Soft $g\zeta^*$-closed sets are $(F,A)_1$, $(F,A)_3$, $(F,A)_9$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$, $(F,A)_{16}$. Here $(F,A)_{12}$ and $(F,A)_{15}$ are soft $g\zeta^*$-closed sets but not soft α-closed sets.

Theorem 3.6. *Every soft $g\zeta^*$-closed set is soft gs-closed set but not conversely.*

Proof. Let (A,E) be a soft $g\zeta^*$-closed set in $(X,\ \tau,\ E)$ and (U,E) be an soft open set containing (A,E). Since every soft open set is soft $^{\#}g\alpha$-open, we have $\tilde{s}scl(A,E)\ \widetilde{\subseteq}\tilde{s}\alpha cl(A,\ E)\widetilde{\subseteq}(U,\ E)$. Therefore $\tilde{s}scl(A,\ E)\ \widetilde{\subseteq}(U,E)$. Hence (A,E) is a soft gs-closed set. □

Converse of the above theorem need not be true by the following example.

Example 3.7. *$U = \{a,b\}$, $A = \{e_1,e_2\}$. Define $(F,A)_1 = \{(e_1,\phi),(e_2,\phi)\}$, $(F,A)_2 = \{(e_1,\phi),(e_2,\{a\})\}$, $(F,A)_3 = \{(e_1,\phi),(e_2,\{b\})\}$, $(F,A)_4 = \{(e_1,\phi),(e_2,\{a,b\})\}$, $(F,A)_5 = \{(e_1,\{a\}),(e_2,\phi)\}$, $(F,A)_6 = \{(e_1,\{a\}),(e_2,\{a\})\}$, $(F,A)_7 = \{(e_1,\{a\}),(e_2,\{b\})\}$, $(F,A)_8 = \{(e_1,\{a\}),(e_2,\{a,b\})\}$, $(F,A)_9 = \{(e_1,\{b\}),(e_2,\phi)\}$, $(F,A)_{10} = \{(e_1,\{b\}),(e_2,\{a\})\}$, $(F,A)_{11} = \{(e_1,\{b\}),(e_2,\{b\})\}$, $(F,A)_{12} = \{(e_1,\{b\}),(e_2,\{a,b\})\}$, $(F,A)_{13} = \{(e_1,\{a,b\}),(e_2,\phi)\}$, $(F,A)_{14} = \{(e_1,\{a,b\}),(e_2,\{a\})\}$, $(F,A)_{15} = \{(e_1,\{a,b\}),(e_2,\{b\})\}$, $(F,A)_{16} = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}$ are all soft sets on universal set U under the parameter set A.*

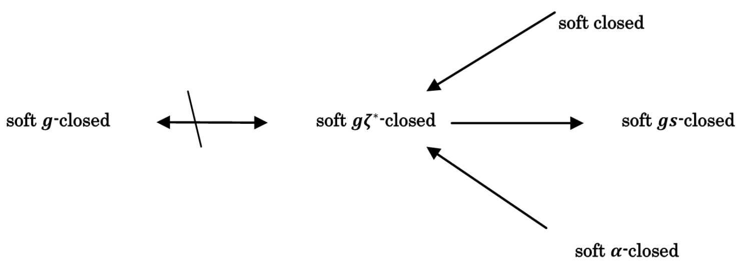
$\widetilde{\tau} = \{(F,A)_1,(F,A)_5,(F,A)_7,(F,A)_8,(F,A)_{16}\}$; $\widetilde{\tau}^c = \{(F,A)_1,(F,A)_9,(F,A)_{10},(F,A)_{12},(F,A)_{16}\}$ Soft gs-closed sets are $(F,A)_1$, $(F,A)_2$, $(F,A)_3$, $(F,A)_4$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$. Soft $g\zeta^*$-closed sets are $(F,A)_1$, $(F,A)_2$, $(F,A)_3$, $(F,A)_4$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{16}$. Here $(F,A)_{13}$, $(F,A)_{14}$ and $(F,A)_{15}$ are soft $g\zeta^*$-closed sets but not soft gs-closed sets.

Remark 3.8. *The concepts of soft g-closed sets and soft $g\zeta^*$-closed sets are independent to each other.*

In example 3.11: Soft g-closed sets in X are $(F,A)_1$, $(F,A)_3$, $(F,A)_5$, $(F,A)_7$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$. Soft $g\zeta^*$-closed sets in X are $(F,A)_1$, $(F,A)_3$, $(F,A)_5$, $(F,A)_7$, $(F,A)_{12}$, $(F,A)_{14}$, $(F,A)_{16}$. The soft sets given above are defined as follows

$(F,A)_1 = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $(F,A)_{12} = \{(e_1,\{b\}),(e_2,\{a,b\})\}$, $(F,A)_3 = \{(e_1,\emptyset),(e_2,\{b\})\}$, $(F,A)_{13} = \{(e_1,\{a,b\}),(e_2,\emptyset)\}$, $(F,A)_5 = \{(e_1,\{a\}),(e_2,\emptyset)\}$, $(F,A)_{14} = \{(e_1,\{a,b\}),(e_2,\{a\})\}$, $(F,A)_7 = \{(e_1,\{a\}),(e_2,\{b\})\}$, $(F,A)_{15} = \{(e_1,\{a,b\}),(e_2,\{b\})\}$, $(F,A)_{11} = \{(e_1,\{b\}),(e_2,\{b\})\}$, $(F,A)_{16} = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}$. Here the soft set $(F,A)_{13}$ is soft g-closed but not soft $g\zeta^*$-closed.

From the above results, the following implication is made:



4. Basic Properties of Soft $g\zeta^*$-closed Sets

Theorem 4.1. *If $(A,\ E)$ and $(B,\ E)$ are soft $g\zeta^*$-closed sets in X, then $(A,\ E)\ \widetilde{\cup}\ (B,\ E)$ is also a soft $g\zeta^*$-closed in X.*

Proof. Suppose that $(A,\ E)$ and $(B,\ E)$ are soft $g\zeta^*$-closed sets in X. Let $(A,\ E)\ \tilde{\cup}\ (B,\ E)\tilde{\subseteq}(U,E)$ and $(U,\ E)$ is soft $^{\#}g\alpha$-open in X. Thus we have $(A,\ E)\ \tilde{\subseteq}(U,\ E)$ and $(B,\ E)\ \tilde{\subseteq}(U,\ E)$. Since (U,E) is soft $^{\#}g\alpha$-open in X and $(A,\ E)$ and $(B,\ E)$ are soft $g\zeta^*$-closed sets, we have $\tilde{s}\alpha cl(A,\ E)\tilde{\subseteq}\ (U,E)$ and $\tilde{s}\alpha cl((B,\ E)\tilde{\subseteq}(U,\ E)$. Hence $\tilde{s}\alpha cl[(A,\ E)\ \tilde{\cup}\ (B,\ E)]\ =\ \tilde{s}\alpha cl(A,\ E)\tilde{\cup}\ \tilde{s}\alpha cl(B,\ E)\tilde{\subseteq}(U,\ E)$. Therefore $(A,\ E)\ \tilde{\cup}\ (B,\ E)$ is also a soft $g\zeta^*$-closed set in X. □

Theorem 4.2. *If $(A,\ E)$ is a soft $g\zeta^*$-closed set in X and $(A,E)\ \tilde{\subseteq}(B,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)$, then $(B,\ E)$ is a soft $g\zeta^*$-closed set.*

Proof. Suppose that $(A,\ E)$ is soft $g\zeta^*$-closed in X and $(A,E)\ \tilde{\subseteq}(B,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)$. Let $(B,\ E)\ \tilde{\subseteq}(U,\ E)$ and $(U,\ E)$ is soft $^{\#}g\alpha$-open in X. Since $(A,E)\ \tilde{\subseteq}(B,\ E)$ and $(B,E)\ \tilde{\subseteq}(U,\ E)$, we have $(A,E)\tilde{\subseteq}\ (U,E)$. Hence $\tilde{s}\alpha cl(A,\ E)\ \tilde{\subseteq}(U,\ E)$ as $(A,\ E)$ is soft $g\zeta^*$-closed. Thus, we have $\tilde{s}cl\,(A,E)\,\tilde{\subseteq}\,(U,\ E)$. Since $(B,E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,E)$, we have $\tilde{s}\alpha cl(B,E)\tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)\ \tilde{\subseteq}(U,\ E)$. Hence (B,E) is a soft $g\zeta^*$-closed set. □

Theorem 4.3. *If a set (A,E) is soft $g\zeta^*$-closed in X then $\tilde{s}\alpha cl(A,\ E)-(A,E)$ contains only null soft $^{\#}g\alpha$-closed set.*

Proof. Suppose that $(A,\ E)$ is a soft $g\zeta^*$-closed set in X.Let $(F,\ E)$ be soft $^{\#}g\alpha$-closed set such that $(F,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)-(A,\ E)$.Since (F, E) is soft $^{\#}g\alpha$-closed, its relative complement $(F,\ E)'$ is soft $^{\#}g\alpha$-open. Since $(F,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)-(A,E)$, we have $(F,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)$ and $(F,\ E)\ \tilde{\subseteq}(A,\ E)'$. Hence $(A,\ E)\ \tilde{\subseteq}(F,\ E)'$,also $\tilde{s}\alpha cl(A,\ E)\subseteq\ (F,\ E)'$ as $(A,\ E)$ is soft $g\zeta^*$-closed set in X. Therefore,$(F,\ E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,\ E)'$.Thus $(F,\ E)\ =\ \phi$. Hence $s\ \ \alpha cl(A,\ E)-(A,\ E)$ contains only null soft $g\zeta^*$-closed set. □

Theorem 4.4. *If (A,E) is soft $^{\#}g\alpha$-open and soft $g\zeta^*$-closed then (A,E) is soft α-closed.*

Proof. Suppose (A,E) is soft $^{\#}g\alpha$-open and soft $g\zeta^*$-closed, then by the definition, $\tilde{s}\alpha cl(A,E)\ \tilde{\subseteq}(A,E)$. But $(A,E)\tilde{\subseteq}\tilde{s}\alpha cl(A,E)$ (by Theorem 4.2). Thus, $(A,E)=\tilde{s}\alpha cl(A,E)$. Therefore, (A,E) is soft α-closed. □

5. Soft $g\zeta^*$-spaces

Definition 5.1. *A soft topological space (X,τ,E) is called soft $\hat{T}_{g\zeta^*}$-space if every soft $g\zeta^*$-closed set in it is a soft α-closed.*

Theorem 5.2. *For a soft topological space (X,τ,E), the following conditions are equivalent.*

(1). X is a soft $\hat{T}_{g\zeta^*}$-space.

(2). Every singleton of X is soft $^{\#}g\alpha$-closed or soft α-open.

(3). Every singleton of X is soft $^{\#}g\alpha$-closed or soft open.

Proof. (1) $\Longrightarrow$ (2) : Let $x\tilde{\in}\ X$. Suppose that $\{x\}$ is not soft $^{\#}g\alpha$-closed set of $(X,\ \tau,\ E)$.Then $X\setminus\{x\}$ is not soft $^{\#}g\alpha$-open.Thus X is the only soft $^{\#}g\alpha$-open set containing $X\setminus\{x\}$. Hence $X\setminus\{x\}$ is soft $g\zeta^*$-closed set of $(X,\ \tau,\ E)$. By (1), $X\setminus\{x\}$ is soft α-closed set of $(X,\ \tau,\ E)$ or equivalently $\{x\}$ is soft α-open. Therefore $\{x\}$ is soft $^{\#}g\alpha$-closed or soft α-open.

(2) $\Longrightarrow$ (3) : Since we know that $\{x\}$ is soft α-open if and only if $\{x\}$ is soft open and hence the proof follows.

(3) $\Longrightarrow$ (2) : Since $\{x\}$ is soft open $\Longrightarrow$ $\{x\}$ is soft α-open, proof is obvious.

(3) $\Longrightarrow$ (1) : Let (A,E) be a soft $g\zeta^*$-closed set. Trivially $(A,E)\ \tilde{\subseteq}\tilde{s}\alpha cl(A,E)$. Let $x\tilde{\in}\tilde{s}\alpha cl(A,E)$. By (2), $\{x\}$ is either soft $^{\#}g\alpha$-closed or soft α-open.

We consider the following two cases:

Case : 1 Let $\{x\}$ be soft $^{\#}g\alpha$ -closed. If a set (A,E) is soft $g\zeta^*$-closed in X then $\tilde{s}\alpha cl(A,E) \setminus (A,E)$ contains only null soft $g\zeta^*$-closed set. If $x \widetilde{\notin} (A,E)$, then $\tilde{s}\alpha cl(A,E) \setminus (A,E)$ contains a nonempty soft $^{\#}g\alpha$-closed set of $\{x\}$ which is a contradiction. Hence (A,E) is soft $g\zeta^*$-closed set and $x \widetilde{\in} (A,E)$.

Case : 2 Let $\{x\}$ be soft α-open. Since $x \widetilde{\in} \tilde{s}\alpha cl(A,E), \{x\}\tilde{n}\ A,E) \neq \phi$. Hence $x \widetilde{\in}(A,E)$ which implies $\tilde{s}\alpha cl(A,E) \widetilde{\subseteq} (A,E)$. Thus $\widetilde{s}\alpha cl(A,E) = (A,E)$. Therefore (A,E) is soft α-closed. Hence X is a soft $\hat{T}_{g\zeta^*}$-space. □

6. Soft $g\zeta^*$-Continuity and Soft $g\zeta^*$-Irresoluteness

Definition 6.1. *A soft mapping $f : X \to Y$ is said to be soft $g\zeta^*$-continuous if the inverse image of each soft open set in Y is a soft $g\zeta^*$-open set in X.*

Theorem 6.2. *Every soft continuous function is soft $g\zeta^*$-continuous.*

Proof. Let $f : X \to Y$ be a soft continuous function. Let (G,B) be a soft open set in Y. Since f is soft continuous, $f^{-1}(G,B)$ is soft open in X and hence $f^{-1}(G,B)$ is soft $g\zeta^*$-open in X since every soft open set is soft $g\zeta^*$-open. Therefore, f is soft $g\zeta^*$-continuous. □

Converse need not be true as shown in the following example.

Example 6.3. *Let $X = \{a,b\}$, $Y = \{m,n\}$, $E = \{e_1,e_2,e_3\}$, $K = \{k_1,k_2,k_3\}$, $A(\subseteq E) = \{e_1,e_2\}$ and $B(\subseteq K) = \{k_1,k_2\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.*

Define $u : X \to Y$ and $p : E \to K$ as $u(a) = \{m\}$, $u(b) = \{n\}$ and $p(e_1) = k_1$, $p(e_2) = k_2$, $p(e_3) = k_3$. Let us consider the soft topology τ on X as $\tau = \{(F,A)_1, (F,A)_5, (F,A)_7, (F,A)_8, (F,A)_{16}\}$. Now consider the soft topology υ on Y as $\upsilon = \{(G,B)_1, (G,B)_6, (G,B)_{16}\}$.

Thus, the inverse images of soft closed sets in Y are $(G,B)_1$, $(G,B)_{11}$, $(G,B)_{16}$. Soft $g\zeta^*$-closed sets in X are $(F,A)_1$, $(F,A)_2$, $(F,A)_3$, $(F,A)_4$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{25}$, $(F,A)_{16}$.

Let $f : (X,\tau,E) \to (Y,\mu,K)$ be a soft mapping. Then by the definition, the inverse image of soft closed set in Y is soft $g\zeta^*$-closed in X but not soft closed in X. Therefore, f is soft $g\zeta^*$-continuous but not soft continuous.

Theorem 6.4. *Every soft α-continuous function is soft $g\zeta^*$-continuous.*

Proof. Let $f : X \to Y$ be a soft α-continuous function. Let (G,B) be a soft open set in Y. Since f is soft continuous, $f^{-1}(G,B)$ is soft α-open in X and hence $f^{-1}(G,B)$ is soft $g\zeta^*$-open in X as every soft α-open set is soft $g\zeta^*$-open . Therefore, f is soft $g\zeta^*$-continuous. □

Converse need not be true as shown in the following example.

Example 6.5. *Let $X = \{a,b\}$, $Y = \{m,n\}$, $E = \{e_1,e_2,e_3\}$, $K = \{k_1,k_2,k_3\}$, $A(\subseteq E) = \{e_1,e_2\}$ and $B(\subseteq K) = \{k_1,k_2\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces. Define $u : X \to Y$ and $p : E \to K$ as $u(a) = \{m\}$, $u(b) = \{n\}$ and $p(e_1) = k_1$, $p(e_2) = k_2$. Let us consider the soft topology τ on X as $\tau = \{(F,A)_1, (F,A)_6, (F,A)_{16}\}$. Now consider the soft topology υ on Y as $\upsilon = \{(G,B)_1, (G,B)_2, (G,B)_5, (G,B)_6, (G,B)_{16}\}$.*

Thus, the inverse images of soft closed sets in Y are $(F,A)_1$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$, $(F,A)_{16}$. Soft α-closed sets in X are $(F,A)_1$, $(F,A)_3$, $(F,A)_9$, $(F,A)_{11}$, $(F,A)_{16}$. Soft $g\zeta^*$-closed sets in X are $(F,A)_1$, $(F,A)_3$, $(F,A)_9$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$, $(F,A)_{16}$.

Let $f : (X,\tau,E) \to (Y,u,K)$ be a soft mapping. Then by the definition, the inverse image of each soft closed set in Y is soft $g\zeta^*$-closed in X but not soft α-closed in X. Therefore, f is soft $g\zeta^*$-continuous but not soft α-continuous.

Theorem 6.6. *Let $f : X \to Y$ be a mapping from soft space to X to soft space Y. Then, the following statements are equivalent*

(1). f is soft $g\zeta^*$-continuous,

(2). The inverse image of each soft closed set in Y is soft $g\zeta^*$-closed in X.

Proof. (1) $\Rightarrow$ (2): Let (G, B) be a soft closed set in Y. Then $(G, B)^C$ is soft open. Since f is soft $g\zeta^*$-continuous, $f^{-1}((G,B)^C)\tilde{\in}\tilde{s}G\zeta^*OS\,(X)$. Hence the inverse image of each soft closed set $f^{-1}((G,B))$ in Y is soft $g\zeta^*$-closed in X.
(2) $\Rightarrow$ (1): Let (O, B) be a soft open set in Y. Then $(O, B)^C$ is soft closed. Since the inverse image of each soft closed set in Y is soft $g\zeta^*$-closed in X, we have $f^{-1}((O,B)^C)\ \tilde{\in}\tilde{s}G\zeta^*CS\,(X)$. Hence $f^{-1}((O,B))$ is a soft $g\zeta^*$-open set in X. Therefore, f is soft $g\zeta^*$-continuous. □

Definition 6.7. *A soft mapping $f : X \to Y$ is said to be soft $g\zeta^*$-irresolute mapping if the inverse image of every soft $g\zeta^*$-open set of Y is a soft $g\zeta^*$-open set in X.*

Theorem 6.8. *Every soft $g\zeta^*$-irresolute mapping is soft $g\zeta^*$-continuous mapping.*

Proof. Let $f : X \to Y$ be a soft $g\zeta^*$-irresolute mapping. Let (G, B) be a soft open set in Y, hence (G,B) is soft $g\zeta^*$-open set in Y. Since f is soft $g\zeta^*$-irresolute mapping, $f^{-1}(G, B)$ is soft $g\zeta^*$-open in X. Therefore, f is soft$g\zeta^*$-continuous mapping. □

Converse need not be true as shown in the following example.

Example 6.9. *Let $X = \{a, b\}$, $Y = \{m, n\}$, $E = \{e_1, e_2, e_3\}$, $K = \{k_1, k_2, k_3\}$, $A\,(\subseteq E) = \{e_1, e_2\}$ and $B(\subseteq K) = \{k_1, k_2\}$ and let (X, τ, E) and (Y, υ, K) be soft topological spaces.*
Define $u : X \to Y$ and $p : E \to K$ as $u(a) = \{m\}$, $u(b) = \{n\}$ and $p(e_1) = k_1$, $p(e_2) = k_2$, $p(e_3) = k_3$. Let us consider the soft topology τ on X as $\tau = \{(F, A)_1, (F, A)_3, (F, A)_7, (F, A)_9, (F, A)_{11}, (F, A)_{15}, (F, A)_{16}\}$. Now consider the soft topology υ on Y as $\upsilon = \{(F, A)_1, (F, A)_3, (F, A)_7, (F, A)_{12}, (F, A)_{16}\}$.
Thus, the inverse images of soft closed sets in Y are $(F, A)_1$, $(F, A)_5$, $(F, A)_{10}$, $(F, A)_{14}$, $(F, A)_{16}$. soft $g\zeta^*$-closed sets in Y are $(F, A)_1$, $(F, A)_2$, $(F, A)_5$, $(F, A)_6$, $(F, A)_9$, $(F, A)_{10}$, $(F, A)_{13}$, $(F, A)_{14}$, $(F, A)_{16}$. Let $f : (X, \tau, E) \to (Y, u, K)$ be a soft mapping. Then by the definition, the inverse image of each soft closed set in Y is soft $g\zeta^*$-closed in X which makes clear that f is soft $g\zeta^*$-continuous. But the inverse image of every soft $g\zeta^*$-closed set in Y is not soft $g\zeta^*$-closed in X. Hence f is not soft $g\zeta^*$-irresolute mapping.

Theorem 6.10. *Let $f : (X, \tau, E) \to (Y, u, K)$, $g : (Y, \upsilon, K) \to (Z, \sigma, T)$ be two functions. Then*

(1). $g \circ f : X \to Z$ is soft $g\zeta^*$-continuous, if f is soft $g\zeta^*$-continuous and g is soft continuous.

(2). $g \circ f : X \to Z$ is soft $g\zeta^*$-irresolute, if f and g are soft $g\zeta^*$-irresolute functions.

(3). $g \circ f : X \to Z$ is soft $g\zeta^*$-continuous, if f is soft $g\zeta^*$-irresolute and g is soft $g\zeta^*$-continuous.

Proof.

(1). Let (H, T) be a soft closed set of Z. Since $g : Y \to Z$ is soft continuous, by the definition, $g^{-1}(H, T)$ is soft closed set of Y. Now $f : X \to Y$ is soft $g\zeta^*$-continuous and $g^{-1}((H, T))$ is soft closed set of Y, so by the definition, $f^{-1}\left(g^{-1}\left(H,T\right)\right) = ((g \circ f)^{-1}(H,T))$ is soft $g\zeta^*$-closed in X. Since the inverse image $(g \circ f)^{-1}(H,T))$ of soft closed set (H, T) in Z is soft $g\zeta^*$-closed in X, $g \circ f : X \to Z$ is soft $g\zeta^*$-continuous.

(2). Let $g : Y \to Z$ is soft $g\zeta^*$-irresolute and Let (H, T) be soft $g\zeta^*$-closed set of Z. Since $g : Y \to Z$ is soft $g\zeta^*$-irresolute then by the definition, $g^{-1}(H, T)$ is soft $g\zeta^*$-closed set of Y. Also $f : X \to Y$ is soft $g\zeta^*$-irresolute and $g^{-1}(H, T)$ is soft $g\zeta^*$-closed set of Y, so $f^{-1}\left(g^{-1}(H, T)\right) = (g \circ f)^{-1}(H, T)$ is soft $g\zeta^*$-closed in X. Since the inverse image $((g \circ f)^{-1}(H, T))$ of soft $g\zeta^*$-closed set (H, T) in Z is soft $g\zeta^*$-closed in X, $g \circ f : X \to Z$ is soft $g\zeta^*$-irresolute.

(3). Let (H, T) be a soft closed set of Z. Since $g : Y \to Z$ is soft $g\zeta^*$-continuous, then by the definition, $g^{-1}(H, T)$ is soft $g\zeta^*$-closed set of Y. Now $f : X \to Y$ is soft $g\zeta^*$-irresolute and $g^{-1}(H, T)$ is soft $g\zeta^*$-closed set of Y, so $f^{-1}\left(g^{-1}(H, T)\right) = (g \circ f)^{-1}(H, T)$ is soft $g\zeta^*$-closed in X. Since the inverse image $((g \circ f)^{-1}(H, T))$ of soft $g\zeta^*$-closed set (H, T) in Z is soft $g\zeta^*$-closed in X, $g \circ f : X \to Z$ is soft $g\zeta^*$- irresolute.

□

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