International Journal of Mathematics And its Applications

# Some Common Fixed Point Theorems in Dislocated Metric Spaces 

## Research Article

Vishnu Bairagi ${ }^{1 *}$ and V. H. Badshah ${ }^{2}$<br>1 Department of Mathematics, Government M. L. B. Girls P. G. College, Indore, Madhya Pradesh, India.<br>2 School of Studies in Mathematics, Vikram University, Ujjain, Madhya Pradesh, India.


#### Abstract

In this paper, we discussed the existence and uniqueness of fixed point. The aim of this paper is to establish some new common fixed point theorems for two pairs of weakly compatible self-mappings in a dislocated metric space, which generalizes and improves similar fixed point theorems.


Keywords: Fixed point, common fixed point, dislocated metric space, weak compatible maps.
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## 1. Introduction and Preliminaries

In 2000, P. Hitzler and A.K. Seda [3], introduced the concept of dislocated topology where the initiation of dislocated metric space is appeared. After the concept of dislocated metric space many authors have established fixed point theorem in dislocated metric space, one can see many results in the field of dislocated metric space [3-11]. P. Hitzler and A.K. Seda [3], generalized the famous Banach contraction principle in this space. C. T. Aage and J. N. Salunke [1] and A. Isufati [5], established some important fixed point theorems for single and pair of mappings in dislocated metric space. G. Jungck [6], already introduced the concept of weak compatibility then many interesting fixed point theorems of compatible and weakly compatible maps under various contractive conditions have been obtained by a number of authors. In 2012, K. Jha and D. Panthi $[7,8]$ have established a common fixed point theorems for two pairs of weakly compatible mappings in dislocated metric space. In 2015 S. Bennani et al. [2], established some common fixed point theorems in dislocated metric spaces. Our result generalizes and improves the result of fixed point theorem established by S. Bennani et al. [2].

Definition 1.1 ([11]). Let $X$ be a nonempty set and let $d: X \times X \rightarrow[0, \infty)$ be a function satisfying the following conditions
(1). $d(x, y)=d(y, x)$
(2). $d(x, y)=d(y, x)=0$ implies $x=y$
(3). $d(x, y)=d(x, z)+d(z, y)$ for all $x, y, z \in X$.

Then $d$ is called dislocated metric (or simply d-metric) on $X$.

[^0]Definition $1.2([3])$. A sequence $\left\{x_{n}\right\}$ in a d-metric space $(X, d)$ is called a Cauchy sequence if for given $\epsilon>0$ there exists $n_{0} \in N$ such that for all $m, n \geq n_{0}$, we have $d\left(x_{m}, x_{n}\right)<\epsilon$.

Definition 1.3 ([3]). A sequence in a d-metric space converges with respect to $d$ if there exists $x \in X$ such that $d\left(x_{n}, x\right) \rightarrow 0$ as $n \rightarrow \infty$. In this case $x$ is called limit point of $\left\{x_{n}\right\}$ in $d$ and we write $x_{n} \rightarrow x$.

Definition 1.4 ([3]). A d-metric space $(X, d)$ is called complete if every Cauchy sequence is convergent in $d$.
Definition 1.5 ([11]). Let $A$ and $S$ be two self-mappings of a d-metric space $(X, d) . A$ and $S$ are said to be weakly compatible if they commute at their coincident point; that is, $A x=S x$ for some $x \in X$ implies $A S x=S A x$.

Definition 1.6 ([4]). Let ( $X, d$ ) be a d-metric space. A map $T: X \rightarrow X$ is called contraction mapping if there exists a number $\lambda$ with $0 \leq \lambda<1$ such that $d(T x, T y) \leq \lambda d(x, y)$ for all $x, y \in X$.

Remark 1.7. It is easy to verify that in a dislocated metric space, we have the following technical properties:
(1). A subsequence of a Cauchy sequence in d-metric space is a Cauchy sequence.
(2). A Cauchy sequence in d-metric space which possesses a convergent subsequence, converges.
(3). Limits in a d-metric space are unique.

Theorem 1.8 ([9]). Let $A, B, T$ and $S$ be four continuous self-mappings of a complete d-metric space ( $X, d$ ) such that
(1). $T X \subset A X$ and $S X \subset B X$;
(2). The pairs $(S, A)$ and $(T, B)$ are weakly compatible and
(3). $d(S x, T y) \leq \alpha d(A x, T y)+\beta d(B y, S x)+\gamma d(A x, B y)$ for all $x, y \in X$, where $\alpha, \beta, \gamma \geq 0$ satisfying $\alpha+\beta+\gamma<\frac{1}{2}$.

Then $A, B, T$ and $S$ have a unique common fixed point in $X$.

Theorem 1.9 ([7]). Let $A, B, T$ and $S$ be four continuous self-mappings of a complete d-metric space $(X, d)$ such that
(1). $T X \subset A X$ and $S X \subset B X$;
(2). The pairs $(S, A)$ and $(T, B)$ are weakly compatible;
(3). $d(S x, T y) \leq \alpha[d(A x, T y)+d(B y, S x)]+\beta[d(A x, S x)+d(B y, T y)]+\gamma d(A x, B y)$ for all $x, y \in X$ where $\alpha, \beta, \gamma \geq 0$ satisfying $\alpha+\beta+\gamma<\frac{1}{4}$.

Then $A, B, T$ and $S$ have a unique common fixed point in $X$.
Theorem 1.10 ([2]). Let $A, B, T$ and $S$ be four self-mappings of a complete $d$-metric space ( $X, d$ ) such that
(1). $T X \subset A X$ and $S X \subset B X$;
(2). The pairs ( $S, A$ ) and ( $T, B$ ) are weakly compatible;
(3). $d(S x, T y) \leq \alpha[d(A x, T y)+d(B y, S x)]+\beta[d(A x, S x)+d(B y, T y)]+\gamma d(A x, B y)$ for all $x, y \in X$, where $\alpha, \beta, \gamma \geq 0$ satisfying $\alpha+\beta+\gamma<\frac{1}{4}$.
(4). The range of one of the mapping $A, B, T$ or $S$ is a complete subspace of $X$.

Then $A, B, T$ and $S$ have a unique common fixed point in $X$.

## 2. Main results

Theorem 2.1. Let $A, B, T$ and $S$ be four self-mappings of a complete $d$-metric space ( $X, d$ ) such that
(1). $T X \subset A X$ and $S X \subset B X$;
(2). The pairs $(S, A)$ and $(T, B)$ are weakly compatible;
(3). For all $x, y \in X$

$$
\begin{equation*}
d(S x, T y) \leq \alpha[d(A x, T y)+d(B y, S x)]+\beta[d(A x, S x)+d(B y, T y)]+\gamma[d(A x, B y)+d(A x, T y)]+\eta d(B y, S x) \tag{1}
\end{equation*}
$$ where $\alpha, \beta, \gamma, \eta \geq 0$ satisfying $\alpha+\beta+\gamma+\eta<\frac{1}{4}$

(4). The range of one of the mapping $A, B, T$ or $S$ is a complete subspace of $X$.

Then $A, B, T$ and $S$ have a unique common fixed point in $X$.

Proof. Let $x_{0}$ be an arbitrary point in X. Choose $x_{1} \in X$ such that $B x_{1}=S x_{0}$. Choose $x_{2} \in X$ such that $A x_{2}=T x_{1}$. Continuing in this way, choose $x_{n} \in X$ such that $S x_{2 n}=B x_{2 n+1}$ and $T x_{2 n+1}=A x_{2 n+2}$ for $n=0,1,2, \ldots$ To simplify, we consider the sequence $\left\{y_{n}\right\}$ defined by $y_{2 n}=S x_{2 n}$ and $y_{2 n+1}=T x_{2 n+1}$ for $n=0,1,2, \ldots$.

We claim that $\left\{y_{n}\right\}$ is a Cauchy sequence. Indeed, by using (1) for $n \geq 1$, we have

$$
\begin{aligned}
d\left(y_{2 n+1}, y_{2 n}\right)= & d\left(S x_{2 n}, T x_{2 n+1}\right) \\
\leq & \alpha\left[d\left(A x_{2 n}, T x_{2 n+1}\right)+d\left(B x_{2 n+1}, S x_{2 n}\right)\right]+\beta\left[d\left(A x_{2 n}, S x_{2 n}\right)+d\left(B x_{2 n+1}, T x_{2 n+1}\right)\right] \\
& +\gamma\left[d\left(A x_{2 n}, B x_{2 n+1}\right)+d\left(A x_{2 n}, T x_{2 n+1}\right)\right]+\eta d\left(B x_{2 n+1}, S x_{2 n}\right) \\
\leq & \alpha\left[d\left(y_{2 n-1}, y_{2 n+1}\right)+d\left(y_{2 n}, y_{2 n}\right)\right]+\beta\left[d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n}, y_{2 n+1}\right)\right] \\
& +\gamma\left[d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n-1}, y_{2 n+1}\right)\right]+\eta d\left(y_{2 n}, y_{2 n}\right) \\
\leq & \alpha\left[d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n}, y_{2 n+1}\right)+d\left(y_{2 n}, y_{2 n+1}\right)+d\left(y_{2 n+1}, y_{2 n}\right)\right]+\beta\left[d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n}, y_{2 n+1}\right)\right] \\
& +\gamma\left[d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n-1}, y_{2 n}\right)+d\left(y_{2 n}, y_{2 n+1}\right)\right]+\eta\left[d\left(y_{2 n}, y_{2 n-1}\right)+d\left(y_{2 n-1}, y_{2 n}\right)\right] \\
\leq & (\alpha+\beta+2 \gamma+2 \eta) d\left(y_{2 n-1}, y_{2 n}\right)+(3 \alpha+\beta+\gamma) d\left(y_{2 n}, y_{2 n+1}\right) \\
d\left(y_{2 n}, y_{2 n+1}\right) \leq & h d\left(y_{2 n-1}, y_{2 n}\right)
\end{aligned}
$$

where $h=\frac{(\alpha+\beta+\gamma+2 \eta)}{(1-3 \alpha-\beta-\gamma)}<1$. This shows that

$$
d\left(y_{n}, y_{n+1}\right) \leq h d\left(y_{n-1}, y_{n}\right) \leq h^{2} d\left(y_{n-2}, y_{n-1}\right) \leq h^{3} d\left(y_{n-3}, y_{n-2}\right) \cdots \leq h^{n} d\left(y_{0}, y_{1}\right)
$$

Thus for every integer $q>0$, we have

$$
\begin{aligned}
d\left(y_{n}, y_{n+q}\right) & \leq d\left(y_{n}, y_{n+1}\right)+d\left(y_{n+1}, y_{n+2}\right)+d\left(y_{n+2}, y_{n+3}\right) \cdots+d\left(y_{n+q-1}, y_{n+q}\right) \\
& \leq h^{n} d\left(y_{0}, y_{1}\right)+h^{n+1} d\left(y_{0}, y_{1}\right)+h^{n+2} d\left(y_{0}, y_{1}\right) \cdots+h^{n+q-1} d\left(y_{0}, y_{1}\right) \\
& \leq h^{n}\left[1+h+h^{2}+h^{3} \cdots+h^{q-1}\right] d\left(y_{0}, y_{1}\right) \\
& \leq \frac{h^{n}}{1-h} d\left(y_{0}, y_{1}\right)
\end{aligned}
$$

Since, $0<h<1, h^{n} \rightarrow 0$ as $n \rightarrow \infty$. So we get $d\left(y_{n}, y_{n+q}\right) \rightarrow 0$. This implies that $\left\{y_{n}\right\}$ is a Cachy sequence in a complete dislocated metric space $(X, d)$ and therefore, according to Remarks 1.7, $\left\{S x_{2 n}\right\},\left\{B x_{2 n+1}\right\},\left\{T x_{2 n+1}\right\}$ and $\left\{A x_{2 n+2}\right\}$ are also Cauchy sequence. Suppose that SX is a complete subspace of X , then the sequence $\left\{S x_{2 n}\right\}$ converges to some Sx such that $x \in X$. According to Remark 1.7, $\left\{y_{n}\right\},\left\{B x_{2 n+1}\right\},\left\{T x_{2 n+1}\right\}$ and $\left\{A x_{2 n+2}\right\}$ converge to Sx. Since $S X \subset B X$, there exists $u \in X$ such that $S x=B u$. We show that $B u=T u$. In fact, by using (1), we have
$d\left(S x_{2 n}, T u\right) \leq \alpha\left[d\left(A x_{2 n}, T u\right)+d\left(B u, S x_{2 n}\right)\right]+\beta\left[d\left(A x_{2 n}, S x_{2 n}\right)+d(B u, T u)\right]+\gamma\left[d\left(A x_{2 n}, B u\right)+d\left(A x_{2 n}, T u\right)\right]+\eta d\left(B u, S x_{2 n}\right)$
and therefore, on letting $n \rightarrow \infty$, we get

$$
\begin{aligned}
d(B u, T u) & \leq \alpha[d(B u, T u)+d(B u, B u)]+\beta[d(B u, B u)+d(B u, T u)+\gamma[d(B u, B u)+d(B u, T u)]+\eta d(B u, B u) \\
& \leq(\alpha+\beta+\gamma+\eta) d(B u, B u)+(\alpha+\beta+\gamma) d(B u, T u) \\
& \leq 2(\alpha+\beta+\gamma+\eta) d(B u, T u)+(\alpha+\beta) d(B u, T u) \\
& \leq(3 \alpha+3 \beta+2 \gamma+2 \eta) d(B u, T u)
\end{aligned}
$$

which implies that $(1-3 \alpha-3 \beta-2 \gamma-2 \eta) d(B u, T u) \leq 0$ and therefore $d(B u, T u)=0$, since $(1-3 \alpha-3 \beta-2 \gamma-2 \eta)<0$, which implies that $T u=B u$. Since $T X \subset A X$, there exists $v \in X$ such that $T u=A v$. We show that $S v=A v$. Indeed, by using (1), we have

$$
\begin{aligned}
d(S v, A v) & =d(S v, T u) \\
& \leq \alpha[d(A v, T u)+d(B u, S v)]+\beta[d(A v, S v)+d(B u, T u)]+\gamma[d(A v, B u)+d(A v, T u)]+\eta d(B u, S v) \\
& \leq \alpha[d(A v, A v)+d(A v, S v)]+\beta[d(A v, S v)+d(A v, A v)]+\gamma[d(A v, A v)+d(A v, A v)]+\eta d(A v, S v) \\
& \leq(\alpha+\beta+2 \gamma) d(A v, A v)+(\alpha+\beta+\eta) d(S v, A v) \\
& \leq 2(\alpha+\beta+2 \gamma) d(S v, A v)+(\alpha+\beta+\eta) d(S v, A v) \\
& \leq(3 \alpha+3 \beta+4 \gamma+\eta) d(A v, S v)
\end{aligned}
$$

which implies that $(1-3 \alpha-3 \beta-4 \gamma-\eta) d(A v, S v) \leq 0$ and therefore $d(A v, S v)=0$, since $1-3 \alpha-3 \beta-4 \gamma-\eta<0$, which implies that $A v=S v$. Hence $B u=T u=A v=S v$. The weak compatibility of $S$ and $A$ implies that $A S v=S A v$, from which it follows that $A A v=A S v=S A v=S S v$. The weak compatibility of $B$ and $T$ implies that $B T u=T B u$, from which it follows that $B B u=B T u=T B u=T T u$. Let us show that Bu is a fixed point of $T$. Indeed, from (1), we get

$$
\begin{aligned}
d(B u, T B u) & =d(S v, T B u) \\
& \leq \alpha[d(A v, T B u)+d(B B u, S v)]+\beta[d(A v, S v)+d(B B u, T B u)]+\gamma[d(A v, B B u)+d(A v, T B u)]+\eta d(B B u, S v) \\
& \leq \alpha[d(B u, T B u)+d(T B u, B u)]+\beta[d(B u, B u)+d(T B u, T B u)]+\gamma[d(B u, T B u)+d(B u, T B u)]+\eta d(T B u, B u) \\
& \leq 2 \alpha d(B u, T B u)+\beta[d(B u, T B u)+d(T B u, B u)+d(T B u, B u)+d(B u, T B u)]+2 \gamma d(B u, T B u)+\eta d(T B u, B u) \\
& \leq(2 \alpha+4 \beta+2 \gamma+\eta) d(B u, T B u)
\end{aligned}
$$

and therefore $d(B u, T B u)=0$, since $1-2 \alpha-4 \beta-2 \gamma-\eta<0$, which implies that $T B u=B u$. Hence Bu is a fixed point of $T$. It follows that $B B u=T B u=B u$, which implies that $B u$ is a fixed point of $B$. On the other hand, in view of (1), we
have

$$
\begin{align*}
d(S B u, B u)= & d(S B u, T B u) \\
\leq & \alpha[d(A B u, T B u)+d(B B u, S B u)]+\beta[d(A B u, S B u)+d(B B u, T B u)] \\
& \quad+\gamma[d(A B u, B B u)+d(A B u, T B u)]+\eta d(B B u, S B u)  \tag{4}\\
\leq & \alpha[d(S B u, B u)+d(B u, S B u)]+\beta[d(S B u, S B u)+d(B u, B u)]+\gamma[d(S B u, B u)+d(S B u, B u)]+\eta d(B u, S B u) \\
\leq & 2 \alpha d(B u, S B u)+\beta[d(B u, S B u)+d(S B u, B u)+d(B u, S B u)+d(S B u, B u)] \\
& \quad+\gamma[d(B u, S B u)+d(S B u, B u)]+\eta d(B u, S B u) \\
\leq & (2 \alpha+4 \beta+2 \gamma+\eta) d(B u, S B u)
\end{align*}
$$

and therefore $d(B u, S B u)=0$, since $1-2 \alpha-4 \beta-2 \gamma<0$, which implies that $S B u=B u$. Hence $B u$ is a fixed point of $S$. It follows that $A B u=S B u=B u$, which implies that $B u$ is also a fixed point of $A$. Thus $B u$ is a common fixed point of $S$, $T, A$ and $B$.

Finally, to prove uniqueness, suppose that there exists $u, v \in X$ such that $S u=T u=A u=B u=u$ and $S v=T v=$ $A v=B v=v$. If $d(u, v) \neq 0$, then, by using (1), we get

$$
\begin{align*}
d(u, v) & =d(S u, T v) \\
& \leq \alpha[d(A u, T v)+d(B v, S u)]+\beta[d(A u, S u)+d(B v, T v)]+\gamma[d(A u, B v)+d(S u, T v)]+\eta d(B v, S u)  \tag{5}\\
& \leq \alpha[d(u, v)+d(u, v)]+\beta[d(u, u)+d(v, v)]+\gamma[d(u, v)+d(u, v)]+\eta d(v, u) \\
& \leq(2 \alpha+4 \beta+2 \gamma+\eta) d(u, v)
\end{align*}
$$

from which it follows that $(1-2 \alpha-4 \beta-2 \gamma-\eta) d(u, v) \leq 0$ which is a contradiction since $1-2 \alpha-4 \beta-2 \gamma-\eta<0$. Hence $d(u, v)=0$ and therefore $u=v$. The proof is similar when $T X$ or $A X$ or $B x$ is a complete subspace of $X$. This completes the proof.

For $A=B$ and $S=T$ in (1), we have the following result.

Corollary 2.2. Let $(X, d)$ be a d-metric space. Let $A$ and $T$ be two self-mappings of $X$ such that
(1). $T X \subset A X$
(2). The pair ( $T, A$ ) is weakly compatible and
(3). $d(T x, T y) \leq \alpha[d(A x, T y)+d(A y, T x)]+\beta[d(A x, T x)+d(A y, T y)]+\gamma[d(A x, A y)+d(A x, T y)]+\eta d(A y, T x) \quad$ for all $x, y \in X$ where $\alpha, \beta, \gamma, \eta \geq 0$ satisfying $\alpha+\beta+\gamma+\eta<\frac{1}{4}$.
(4). TX or $A X$ is a complete subspace of $X$.

Then $A$ and $T$ have a unique common fixed point in $X$.

For $A=B=I d_{X}$ in (1), we get the following corollary.

Corollary 2.3. Let $(X, d)$ be a d-metric space. Let $T$ and $S$ be two self-mappings of $X$ such that
(1). $d(S x, T y) \leq \alpha[d(x, T y)+d(y, S x)]+\beta[d(x, S x)+d(y, T y)]+\gamma[d(x, y)+d(x, T y)]+\eta d(y, S x) \quad$ for $\quad$ all $\quad x, y \in X \quad$ where $\alpha, \beta, \gamma, \eta \geq 0$ satisfying $\alpha+\beta+\gamma+\eta<\frac{1}{4}$.
(2). $T X$ or $S X$ is a complete subspace of $X$.

Then $T$ and $S$ have a unique common fixed point in $X$.

For $S=T=I d_{X}$ in (1), we have the following result.

Corollary 2.4. Let $(X, d)$ be a complete d-metric space. Let $A$ and $B$ be two surjective self-mappings of $X$ such that

$$
d(x, y) \leq \alpha[d(A x, y)+d(B y, x)]+\beta[d(A x, x)+d(B y, y)]+\gamma[d(A x, B y)+d(A x, y)]+\eta d(B y, x)
$$

for all $x, y \in X$ where $\alpha, \beta, \gamma, \eta \geq 0$ satisfying $\alpha+\beta+\gamma+\eta<\frac{1}{4}$. Then $A$ and $B$ have a unique common fixed point in $X$.

Remark 2.5. Following the procedure used in the proof of Theorem in 2.1, we have the next new result in which we replace the condition $\alpha+\beta+\gamma+\eta<\frac{1}{4}$ by $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$, for $\alpha, \beta, \gamma, \eta>0$.

Theorem 2.6. Let $A, B, T$ and $S$ be four self-mappings of a d-metric space ( $X, d$ ) such that
(1). $T X \subset A X$ and $S X \subset B X$
(2). The pairs $(S, A)$ and $(T, B)$ are weakly compatible and
(3). $d(S x, T y) \leq \alpha[d(A x, T y)+d(B y, S x)]+\beta[d(A x, S x)+d(B y, T y)]+\gamma[d(A x, B y)+d(A x, T y)]+\eta d(B y, S x)$ for all $x, y \in$ $X, \alpha, \beta, \gamma$ and $\eta>0$ satisfying $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$.
(4). The range of one of the mappings $A, B, S$ or $T$ is a complete subspace of $X$.

Then $A, B, T$ and $S$ have a unique common fixed point in $X$.

Proof. Theorem 2.6 may be proved by putting $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$ instead of condition $\alpha+\beta+\gamma+\eta<\frac{1}{4}$ for $\alpha$, $\beta$, $\gamma$ and $\eta>0$.

For $A=B$ and $S=T$ in Theorem 2.6, we have the following result.

Corollary 2.7. Let $(X, d)$ be a d-metric space. Let $A$ and $T$ be two self-mappings of $X$ such that
(1). $T X \subset A X$
(2). The pair (T, A) is weakly compatible and
(3). $d(T x, T y) \leq \alpha[d(A x, T y)+d(A y, T x)]+\beta[d(A x, T x)+d(A y, T y)]+\gamma[d(A x, A y)+d(A x, T y)]+\eta d(A y, T x)$ for all $x, y \in X$ where $\alpha, \beta, \gamma$ and $\eta>0$ satisfying $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$.
(4). TX or $A X$ is a complete subspace of $X$.

Then $A$ and $T$ have a unique common fixed point in $X$.

For $A=B=I d_{X}$ in (1), we get the following corollary.

Corollary 2.8. Let $(X, d)$ be a d-metric space. Let $T$ and $S$ be two self-mappings of $X$ such that

$$
d(S x, T y) \leq \alpha[d(x, T y)+d(y, S x)]+\beta[d(x, S x)+d(y, T y)]+\gamma[d(x, y)+d(x, T y)]+\eta d(y, S x)
$$

for all $x, y \in X$ where $\alpha, \beta, \gamma$ and $\eta>0$ satisfying $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$. Then $T$ and $S$ have a unique common fixed point in $X$.

For $S=T=I d_{X}$ in Theorem 2.6, we have the following result.

Corollary 2.9. Let $(X, d)$ be a complete d-metric space. Let $A$ and $B$ be two surjective self-mappings of $X$ such that

$$
d(x, y) \leq \alpha[d(A x, y)+d(B y, x)]+\beta[d(A x, x)+d(B y, y)]+\gamma[d(A x, B y)+d(A x, y)]+\eta d(B y, x)
$$

for all $x, y \in X$ where $\alpha, \beta, \gamma$ and $\eta>0$ satisfying $\alpha+\beta+\gamma+\eta \leq \frac{1}{4}$. Then $A$ and $B$ have a unique common fixed point in $X$.

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[^0]:    * E-mail: vishnuprasadbairagi@yahoo.in

