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Properties of Double Fuzzy t-irresolute Multifunctions

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Abstract: In this paper, we introduce and study the concept of upper and lower double fuzzy t-irresolute multifunction via fuzzy t-open set.

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1.   Introduction

The notion of fuzzy set was introduced by L.A. Zadeh in the year 1965. Since than it has been applied in almost all the branches of science and technology, where set theory and mathematical logic play an important role. This newly introduced concept opened lot of scope and directions for investigations in many ways in all the branches for research. Recently fuzzy set theory has been applied and fuzzy topological spaces have been studied by Kubiak [10] and Sostak [16] introduced the notion of (L-)fuzzy topological space as a generalization of L-topological spaces (originally called (L-)fuzzy topological spaces by Chang [5] and Goguen [9]). Berge [4] introduced the concept multimapping $F : (X, \tau) \to (Y, \sigma)$ where X and Y are topological spaces. After Chang introduced the concept of fuzzy topology [5], continuity of multifunctions in fuzzy topological spaces have been defined and studied by many authors from different view points (e.g. see [2, 3, 12–14]). Tsiporkova et. al., [17, 18] introduced the continuity of fuzzy multivalued mappings in the Chang's fuzzy topology [5]. On the other hand, as a generalization of fuzzy topological spaces Samanta and Mondal [15], introduced the concept of intuitionistic gradation of openness. In 2005, the term intuitionistic is ended by Garcia and Rodabaugh [8]. They proved that the term intuitionistic is unsuitable in mathematics and applications and they replaced it by double. In this paper, we introduce and study the concept of upper and lower t-irresolute double fuzzy multifunction via fuzzy t-open set.

2.   Preliminaries

Throughout this paper, Let X be a non-empty set, I the unit interval $[0, 1]$, $I_0 = (0, 1]$ and $I_1 = [0, 1)$. The family of all fuzzy sets on X is denoted by I^X. By $\bar{0}$ and $\bar{1}$, we denote the smallest and the greatest fuzzy sets on X. For a fuzzy set $\lambda \in I^X$, $\bar{1} - \lambda$ denotes its complement.

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Definition 2.1. *A fuzzy point x_t in X is a fuzzy set taking value $t \in I_0$ at x and zero elsewhere, $x_t \in \lambda$ if and only if $t \leq \lambda(x)$. A fuzzy set λ is quasicoincident with a fuzzy set μ, denoted by $\lambda q \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. Otherwise $\lambda \bar{q} \mu$.*

Definition 2.2 ([6, 15])**.** *A double fuzzy topology on X is a pair of maps $\tau, \tau^\star : I^X \to I$, which satisfies the following properties:*

(1). $\tau(\lambda) \leq \bar{1} - \tau^\star(\lambda)$ for each $\lambda \in I^X$.

(2). $\tau(\lambda_1 \wedge \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$ and $\tau^\star(\lambda_1 \wedge \lambda_2) \leq \tau^\star(\lambda_1) \vee \tau^\star(\lambda_2)$ for each $\lambda_1, \lambda_2 \in I^X$.

(3). $\tau(\bigvee_{i \in \Gamma} \lambda_i) \geq \bigwedge_{i \in \Gamma} \tau(\lambda_i)$ and $\tau^\star(\bigvee_{i \in \Gamma} \lambda_i) \leq \bigvee_{i \in \Gamma} \tau^\star(\lambda_i)$ for each $\lambda_i \in I^X, i \in \Gamma$.

The triplet $(X, \tau, \tau^\star)$ is called a double fuzzy topological space.

Definition 2.3 ([6, 15])**.** *Let $(X, \tau, \tau^\star)$ be a double fuzzy topological space and $\lambda \in I^X$, $r \in I_0$ and $s \in I_1$ such that $r + s \leq 1$. Then the fuzzy set λ is called an (r, s)-fuzzy open if $\tau(\lambda, r, s) \geq r$ and $\tau^\star(\lambda, r, s) \leq s$, λ is called an (r, s)-fuzzy closed if, and only if $\bar{1} - \lambda$ is an (r, s)-fuzzy open set.*

Theorem 2.4 ([7, 11])**.** *Let $(X, \tau, \tau^\star)$ be a double fuzzy topological space. Then double fuzzy closure operator and double fuzzy interior operator of $\lambda \in I^X$ are defined by*

$$\mathrm{Cl}(\lambda, r, s) = \bigwedge \{\mu \in I^X \mid \lambda \leq \mu, \tau(\bar{1} - \mu) \geq r, \tau^\star(\bar{1} - \mu) \leq s\},$$
$$\mathrm{Int}(\lambda, r, s) = \bigvee \{\mu \in I^X \mid \mu \leq \lambda, \tau(\mu) \geq r, \tau^\star(\mu) \leq s\},$$

where $r \in I_0$ and $s \in I_1$ such that $r + s \leq 1$.

Definition 2.5 ([19])**.** *A subset A of a topological space (X, τ) is said to be a t-set if $\mathrm{Int}(\mathrm{Cl}(A)) = \mathrm{Int}(A)$.*

In this paper we will use the term t-open set instead of t-set. The complement of t-open set is called a t-closed set. Also the notions $t\,\mathrm{Cl}(A)$ and $t\,\mathrm{Int}(A)$ for a subset A of a topological space (X, τ) is defined as in the similar manner of $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

3. Double Fuzzy t-irresolute Multifunctions

In this paper, we introduce and study the concept of upper and lower t-irresolute double fuzzy multifunction via fuzzy t-open set.

Definition 3.1. *Let (X, τ) be a topological space in the classical sense and $(Y, \sigma, \sigma^\star)$ be a double fuzzy topological space. A function $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ is called a double fuzzy multifunction if for each $x \in X$, $F(x) \in I^Y$.*

Definition 3.2. *For a double fuzzy multifunction $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ the upper inverse $F^+(\lambda)$ and the lower inverse $F^-(\lambda)$ of $\lambda \in I^Y$ are defined as follows: $F^+(\lambda) = \{x \in X : F(x) \leq \lambda\}$ and $F^-(\lambda) = \{x \in X : F(x) + \lambda(x) > 1\}$.*

Definition 3.3. *The fuzzy subset $\lambda \in I^Y$ in a double fuzzy topological space $(Y, \sigma, \sigma^\star)$ is said to be an (r, s)-fuzzy t-open set if $\mathrm{Int}(\lambda, r, s) = \mathrm{Int}(\mathrm{Cl}(\lambda, r, s), r, s)$. The complement of an (r, s)-fuzzy t-open set is an (r, s)-fuzzy t-closed set.*

Definition 3.4. *Let $(Y, \sigma, \sigma^\star)$ ba a double fuzzy topological space. For any $\lambda \in I^Y$, the (r, s)-fuzzy t-interior of λ is defined by, $t\,\mathrm{Int}(\lambda, r, s) = \vee\{\mu : \mu \leq \lambda, \mu$ is (r, s)-fuzzy t-open$\}$.*

Definition 3.5. *Let* $(Y, \sigma, \sigma^\star)$ *ba a double fuzzy topological space. For any* $\lambda \in I^Y$, *the* (r, s)-*fuzzy* t-*closure of* λ *is defined by,* $t \operatorname{Int}(\lambda, r, s) = \wedge\{\mu : \mu \geq \lambda, \, \mu \text{ is } (r, s)\text{-fuzzy } t\text{-closed}\}$.

Definition 3.6. *Let* $(Y, \sigma, \sigma^\star)$ *be a double fuzzy topological space and* $\lambda \in I^Y$. *Then* λ *is called a double fuzzy* t-*neighborhood of a fuzzy point* y_α *in* Y *if there exists an* (r, s)-*fuzzy* t-*open set* $\mu \in I^Y$ *such that* $y_\alpha \in \mu \leq \lambda$.

Definition 3.7. *A double fuzzy multifunction* $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ *is said to be:*

(1). double fuzzy upper t-*irresolute at a point* $x \in X$ *if for each* (r, s)-*fuzzy* t-*open set* $\lambda \in I^Y$ *with* $F(x) \leq \lambda$, *there exists a* t-*open set* U *in* X *with* $x \in U$ *such that* $F(U) \leq \lambda$.

(2). double fuzzy lower t-*irresolute at a point* $x \in X$ *if for each* (r, s)-*fuzzy* t-*open set* $\lambda \in I^Y$ *with* $F(x) \, q \, \lambda$, *there exists a* t-*open set* U *in* X *with* $x \in U$ *such that* $U \subset F^-(\lambda)$.

(3). double fuzzy upper t-*irresolute (resp. double fuzzy lower* t-*irresolute) if it is double fuzzy upper* t-*irresolute (resp. double fuzzy lower* t-*irresolute) at each* $x \in X$.

Proposition 3.8. *For a fuzzy multifunction* $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$, *the following statements are equivalent:*

(1). F is double fuzzy upper t-*irresolute at a point* $x \in X$.

(2). For each (r, s)-*fuzzy* t-*open set* $\lambda \in I^Y$ *with* $F(x) \leq \lambda$, $x \in t \operatorname{Int}(F^+(\lambda))$.

(3). For each (r, s)-*fuzzy* t-*open set* $\lambda \in I^Y$ *with* $F(x) \leq \lambda$, *there exists a* t-*open set* U *in* X *such that* $x \in U \subset t \operatorname{Cl}(F^+(\lambda))$.

Proof. $(1) \Leftrightarrow (2)$: Let $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set with $F(x) \leq \lambda$. Since F is double fuzzy upper t-irresolute at a point $x \in X$, there exists a t-open set U in X with $x \in U$ such that $F(U) \leq \lambda$. That is, $U \leq F^+(\lambda)$. Now, $x \in U = t \operatorname{Int}(U)$, implies that $x \in t \operatorname{Int}(F^+(\lambda))$. Conversely, let λ be an (r, s)-fuzzy t-open set with $F(x) \leq \lambda$. Then $x \in t \operatorname{Int}(F^+(\lambda)) = U$. Now, U is a t-open set in X with $x \in U$. Now $F(U) = \bigvee_{x \in U} F(x) \leq \lambda$. This implies that F is double fuzzy upper t-irresolute at a point $x \in X$.

$(3) \Leftrightarrow (1)$: Let λ be an (r, s)-fuzzy t-open set with $F(x) \leq \lambda$. This implies $x \in F^+(\lambda)$. By (2), it follows that $x \in t \operatorname{Int}(F^+(\lambda)) = U$. Now, U is a t-open set in X with $x \in U$ and $U \subset F^+(\lambda)$. Hence it follows that $x \in U \subset t \operatorname{Cl}(F^+(\lambda))$. Conversely, let $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set with $F(x) \leq \lambda$. Then by (3), there exists a t-open set U in X with $x \in U \subset t \operatorname{Int}(U) \subset t \operatorname{Int}(F^+(\lambda))$. Hence $x \in t \operatorname{Int}(F^+(\lambda))$. $\qquad \square$

Theorem 3.9. *Let* $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ *be a double fuzzy multifunction. Then the following statements are equivalent:*

(1). F is double fuzzy lower t-*irresolute at a point* $x \in X$.

(2). For each (r, s)-*fuzzy* t-*open set* λ *with* $F(x) \, q \, \lambda$, $x \in t \operatorname{Int}(F^-(\lambda))$.

(3). For each (r, s)-*fuzzy* t-*open set* λ *with* $F(x) \, q \, \lambda$, *there exists a* t-*open set* U *in* X *such that* $x \in U \subset t \operatorname{Cl}(F^-(\lambda))$.

Proof. $(1) \Rightarrow (2)$: Let λ be a (r, s)-fuzzy t-open set with $F(x) \, q \, \lambda$. Since F is double fuzzy lower t-irresolute at a point $x \in X$, there exists a t-open set U in X with $x \in U$ such that $U \subset F^-(\lambda)$. Now $x \in U = t \operatorname{Int}(U)$ implies that $x \in t \operatorname{Int}(F^-(\lambda))$.

$(2) \Rightarrow (3)$: Let λ be an (r, s)-fuzzy t-open set with $F(x) \, q \, \lambda$. This implies that $x \in F^-(\lambda)$. By (2), it follows that $x \in t \operatorname{Int}(F^-(\lambda)) = U$. Now U is a t-open set in X with $x \in U$ and $U \subset F^-(\lambda)$. Hence $x \in U \subset t \operatorname{Cl}(F^-(\lambda))$.

$(3) \Rightarrow (1)$: Let $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set with $F(x) \, q \, \lambda$. Then by (3), there exists a t-open set U in X with $x \in U \subset t \operatorname{Cl}(F^-(\lambda))$. Now, $F(U) = \bigvee_{x \in U} F(x) \geq \bar{1} - \lambda$. This implies that F is double fuzzy lower t-irresolute at a point $x \in X$. $\qquad \square$

Theorem 3.10. *Let $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ be a double fuzzy multifunction. Then the following statements are equivalent:*

(1). F is a double fuzzy upper t-irresolute function.

(2). For each point x of X and each double fuzzy $Q^\ast t$-neighborhood λ of $F(x)$, $F^+(\lambda)$ is a t-neighborhood of x.

(3). For each point x of X and each double fuzzy $Q^\ast t$-neighborhood λ of $F(x)$, there exists a t-neighborhood U of x such that $F(U) \leq \lambda$.

(4). $F(\lambda)$ is a t-open set in X for each (r, s)-fuzzy t-open set $\lambda \in I^Y$.

(5). $F^-(\delta)$ is a t-closed set in X for each (r, s)-fuzzy t-closed set $\delta \in I^Y$.

(6). $t\operatorname{Cl}(F^-(\mu)) \subset F^-(t\operatorname{Cl}(\mu, r, s))$ for every $\mu \in I^Y$.

Proof. (1)$\Rightarrow$(2): Let $x \in X$ and λ be a double fuzzy $Q^\ast t$-neighborhood of $F(x)$. Then there exists an (r, s)-fuzzy t-open set μ such that $F(x) \leq \mu \leq \lambda$. Since F is double fuzzy upper t-irresolute, there exists a t-open set U in X with $x \in U$ such that $F(U) \leq \lambda$. That is, $U \subset F^+(\lambda)$. Now $x \in U \subset F^+(\lambda)$. Hence $F^+(\lambda)$ is a t-neighborhood of x.

(2)$\Rightarrow$(3): Let $x \in X$ and λ be a double fuzzy $Q^\ast t$-neighborhood of $F(x)$. By (2), $F^+(\lambda)$ is a t-neighborhood of x. Put $U = F^+(\lambda)$. Now $F(U) = \bigvee\limits_{x \in U} F(x) \leq \lambda$.

(3)$\Rightarrow$(4): Let $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set. Let $x \in F^+(\lambda)$. By (3), there exists a t-neighborhood G of x such that $F(G) \leq \lambda$. Now there exists a t-open set U in X such that $x \in U \subset G$. Then $F(U) \leq F(G) \leq \lambda$. That is, $x \in U \subset F^+(\lambda)$. Hence $F(\lambda)$ is a t-open set in X.

(4)$\Rightarrow$(5): Let $\delta \in I^Y$ be an (r, s)-fuzzy t-closed set. Then $1 - \delta$ is an (r, s)-fuzzy t-open set. By (4), $F^+(1 - \delta)$ is a t-open set in X. Hence $F^-(\delta)$ is a t-closed set in X.

(5)$\Rightarrow$(6): Let $\mu \in I^Y$. Then $t\operatorname{Cl}(\mu, r, s)$ is an (r, s)-fuzzy t-closed set. By (5), $F(t\operatorname{Cl}(\mu, r, s))$ is an (r, s)-fuzzy t-closed set. Now $\mu \leq t\operatorname{Cl}(\mu, r, s)$. Hence $F^-(\mu) \subset F^-(t\operatorname{Cl}(\mu, r, s))$. Now $t\operatorname{Cl}(F^-(\mu)) \subset t\operatorname{Cl}(F^-(t\operatorname{Cl}(\mu, r, s))) = F^-(t\operatorname{Cl}(\mu, r, s))$. So $t\operatorname{Cl}(F^-(\mu)) \subset F^-(t\operatorname{Cl}(\mu, r, s))$.

(6)$\Rightarrow$(1): Let $x \in X$ and $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set such that $F(x) \leq \lambda$. Now $F^-(\bar{1} - \lambda) = \{x \in X : F(x) \, q \, \bar{1} - \lambda\}$. Suppose $x \notin F^-(\bar{1} - \lambda)$, then $F(x) + (\bar{1} - \lambda) \leq \bar{1}$. That is, $F(x) \leq \lambda$. Therefore, $x \notin F^-(\bar{1} - \lambda)$. That is, $x \notin F^-(t\operatorname{Cl}(\bar{1} - \lambda, r, s))$. By (6), it follows that $x \notin t\operatorname{Cl}(F^-(\bar{1} - \lambda, r, s))$. Then there exists a t-open set U in X with $x \in U$ such that $U \cap (F^-(\bar{1} - \lambda) = \emptyset$. That is, $U \cap (X \backslash F^+(\lambda)) = \emptyset$. Therefore, $F(U) = \bigvee\limits_{x \in U} F(x) \leq \lambda$. Hence F is a double fuzzy upper t-irresolute function. $\qquad\square$

Theorem 3.11. *Let $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ be a double fuzzy multifunction. Then the following statements are equivalent:*

(1). F is a double fuzzy lower t-irresolute function.

(2). For each (r, s)-fuzzy t-open set λ and each $x \in F^-(\lambda)$, there exists a t-open set U in X with $x \in U$ such that $U \subset F^-(\lambda)$.

(3). $F^-(\lambda)$ is a t-open set in X for each (r, s)-fuzzy t-open set $\lambda \in I^Y$.

(4). $F^+(\delta)$ is a t-closed set in X for each (r, s)-fuzzy t-closed set $\delta \in I^Y$.

(5). $t\operatorname{Cl}(F^+(\mu)) \subset F^+(t\operatorname{Cl}(\mu, r, s))$ for every $\mu \in I^Y$.

Definition 3.12. *For a given double fuzzy multifunction $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ a double fuzzy multifunction $t\operatorname{Cl}F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ is defined as $(t\operatorname{Cl}F)^-(x) = t\operatorname{Cl}(F(x), r, s)$ for each $x \in X$.*

Lemma 3.13. *If $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ is a double fuzzy multifunction, then $(t\operatorname{Cl}F)^-(\lambda) = F^-(\lambda)$ for each (r, s)-fuzzy t-open set $\lambda \in I^Y$.*

Proof.   Let λ be an (r, s)-fuzzy t-open set. Let $x \in F^-(\lambda)$. Then $F(x) \, q \, \lambda$. That is, $F(x) + \lambda > \bar{1}$. Then $t\operatorname{Cl}(F(x), r, s) + \lambda > \bar{1}$, then $x \in (t\operatorname{Cl}F)^-(\lambda)$. Hence $F^-(\lambda) \subset (t\operatorname{Cl}F)^-(\lambda)$. Similarly, $(t\operatorname{Cl}F)^-(\lambda) \subset F(\lambda)$. Hence $(t\operatorname{Cl}F)^-(\lambda) = F^-(\lambda)$. $\square$

Proposition 3.14. *A double fuzzy multifunction $F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ is a double fuzzy lower t-irresolute function if and only if $t\operatorname{Cl}F : (X, \tau) \to (Y, \sigma, \sigma^\star)$ is a double fuzzy lower t-irresolute function.*

Proof.   Suppose F is a double fuzzy lower t-irresolute function. Let λ be an (r, s)-fuzzy t-open set such that $(t\operatorname{Cl}F)(x) \, q \, \lambda$. By Lemma 3.13, it follows that $x \in (t\operatorname{Cl}F)(\lambda) = F(\lambda)$. Since F is a double fuzzy lower t-irresolute, there exists a t-open set U in X with $x \in U$ such that $U \subset F(\lambda)$. By Lemma 3.13, it follows that $(t\operatorname{Cl}F)^-(\lambda)$ is a t-open set in X and hence $(t\operatorname{Cl}F)(x) \, q \, \lambda$. Hence $(t\operatorname{Cl}F)$ is a double fuzzy lower t-irresolute function. Conversely, suppose that $(t\operatorname{Cl}F)$ is a double fuzzy lower t-irresolute function. Let $\lambda \in I^Y$ be an (r, s)-fuzzy t-open set such that $F(x) \, q \, \lambda$. Then $x \in F^-(\lambda)$. By Lemma 3.13, $x \in t\operatorname{Cl}F(\lambda)$. That is, $(t\operatorname{Cl}F)(x) \, q \, \lambda$. Since $(t\operatorname{Cl}F)$ is a double fuzzy lower t-irresolute, there exists a t-open set U in X with $x \in U$ such that $U \subset (t\operatorname{Cl}F)(\lambda)$ that is, $U \subset F^-(\lambda)$. Hence F is a double fuzzy lower t-irresolute function. $\square$

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