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On 0-Edge Magic Labeling of Some Graphs

Research Article

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Abstract: A graph $G = (V, E)$ where $V = \{v_i, 1 \leq i \leq n\}$ and $E = \{v_i v_{i+1}, 1 \leq i \leq n\}$ is 0-edge magic if there exists a bijection $f : V(G) \rightarrow \{1, -1\}$ then the induced edge labeling $f : E \rightarrow \{0\}$, such that for all $uv \in E(G)$, $f^*(uv) = f(u) + f(v) = 0$. A graph G is called *0-edge magic* if there exists a 0-edge magic labeling of G. In this paper, we determine the 0-edge magic labeling of the cartesian graphs $P_m \times P_n$ and $C_m \times C_n$, and the generalized Petersen graph $P(m, n)$.

MSC: 05C78.

Keywords: Graph Labeling, 0-Edge Magic Labeling, Magic Labeling.

1. Introduction

In 1967, the concept of graph labeling was introduced by Alex Rosa [3]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs are becoming more useful for Mathematical Methods for a broad range of applications such as coding theory, circuit design, x-ray crystallography, radar, astronomy, communication network addressing, data base managment, etc. Thus, in the preceding years, various labeling of graphs such as graceful labeling, harmonious labeling, mean labeling, arithmetic labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, etc., have been studied in over 1800 papers.

On the other hand, magic labelings were introduced by Sedlacek in 1963 [4]. For a magic type of labeling in general, we require the sum of labels related to a vertex (a vertex magic labeling) or to an edge (an edge magic labeling) to be constant all over the graph. A helpful survey to know about numerous graph labeling is the one by J.A. Gallian [1].

A graph $G = (V, E)$ where $V = \{v_i, 1 \leq i \leq n\}$ and $E = \{v_i v_{i+1}, 1 \leq i \leq n\}$ is 0-edge magic if there exists a bijection $f : V(G) \rightarrow \{1, -1\}$ then the induced edge labeling $f : E \rightarrow \{0\}$, such that for all $uv \in E(G)$, $f^*(uv) = f(u) + f(v) = 0$. A graph G is called *0-edge magic* if there exists a 0-edge magic labeling of G. In early studies it was shown that some graphs like P_n, C_n, C_n^+, G^+, the complete n-ary pseudo tree , two dimensional cylindrical meshes $P_m \times C_n$ where $n \equiv (0 \mod 2)$, n-dimensional hypercube Q_n, the graph C_m attached to mK_1, $n(m \equiv (0 \mod 2))$, the circular graph, friendship graph $C_n^{(m)}$ and the graph $P_m \times P_m \times P_m$ are 0-Edge Magic Graphs [5]. Also, some of the Splitting graphs such as *spl* (P_n), *spl* (C_n), *spl* $(K_{1,n})$, *spl* $(B_{m,n})$, and splitting graph of any tree admits 0-Edge Magic Labeling [2].

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2. Preliminaries

Definition 2.1. *A Cartesian product, denoted by $G \times H$, of two graphs G and H, is the graph with vertex $V(G \times H) = V(G) \times V(H)$ and the edge set $E(G \times H)$ satisfying the following conditions $(u_1, u_2)(v_1, v_2) \in E(G \times H)$ if and only if either $u_1 = v_1$ and $u_2v_2 \in E(H)$ or $u_2 = v_2$ and $u_1v_1 \in E(G)$.*

Definition 2.2. *The generalized Petersen graph $P(m, n)$, $m \geq 3$ and $1 \leq n \leq \left\lfloor \frac{m-1}{2} \right\rfloor$, consists of an outer m-cycle $u_0u_1 \ldots u_{m-1}$, a set of m spokes u_iv_i, $0 \leq i \leq m-1$, and m inner edges v_iv_{i+m} with indices taken modulo m.*

Definition 2.3. *A graph $G = (V, E)$ where $V = \{v_i, 1 \leq i \leq n\}$ and $E = \{v_iv_{i+1}, 1 \leq i \leq n\}$ is 0-edge magic if there exists a bijection $f : V(G) \to \{1, -1\}$ then the induced edge labeling $f : E \to \{0\}$, such that for all $uv \in E(G)$, $f^*(uv) = f(u) + f(v) = 0$. A graph G is called 0-edge magic if there exists a 0-edge magic labeling of G.*

Theorem 2.4 ([3])**.** *The circular ladder graph is 0-edge magic graph.*

3. Main Section

Theorem 3.1. *The cartesian graph $P_m \times P_n$ is 0-edge magic, for all $m, n \geq 1$.*

Proof. The cartesian graph $P_m \times P_n$ has the following vertex set and edge set. Let $V(P_m \times P_n)=\left\{v_{(1,1)}, v_{(1,2)}, ..., v_{(1,n)}\right\} \cup \left\{v_{(2,1)}, v_{(2,2)}, ..., v_{(2,n)}\right\} \cup ... \cup \left\{v_{(m,1)}, v_{(m,2)}, ..., v_{(m,n)}\right\}$ be the vertex set of $P_m \times P_n$. Let

$$A = \bigcup_{i=1}^{m} \left(\bigcup_{j=1}^{n-1} \left\{v_{(i,j)}v_{(i,j+1)}\right\} \right)$$

be the set of horizontal edges and

$$B = \bigcup_{j=1}^{n} \left(\bigcup_{i=1}^{m-1} \left\{v_{(i,j)}v_{(i+1,j)}\right\} \right)$$

be the set of vertical edges. Then, $E(P_m \times P_n)=A \cup B$ is the edge set of $P_m \times P_n$. Define the function $f : V \to \{-1, 1\}$ by

$$f(v_{i,j}) = \begin{cases} 1, & \text{if } i \text{ is odd and } j \text{ is odd or} \\ & \text{if } i \text{ is even and } j \text{ is even} \\ -1, & \text{otherwise} \end{cases}$$

for all $1 \leq i \leq m$ and $1 \leq j \leq n$. Then the induced edge labeling is defined as follows:

Case 1: m is odd and n is even.

For vertical edges $v_{(i,j)}v_{(i+1,j)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n$; i is odd, $1 \leq i \leq m-2$ and j is even, $1 \leq j \leq n$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= 1 + (-1) \\ &= 0. \end{aligned}$$

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,j)}v_{(i,j+1)}$ where i is odd, $1 \leq i \leq m$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-2$; i odd, $1 \leq i \leq m$ and j is even, $1 \leq j \leq n-1$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges of $P_m \times P_n$ are labeled with zero. Hence, the cartesian graph $P_m \times P_n$ is 0-edge magic if m is odd and n is even.

Case 2: m is odd and n is odd.

For vertical edges $v_{(i,j)}v_{(i+1,j)}$ where i is odd, $1 \leq i \leq m-2$ and j is odd, $1 \leq j \leq n$; i is even, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n$; i is even, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m-2$ and j is even, $1 \leq j \leq n-1$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,j)}v_{(i,j+1)}$ where i is odd, $1 \leq i \leq m$ and j is odd, $1 \leq j \leq n-2$; i is even, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-2$; i is even, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m$ and j is even, $1 \leq j \leq n-1$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges of $P_m \times P_n$ are labeled with zero. Hence, the cartesian graph $P_m \times P_n$ is 0-edge magic if m is odd and n is odd.

Case 3: m is even and n is even.

For vertical edges $v_{(i,j)}v_{(i+1,j)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-2$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-2$ and j is even, $1 \leq j \leq n$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,j)}v_{(i,j+1)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-2$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges of $P_m \times P_n$ are labeled with zero. Hence, the cartesian graph $P_m \times P_n$ is 0-edge magic if m is even and n is even.

Case 4: m is even n is odd

For vertical edges $v_{(i,j)}v_{(i+1,j)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n$; i is even, $1 \leq i \leq m-2$ and j is odd, $2 \leq j \leq n$; i is even, $1 \leq i \leq m-2$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-1$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,j)}v_{(i,j+1)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-2$; i is even, $i \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-2$; i is even, $1 \leq i \leq m$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-1$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges of $P_m \times P_n$ are labeled with zero. Hence, the cartesian graph $P_m \times P_n$ is 0-edge magic if m is even and n is odd. Considering all the cases above, we can say, that $P_m \times P_n$ is 0-edge magic for all $m, n \geq 1$. □

Theorem 3.2. *The cycle graph C_n is not 0-edge magic for all $n \geq 3$, n is odd.*

Proof. Let $V(C_n) = \{v_i | 1 \leq i \leq n\}$ and $E(C_n) = \{v_i v_{i+1} | 1 \leq i \leq n,\ n+1 = 1\}$. The order and the size of the cycle graph C_n is n, respectively. To prove the theorem, we will consider the following cases:

Case 1: Define the function $f : V \to \{-1, 1\}$ by

$$f(v_i) = \begin{cases} 1, & \text{if } i \text{ is odd} \\ -1, & \text{if } i \text{ is even.} \end{cases}$$

for all $1 \leq i \leq n$. The induced edge labeling is defined as follows:

For the edges $v_i v_{i+1}$ where i is odd, $1 \leq i \leq n-2$; i is even $1 \leq i \leq n-1$, we have;

$$\begin{aligned} f^*(v_i v_{i+1}) &= f(v_i) + f(v_{i+1}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_i v_{i+1}) &= f(v_i) + f(v_{i+1}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For the edges $v_n v_1$ we have;

$$\begin{aligned} f^*(v_n v_1) &= f(v_n) + f(v_1) \\ &= 1 + 1 \\ &= 2. \end{aligned}$$

Observe that not all the edges are labeled with zero.

Case 2: Define the function $f : V \to \{-1, 1\}$ by

$$f(v_i) = \begin{cases} -1, & \text{if } i \text{ is odd} \\ 1, & \text{if } i \text{ is even.} \end{cases}$$

For the edges v_iv_{i+1} where$i$ is odd, $1 \leq i \leq n-2$; i is even, $1 \leq i \leq n-1$, we have;

$$\begin{aligned} f^*(v_iv_{i+1}) &= f(v_i) + f(v_{i+1}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_iv_{i+1}) &= f(v_i) + f(v_{i+1}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For the edges v_nv_1 we have;

$$\begin{aligned} f^*(v_nv_1) &= f(v_n) + f(v_1) \\ &= -1 + (-1) \\ &= -2. \end{aligned}$$

Observe that not all the edges are labeled with zero. Considering all the cases above, we can say, that the cycle graph C_n is not 0-edge magic for all $n \geq 3$, n is odd. □

Theorem 3.3. *The cartesian graph $C_m \times C_n$ is 0-edge magic if and only if m and n are even.*

Proof. Suppose that the cartesian graph $C_m \times C_n$ is 0-edge magic, such that C_m and C_n are 0-edge magic. And m and n are not even, that is, m and n are odd, or m is even and n is odd or m is odd and n is even. For every cases of m and n mentioned, the cartesian graph $C_m \times C_n$ will always create odd cycles, such that C_m and/or C_n is an odd cycle. By Theorem 3.1.2, a Cycle graph C_n where n is odd, $n \geq 3$ is not 0-edge magic, hence, a contradiction to our assumption. Thus, the cartesian graph $C_m \times C_n$ is 0-edge magic if m and n are even.

Conversely, suppose that m and n are even. Let $V(C_m \times C_n) = \left\{v_{(i,j)} | 1 \leq 1 \leq m, 1 \leq j \leq n\right\}$. The order and size of the graph $C_m \times C_n$ are mn and $2mn$, respectively. Define the function $f : V \to \{-1, 1\}$ by

$$f(v_{i,j}) = \begin{cases} 1, & \text{if } i \text{ is odd and } j \text{ is odd or} \\ & \text{if } i \text{ is even and } j \text{ is even} \\ -1, & \text{otherwise.} \end{cases}$$

for all $1 \leq i \leq m$ and $1 \leq j \leq n$. Then the induced edge labeling is defined as follows:

For vertical edges $v_{(i,j)}v_{(i+1,j)}$ where i is odd, $1 \leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-2$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m-2$ and j is even, $1 \leq j \leq n$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i+1,j)}) &= f(v_{(i,j)}) + f(v_{(i+1,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For vertical edges $v_{(1,j)}v_{(m,j)}$ where j is odd, $1 \leq j \leq n-1$; j is even, $1 \leq j \leq n$, we have;

$$\begin{aligned} f^*(v_{(1,j)}v_{(m,j)}) &= f(v_{(1,j)}) + f(v_{(m,j)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(1,j)}v_{(m,j)}) &= f(v_{(1,j)}) + f(v_{(m,j)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,j)}v_{(i,j+1)}$ where i is odd, $\leq i \leq m-1$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m$ and j is odd, $1 \leq j \leq n-1$; i is even, $1 \leq i \leq m$ and j is even, $1 \leq j \leq n-1$; i is odd, $1 \leq i \leq m-1$ and j is even, $1 \leq j \leq n-2$, we have;

$$\begin{aligned} f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,j)}v_{(i,j+1)}) &= f(v_{(i,j)}) + f(v_{(i,j+1)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. For horizontal edges $v_{(i,1)}v_{(i,n)}$ where i is odd, $1 \leq i \leq m-1$; i is even, $1 \leq i \leq m$, we have;

$$\begin{aligned} f^*(v_{(i,1)}v_{(i,n)}) &= f(v_{(i,1)}) + f(v_{(i,n)}) \\ &= 1 + (-1) \\ &= 0. \\ f^*(v_{(i,1)}v_{(i,n)}) &= f(v_{(i,1)}) + f(v_{(i,n)}) \\ &= -1 + 1 \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges of $C_m \times C_n$ are labeled with zero. Hence, $C_m \times C_n$ is 0-edge magic for all m, n are even. Thus, the cartesian graph $C_m \times C_n$ is 0-edge magic if and only if m and n are even. ☐

Theorem 3.4. *The generalized Petersen graph $P(m,n)$ is 0-edge magic for all $m = 4k$, $k \in \mathbb{Z}^+$ and n is odd.*

Proof. Let $V(P(m,n)) = \{v_1, v_2, ..., v_{2m}\}$ where v_i, $1 \leq i \leq m$ are vertices of the outer cycle and v_i, $m+1 \leq i \leq 2m$ are the vertices of the inner cycle. The order and size of the generalized Petersen graph $P(m,n)$ are $2m$ and $3m$, respectively. Define the function $f : V \to \{-1, 1\}$ as follows:

$$f(v_i) = \begin{cases} 1, & \text{if } 1 \leq i \leq m-1, i \text{ is odd or} \\ & \text{if } m+1 \leq i \leq 2m-1, i \text{ is odd} \\ -1, & \text{otherwise} \end{cases}$$

The induced edge labels are defined as follows:

Case 1: $m = 4k$, $k \in \mathbb{Z}^+$ and $n = 1$.

If $m = 4k$, $k \in \mathbb{Z}^+$ and $n = 1$, then $P(m, 1) \cong$ Circular Ladder graph. By Theorem 2.4, $P(m, 1)$ is 0-edge magic.

Case 2: $m = 4k$, $k \in \mathbb{Z}^+$ and $n \geq 3$, n is odd.

For the edges $v_i v_{i+1}$ on the outer cycle where i is odd, $1 \leq i \leq m - 1$; i is even, $2 \leq i \leq m - 2$, we have;

$$
\begin{aligned}
f^*(v_i v_{i+1}) &= f(v_i) + f(v_{i+1}) \\
&= 1 + (-1) \\
&= 0. \\
f^*(v_i v_{i+1}) &= f(v_i) + f(v_{i+1}) \\
&= -1 + 1 \\
&= 0.
\end{aligned}
$$

respectively. For the edges $v_1 v_m$ on the outer cycle, we have;

$$
\begin{aligned}
f^*(v_1 v_m) &= f(v_1) + f(v_m) \\
&= 1 + (-1) \\
&= 0.
\end{aligned}
$$

For the spokes $v_i v_{i+m+1}$ where i is odd, $1 \leq i \leq m - 1$; i is even, $2 \leq i \leq m - 2$, we have;

$$
\begin{aligned}
f^*(v_i v_{i+m+1}) &= f(v_i) + f(v_{i+m+1}) \\
&= 1 + (-1) \\
&= 0. \\
f^*(v_i v_{i+1}) &= f(v_i) + f(v_{i+m+1}) \\
&= -1 + 1 \\
&= 0.
\end{aligned}
$$

respectively. For the spokes $v_m v_{m+1}$, we have;

$$
\begin{aligned}
f^*(v_m v_{m+1}) &= f(v_m) + f(v_{m+1}) \\
&= -1 + 1 \\
&= 0.
\end{aligned}
$$

For the edges $v_i v_{i+n}$ on the inner cycle where i is odd, $m + 1 \leq i \leq 2m - n$; i is even, $m + 2 \leq i \leq 2m - n - 1$, we have;

$$
\begin{aligned}
f^*(v_i v_{i+n}) &= f(v_i) + f(v_{i+n}) \\
&= 1 + (-1) \\
&= 0. \\
f^*(v_i v_{i+n}) &= f(v_i) + f(v_{i+n}) \\
&= -1 + 1 \\
&= 0.
\end{aligned}
$$

respectively. For the edges v_iv_{i+n-m} on the inner cycle where i is even, $2m-n+1 \leq i \leq 2m$; i is odd, $2m-n+2 \leq i \leq 2m-1$, we have;

$$\begin{aligned} f^*(v_iv_{i+n-m}) &= f(v_i) + f(v_{i+n-m}) \\ &= -1 + 1 \\ &= 0. \\ f^*(v_iv_{i+n-m}) &= f(v_i) + f(v_{i+n-m}) \\ &= 1 + (-1) \\ &= 0. \end{aligned}$$

respectively. Observe that all the edges are labeled with zero. Hence, th generalized Petersen graph $P(m,n)$ is 0-edge magic if $m = 4k$, $k \in \mathbb{Z}^+$ and n is odd, $n \geq 3$. Considering all the cases above, therefore, the generalized Petersen graph $P(m,n)$ is 0-edge magic for all $m = 4k$, $k \in \mathbb{Z}^+$ and n is odd. □

4. Conclusion

In this paper we have identified the 0-edge magic labeling of these graphs: $P_m \times P_n$, $C_m \times C_n$ and the generalized Petersen graph $P(m,n)$. Also the cycle graph C_n, where n is odd, $n \geq 3$ is not 0-edge magic.

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