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Analytical Solution of the Problem on the Three-Dimensional Transient Heat Conduction in a Multilayer Rectangular Region

Research Article

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Abstract: An exact analytical solution of the problem on the three-dimensional transient heat conduction in a Rectangular with multiple layers, in which time-dependent, spatially non-uniform internal volume heat sources are installed, is presented. The transient temperature distribution in this rectangle was determined with the use of the eigenfunction expansion method. The solution obtained is valid for any combination of homogenous first- and second-kind boundary conditions in the Y and Z directions of the rectangle and for the nonhomogeneous third-kind boundary conditions in the X direction. As a partial case, the problem on the heat conduction in a three-layer rectangle region was solved.

Keywords: Heat conduction, exact analytical solution, transient problem, multiple layers.

1. Introduction

Multilayer transient heat conduction is characteristic of composite materials consisting of several layers. The interest shown in these materials is explained by the fact that they combine physical, mechanical, and thermal properties of different substances. The indicated materials are used in the aerospace, automobile, chemical, power, and civil engineering, biomedical industry, thermodynamic and solidification processes, and high-density microelectronics as well as for production of fiber-insulated heaters, multilayer insulators, nuclear fuel rods, fuel cells, electrochemical reactors, building structures, and also widely used in investigating the thermal properties of composite materials. The analytical methods are the method of separation of variables, the Laplace-transform method, the method of finite integral transforms, and the eigenfunction-expansion and Green's function methods.

H.Salt [1] used the orthogonal expansion method in investigating the unsteady 2-D heat conduction in a Cartesian slab. N.Dalir and S.S.Nourazar[3] use separation of variable method for solving 3-D transient heat conduction in a multilayer cylinder. A. Haji-Sheikh and J. Beck [4] used the Green's function approach to determine the 3-D temperature distribution in a two-layer orthotropic slab. X. Lu and his collaborators [5-8] used, in combination, the Laplace-transform method and the method of separation of variables to investigate the 2-D temperature distribution in rectangular, cylindrical, and spherical bodies. F. de Monte [9, 10] used the eigenfunction expansion method to solve the problem on the 2-D unsteady heat conduction in a two-layer isotropic slab at homogenous boundary conditions. P. Jain and his collaborators [11-13]

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used, in combination, the method of separation of variables and the eigenfunction expansion method in investigating the 2-D unsteady multilayer heat conduction in a sphere. S. Singh and his collaborators [14] used the finite integral transform method to determine the asymmetric heat conduction in a multilayer annulus.

The methods of investigating the 2-D transient heat conduction in multilayer bodies were developed on the basis of the works of P. Jain, S. Singh, and R. Uddin [11-14], in which exact analytical solutions of the problems of 1) the transient heat conduction in polar bodies with multiple radial layers [11], 2) the transient asymmetric heat conduction in a multilayer annulus [12], and 3) the 2-D heat conduction in a multilayer sphere [13] were obtained. The authors of the indicated works solved the indicated problems by the method of partial solutions that involves the division of a nonhomogeneous Transient problem into a homogeneous transient problem and a nonhomogeneous steady-state problem. The homogeneous transient problem is solved using the method of separation of variables, and the nonhomogeneous steady-state problem is solved by the eigenfunction expansion method

The above-presented brief survey of the literature data shows that there is no an exact analytical method for solving the problem of the 3-D transient temperature distribution in a rectangle region with multiple layers. In this connection, the aim of the present work is to obtain an exact analytical triple-series solution of the problem on the 3-D transient heat conduction in a rectangle with multiple layers, in which time-dependent, spatially nonuniform volume heat sources are installed. The boundary-value problem of the 3-D transient heat conduction in the X ,Y ,Z multilayer region having a rectangle or a partially rectangular geometry with time-dependent, spatially nonuniform volume heat sources was solved. The nonhomogeneous boundary conditions of the first, second, or third kind were set on the inner and outer x layer boundaries of the computational region, and the homogeneous boundary conditions of the first or second kind were set on the surfaces y = constant and z = constant.

2. Mathematical Formulation of the Problem

We will consider a n-layer composite rectangular slab with coordinates $x_0 \leq x \leq x_n$, $0 \leq y \leq b$, and $0 \leq z \leq c$. It is assumed that all the layers are thermally isotropic and make a perfect thermal contact. At $t = 0$, the i$^{\text{th}}$ layer has a temperature $f_i(x, y, z)$. At $t > 0$, homogenous boundary conditions of the first or second kind are set on the surfaces $y = 0$ and $y = b$ and on the surfaces $z = 0$ and $z = c$. All these boundary conditions can be used for the inner $(i = 1, x = x0)$ and outer $(i = n, x = x_n)$ surfaces. The time-dependent heat sources $g_i(x, y, z, t)$ are actuated in each layer at $t = 0$. The governing differential equation for the 3-D transient heat conduction in a multilayer rectangular has the form

$$\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} + \frac{\partial^2 T_i}{\partial z^2} + \frac{1}{k_i} g_i\left(x, y, z, t\right) = \frac{1}{\propto_i} \frac{\partial T_i}{\partial t} \tag{1}$$

$T_i = T_i(x, y, z, t)$; $x_0 \leq x \leq x_n$; $x_{i-1} \leq x \leq x_i$; $1 \leq i \leq n$; $0 \leq y \leq b$; $0 \leq z \leq c$, $t = 0$. This equation is used with the following boundary conditions for the inner surface of the 1$^{\text{st}}$ layer $(i = 1)$:

$$A_{in} \frac{\partial T_1(x, y, z, t)}{\partial x} + B_{in} T_1\left(x, y, z, t\right) = C_{in} \tag{2}$$

The outer surface of the n$^{\text{th}}$ layer $(i = n)$

$$A_{out} \frac{\partial T_n(x, y, z, t)}{\partial x} + B_{out} T_n\left(x, y, z, t\right) = C_{out} \tag{3}$$

Surface $y = 0$ $(i = 1, 2, \ldots, n)$

$$T_i\left(x,\ y = 0,\ z,\ t\right) = 0 \ \ or \ \ \frac{\partial T_i(x,\ y = 0,\ z,\ t)}{\partial y} = 0 \tag{4}$$

Surface $y = b$ $(i = 1, 2, \ldots, n)$

$$T_i\,(x,\ y = b,\ z,\ t) = 0 \ \ or \ \ \frac{\partial T_i(x,\ y = b,\ z,\ t)}{\partial y} = 0 \tag{5}$$

Surface $z = 0$ $(i = 1, 2, \ldots, n)$

$$T_i\,(x,\ y,\ z = 0,\ t) = 0 \ \ or \ \ \frac{\partial T_i(x,\ y,\ z = 0,\ t)}{\partial z} = 0 \tag{6}$$

Surface $z = c$ $(i = 1, 2, \ldots, n)$

$$T_i\,(x,\ y,\ z = c,\ t) = 0 \ \ or \ \ \frac{\partial T_i(x,\ y,\ z = c,\ t)}{\partial z} = 0 \tag{7}$$

Inner interface of the i$^{\text{th}}$ layer $(i = 2, 3, \ldots, n)$

$$T_i(x_{i-1}, y, z, t) = T_{i-1}(x_{i-1}, y, z, t), \tag{8}$$

$$k_i \frac{\partial T_i(x_{i-1},\ y,\ z,\ t)}{\partial z} = k_{i-1} \frac{\partial T_{i-1}(x_{i-1},\ y,\ z,\ t)}{\partial z} \tag{9}$$

Outer interface of the i$^{\text{th}}$ layer $(i = 1, 2, 3 \ldots, n-1)$

$$T_i(x_i, y, z, t) = T_{i+1}(x_i, y, z, t), \tag{10}$$

$$k_i \frac{\partial T_i(x_i,\ y,\ z,\ t)}{\partial z} = k_{i+1} \frac{\partial T_{i+1}(x_i,\ y,\ z,\ t)}{\partial z} \tag{11}$$

Where A_{in}, A_{out}, B_{in}, B_{out}, C_{in}, and C_{out} are coefficients. The initial condition is as follows:

$$T_i\,(x, y, z, t = 0) = f_i(x, y, z) \tag{12}$$

It should be noted that the boundary conditions of the first, second, and third kind (2) and (3) with appropriate coefficients can be used in the cases where $x = x_0$ and $x = x_n$ and that a multilayer rectangle with a zero inner surface ($x_0 = 0$) can be simulated by assigning zero values to B_{in} and C_{in} in the boundary condition (2).

3. Method of Solving the Problem

The problem posed is solved using the eigenfunction expansion method. First, the eigenfunctions are obtained for all the spatial directions of the rectangle being considered with the use of the associated (or equivalent) eigenvalue problem that is solved using the method of separation of variables. Then a dependent variable or a problem solution (e.g., the temperature in the heat-conduction problem) is written as a series expansion of all the obtained eigenfunctions. The nonhomogeneity in the governing differential equation of the heat-conduction problem (e.g., an internal volume heat source) is also written as a series expansion of the eigenfunctions. These two series expansions are then substituted into the differential equation of the problem. Finally, with the use of some simplification, an ordinary differential equation (ODE) of the first or second order can be obtained for an independent time variable. The solution of this ODE completes the solution of the problem, and the indicated ODE is a first-order equation in the transient heat conduction problem and a second-order equation in the wave problem.

It should be noted that the method of partial solutions, by which the 2-D transient heat conduction problem was solved in [2], cannot be used for solving the transient heat conduction problem being considered, because, in [2], the heat source independent of time was considered. In the method of partial solutions, the nonhomogeneous transient heat conduction problem is divided into two subproblems: a homogeneous transient problem and a nonhomogeneous steady-state problem.

Since the heat source in the problem being considered depends on the time, a heat-source term cannot be involved in the partial solution of the steady-state subproblem and, consequently, the method of partial solutions cannot be used in this case. Thus, the eigenfunction expansion method can be considered as the most efficient analytical method of the methods known to the authors of this work, which can be used for solving the transient conduction problem. An associated eigenvalue problem is defined by the relation

$$\nabla^2 \emptyset = -\lambda^2 \emptyset$$

$$\frac{\partial^2 \emptyset_i}{\partial x^2} + \frac{\partial^2 \emptyset_i}{\partial y^2} + \frac{\partial^2 \emptyset_i}{\partial z^2} = \lambda^2 \emptyset_i \tag{13}$$

$$\emptyset_i\left(x, y, z\right) = X_i\left(x\right) Y_i\left(y\right) Z_i(z) \tag{14}$$

$$\frac{X_i^{''} Y_i Z_i}{X_i Y_i Z_i} + \frac{X_i Y_i^{''} Z_i}{X_i Y_i Z_i} + \frac{X_i Y_i Z_i^{''}}{X_i Y_i Z_i} = -\lambda^2 \frac{X_i Y_i Z_i}{X_i Y_i Z_i} \tag{15}$$

$$\frac{X_i^{''}}{X_i} + \frac{Y_i^{''}}{Y_i} + \frac{Z_i^{''}}{Z_i} = -\lambda^2 \tag{16}$$

$$\frac{X_i^{''}}{X_i} + \frac{Y_i^{''}}{Y_i} = -\frac{Z_i^{''}}{Z_i} - \lambda^2 = -\mu^2 \tag{17}$$

$$Z_i^{''} + \left(\lambda^2 - \mu^2\right) Z_i = 0 \tag{18}$$

Where $\lambda^2 - \mu^2 = \gamma^2$

$$Z_{il}\left(z\right) = c_1 \sin \gamma_{il} z + c_2 \cos \gamma_{il} z \tag{19}$$

$$\frac{X_i^{''}}{X_i} + \frac{Y_i^{''}}{Y_i} = -\mu^2 \tag{20}$$

$$\frac{X_i^{''}}{X_i} = -\frac{Y_i^{''}}{Y_i} - \mu^2 = -\beta^2 \tag{21}$$

$$Y_i^{''} + \left(\mu^2 - \beta^2\right) Y_i = 0 \tag{22}$$

$$Y_{im}\left(y\right) = c_3 \sin \eta_{im} y + c_4 \cos \eta_{im} y \tag{23}$$

where $\eta_{im} = \mu^2 - \beta^2$

$$X_{in}\left(x\right) = c_5 \sin \beta_{in} x + c_6 \cos \beta_{in} x \tag{24}$$

the continuity of the heat flows at the interfaces of the layers implies the fulfillment of the conditions.

$$\beta_{in}^2 \alpha_i = \beta_{1n}^2 \alpha_1$$

The eigenfunctions obtained are represented in the form of a triple-series expansion

$$T_i\left(x, y, z, t\right) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} T_{ilmn}\left(t\right) X_{in}\left(x\right) Y_{im}\left(y\right) Z_{il}(z) \tag{25}$$

where T_{ilmn} is a coefficient determined by the initial conditions. The heat-source term is represented in the analogous form:

$$g_i\left(x, y, z, t\right) = \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} g_{ilmn}\left(t\right) X_{in}\left(x\right) Y_{im}\left(y\right) Z_{il}(z) \tag{26}$$

where g_{ilmn} is a coefficient. Using the orthogonality property of the indicated eigenfunctions, we obtain

$$g_{ilmn}\left(t\right) = \frac{\int_0^c \int_0^b \int_{x_{i-1}}^{x_i} g_i\left(x, y, z, t\right) X_{in}\left(x\right) Y_{im}\left(y\right) Z_{il}(z)\, dxdydz}{\int_0^c \int_0^b \int_{x_{i-1}}^{x_i} X_{in}^2(x)\, Y_{im}^2(y)\, Z_{il}^2(z) dxdydz} \tag{27}$$

Substitution of the triple-series expansions (25) and (26) for T_i and g_i into the differential equation of the problem (1) gives

$$T_{ilmn}(t)\,X''_{in}(x)\,Y_{im}(y)\,Z_{il}(z)+T_{ilmn}(t)\,X_{in}(x)\,Y''_{im}(y)\,Z_{il}(z)+T_{ilmn}(t)\,X_{in}(x)\,Y_{im}(y)\,Z''_{il}(z)$$
$$+\frac{1}{k_i}g_{ilmn}(t)\,X_{in}(x)\,Y_{im}(y)\,Z_{il}(z)=\frac{1}{\propto_i}T'_{ilmn}X_{in}(x)\,Y_{im}(y)\,Z_{il}(z) \tag{28}$$

$$\frac{X''_{in}(x)}{X_{in}(x)}+\frac{Y''_{im}(y)}{Y_{im}(y)}+\frac{Z''_{il}(z)}{Z_{il}(z)}+\frac{1}{k_i}\frac{g_{ilmn}(t)}{T_{ilmn}(t)}=\frac{1}{\propto_i}\frac{T'_{ilmn}}{T_{ilmn}(t)} \tag{29}$$

$$\frac{dT_{ilmn}(t)}{dt}+(-\propto_i)\left(\frac{X''_{in}(x)}{X_{in}(x)}+\frac{Y''_{im}(y)}{Y_{im}(y)}+\frac{Z''_{il}(z)}{Z_{il}(z)}\right)T_{ilmn}(t)=\frac{\propto_i}{k_i}g_{ilmn}(t) \tag{30}$$

where $W_{ilmn}=(-\propto_i)\left(\frac{X''_{in}(x)}{X_{in}(x)}+\frac{Y''_{im}(y)}{Y_{im}(y)}+\frac{Z''_{il}(z)}{Z_{il}(z)}\right)$

$$e^{W_{ilmn}t}T_{ilmn}(t)=\int_{\tau=0}^{\tau=t}\frac{\propto_i}{k_i}g_{ilmn}(\tau)\,e^{W_{ilmn}\tau}d\tau+c_{i1} \tag{31}$$

$$T_{ilmn}(t)=\frac{\propto_i}{k_i}e^{-W_{ilmn}t}\int_{\tau=0}^{\tau=t}g_{ilmn}(\tau)\,e^{W_{ilmn}\tau}d\tau+c_{i1}e^{-W_{ilmn}t} \tag{32}$$

Using initial condition

$$T_i(x,y,z,t=0)=f_i(x,y,z)=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}T_{ilmn}(0)\,X_{in}(x)\,Y_{im}(y)\,Z_{il}(z) \tag{33}$$

Applying the orthogonality property to the eigenfunctions obtained, we write the coefficient c_{i1} from Equation (33) in the following form

$$C_{i1}=T_{ilmn}(0)=\frac{\int_0^c\int_0^b\int_{x_{i-1}}^{x_i}f_i(x,y,z)\,X_{in}(x)\,Y_{im}(y)\,Z_{il}(z)\,dxdydz}{\int_0^c\int_0^b\int_{x_{i-1}}^{x_i}X_{in}^2(x)\,Y_{im}^2(y)\,Z_{il}^2(z)\,dxdydz} \tag{34}$$

$$T_{ilmn}(t)=\frac{\propto_i}{k_i}e^{-W_{ilmn}t}\int_{\tau=0}^{\tau=t}g_{ilmn}(\tau)\,e^{W_{ilmn}\tau}d\tau+\frac{\int_0^c\int_0^b\int_{x_{i-1}}^{x_i}f_i(x,y,z)\,X_{in}(x)\,Y_{im}(y)\,Z_{il}(z)\,dxdydz}{\int_0^c\int_0^b\int_{x_{i-1}}^{x_i}X_{in}^2(x)\,Y_{im}^2(y)\,Z_{il}^2(z)\,dxdydz}e^{-W_{ilmn}t} \tag{35}$$

Now the solution of the problem is completed.

4. Partial Case of the Problem Being Investigated

A three-layer rectangular region with coordinates $0\le x\le x_3$, $0\le y\le b$, and $0\le z\le c$ is considered. At the initial instant of time ($t=0$), the rectangle has a uniform temperature distribution. At $t>0$, the temperature of the surfaces $y=0$, $y=b$, $z=0$, and $z=c$ is equal to zero; however, heat is transferred from the outer surface ($x=x_3$) at zero ambient temperature. These boundary conditions are defined by the relations $A_{in}=k_1$, $B_{in}=h$, $C_{in}=0$, $A_{out}=k_3$, $B_{out}=h$, and $C_{out}=0$. A uniform internal heat source g_i ($i=1,\ldots,3$) is activated at $t=0$ in each layer. The governing differential equation for the 3-D transient heat conduction in the indicated three-layer rectangle has the form

$$\frac{\partial^2T_i}{\partial x^2}+\frac{\partial^2T_i}{\partial y^2}+\frac{\partial^2T_i}{\partial z^2}+\frac{1}{k_i}g_i(x,y,z,t)=\frac{1}{\propto_i}\frac{\partial T_i}{\partial t} \tag{36}$$

$$T_i=T_i(x,y,z,t)$$

$x_1\le x\le x_3$, $x_{i-1}\le x\le x_i$, $1\le i\le 3$, $0\le y\le b$, $0\le z\le c$. This equation is used with the boundary conditions: Inner surface at 1^{st} layer ($i=1$)

$$k_1\frac{\partial T_1(x_1,y,z,t)}{\partial x}+h_1T_1(x_1,y,z,t)=0 \tag{37}$$

Outer surface at 3^{rd} layer $(i=3)$

$$k_3 \frac{\partial T_3(x_3,y,z,t)}{\partial x} + h_3 T_3\left(x_3,y,z,t\right) = 0 \tag{38}$$

At the layer $i=1,2,3$

$$T_i\left(x,\ y=0,\ z,\ t\right)=0,\quad T_i\left(x,\ y=b,\ z,\ t\right)=0,\quad T_i\left(x,\ y,\ z=0,\ t\right)=0,\quad T_i\left(x,\ y,\ z=c,\ t\right)=0 \tag{39}$$

The inner interface surface of the i^{th} layer $(i=2,3)$:

$$T_i(x_{i-1},y,z,t) = T_{i-1}(x_{i-1},y,z,t), \tag{40}$$

$$k_i \frac{\partial T_i(x_{i-1},\ y,\ z,\ t)}{\partial x} = k_{i-1} \frac{\partial T_{i-1}(x_{i-1},\ y,\ z,\ t)}{\partial x} \tag{41}$$

The outer interface surface of the i^{th} layer $(i=1,2)$:

$$T_i(x_i,y,z,t) = T_{i+1}(x_i,y,z,t), \tag{42}$$

$$k_i \frac{\partial T_i(x_i,\ y,\ z,\ t)}{\partial x} = k_{i+1} \frac{\partial T_{i+1}(x_i,\ y,\ z,\ t)}{\partial x} \tag{43}$$

And the initial condition has the form

$$T_i\left(x,y,z,t=0\right) = 1;\ 1 \le i \le 3 \tag{44}$$

With the use of the eigenfunction expansion method, the associated eigenvalue problem is solved in the z , y and x directions

$$\begin{aligned} Z_{il}\left(z\right) &= c_1 \cos \gamma_{il} z + c_2 \sin \gamma_{il} z \\ Y_{im}\left(y\right) &= c_3 \cos \eta_{im} y + c_4 \sin \eta_{im} y \\ X_{in}\left(x\right) &= c_5 \cos \beta_{in} x + c_6 \sin \beta_{in} x \end{aligned} \tag{45}$$

the use of relevant boundary conditions in each direction. In this case, for the z direction, we obtain the eigenvalues and eigenfunction in the z direction are as follows

$$\begin{aligned} Z_{il}\left(z\right) &= \sin \gamma_{il} z \\ Z_{il}\left(z\right) &= \sin \frac{l\pi}{c} z \end{aligned} \tag{46}$$

The eigenvalues and eigenfunction in the y direction are as follows

$$Y_{im}\left(y\right) = \sin \frac{m\pi}{b} y \tag{47}$$

The eigenvalues and eigenfunction in the x direction are as follows

$$X_{in}\left(x\right) = \beta_{in} \cos \beta_{in} x + H_1 \sin \beta_{in} x \tag{48}$$

The coefficients $g_{ilmn}(t)$ and W_{ilmn} are determined as

$$g_{ilmn}(t) = \frac{\int_0^c \int_0^b \int_{x_{i-1}}^{x_i} \left(g_i(x,y,z,t)(\beta_{in}\cos\beta_{in}x + H_1\sin\beta_{in}x)\sin\frac{m\pi}{b}y\sin\frac{l\pi}{c}z\right)dxdydz}{\int_0^c \int_0^b \int_{x_{i-1}}^{x_i} \left((\beta_{in}\cos\beta_{in}x + H_1\sin\beta_{in}x)(\sin\frac{m\pi}{b}y)\sin\frac{l\pi}{c}z\right)^2 dxdydz} \tag{49}$$

$$W_{ilmn} = (-\propto_i)\left(\frac{X''_{in}(x)}{X_{in}(x)} + \frac{Y''_{im}(y)}{Y_{im}(y)} + \frac{Z''_{il}(z)}{Z_{il}(z)}\right) \tag{50}$$

$$W_{ilmn} = (-\propto_i)\left(-\beta_{in}^2 + \frac{m^2\pi^2}{b^2} + \frac{l^2\pi^2}{c^2}\right) \tag{51}$$

$$T_i(x,y,z,t) = \sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{ilmn}(t)\left(\beta_{inm}\cos\beta_{inm}x + H_1\sin\beta_{inm}x\right)\ \sin\frac{m\pi}{b}y\sin\frac{l\pi}{c}z \tag{52}$$

$$T_{ilmn}(t) = \frac{\propto_i}{k_i}e^{-W_{ilmn}t}g_{ilmn}\frac{1}{W_{ilmn}}\left(e^{W_{ilmn}t} - 1\right) + c_{i1}e^{-W_{ilmn}t} \tag{53}$$

$$T_{ilmn}(t) = g_{ilmn}\left[\frac{\propto_i}{k_iW_{ilmn}} + \left(\frac{1}{g_i}\frac{\propto_i}{k_iW_{ilmn}}\right)e^{-W_{ilmn}t}\right] \tag{54}$$

5. Conclusions

The temperature distribution in a multilayer Rectangle was determined on the basis of the exact analytical solution of the problem of the 3-D transient heat conduction in this rectangle by the eigenfunction expansion method with account for the action of the time-dependent, nonuniform volume heat sources in the x layers of the rectangle with the use of the homogenous boundary conditions of the first and second kind in the y and z directions and the nonhomogeneous boundary conditions of the third kind in the x direction. The partial problem on the heat conduction in a three-layer rectangle was solved, and the temperature distribution in this rectangle was determined.

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