

Subdivisions of Contra Harmonic Mean Graphs

Research Article

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Abstract: A graph $G(V, E)$ is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $0, 1, \dots, q$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ with distinct edge labels. The mapping f is called Contra Harmonic mean labeling of G .

Keywords: Graph, Contra Harmonic mean graph, Path, Comb, Cycle, Triangular Snake, Quadrilateral snake.

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1. Introduction

All graphs in this paper are simple, finite, undirected. Let $G(V, E)$ be a graph with p vertices and q edges. For a detail survey of graph labeling we refer to [1]. A general reference for graph-theoretic ideas is [2]. The concept of mean labeling was introduced by S. Somasundaram and R. Ponraj [3] in 2004. S. Somasundaram and S.S. Sandhya introduced Harmonic mean labeling [4] in 2012. In this paper we investigate the Subdivision of Contra Harmonic mean labeling for some graphs. The following definition are useful for our present study.

Definition 1.1. A graph $G(V, E)$ is called a Contra Harmonic mean graph with p vertices and q edges, if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $0, 1, \dots, q$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rceil$ or $\left\lfloor \frac{f(u)^2 + f(v)^2}{f(u) + f(v)} \right\rfloor$ with distinct edge labels. The mapping f is called Contra Harmonic mean labeling of G .

Definition 1.2. A Triangular snake T_n is obtained from a path $u_1 \dots u_n$ by joining u_i and to a vertex v_i for $1 \leq i \leq n - 1$.

Definition 1.3. A Quadrilateral snake Q_n is obtained from a path $u_1 \dots u_n$ by joining u_i and u_{i+1} to new vertices v_i, w_i , $1 \leq i \leq n - 1$.

Definition 1.4. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$.

Definition 1.5. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2 .

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Definition 1.6. If $e = uv$ is an edge of G and w is not a vertex of G then e is said to be subdivided when it is replaced by the edges uw and wv . The graph obtained by subdividing each of a graph G is called the subdivision of G and is denoted by $S(G)$.

2. Main Results

Theorem 2.1. $S(P_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof. Let $u_1u_2 \dots u_n$ be a path P_n of length n . Let v_i be the vertex which is joined to the vertex u_i , $1 \leq i \leq n$ of the path P_n . The resulting graph is a comb $P_n \odot K_1$. Let G be graph obtained by subdividing the edges. Here we consider the following cases.

Case (i): G is obtained by subdividing each edge of the path. Let $t_1t_2 \dots t_{n-1}$ be the vertices which subdivide the edges of u_iu_{i+1} . Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 3i - 3, \quad 1 \leq i \leq n \\ f(v_i) &= 3i - 2, \quad 1 \leq i \leq n \\ f(t_i) &= 3i - 1, \quad 1 \leq i \leq n - 1 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_iv_i) &= 3i - 2, \quad 1 \leq i \leq n \\ f(u_it_i) &= 3i - 1, \quad 1 \leq i \leq n - 1 \\ f(t_iu_{i+1}) &= 3i, \quad 1 \leq i \leq n - 1 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The Contra Harmonic mean labeling pattern of $S(P_7 \odot K_1)$ is

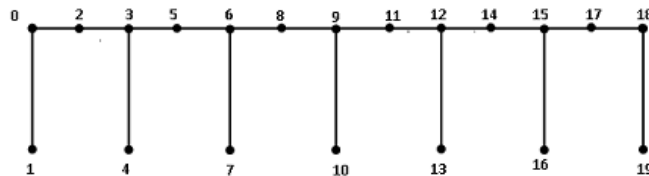


Figure 1.

Case (ii): G is obtained by subdividing the edges u_iv_i . Let w_i be the vertices which subdivide the edges u_iv_i , $1 \leq i \leq n$. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned} f(u_1) &= 0, \quad f(u_2) = 3, \quad f(u_i) = 3i - 2, \quad 3 \leq i \leq n. \\ f(v_1) &= 2, \quad f(v_2) = 5, \quad f(v_i) = 3i, \quad 3 \leq i \leq n. \\ f(w_1) &= 1, \quad f(w_2) = 4, \quad f(w_3) = 8, \quad f(w_4) = 11, \quad f(w_i) = 3i - 1, \quad 5 \leq i \leq n. \end{aligned}$$

Then the distinct edges labels are

$$\begin{aligned} f(u_iu_{i+1}) &= 3i, \quad 1 \leq i \leq n - 1 \\ f(u_iw_i) &= 3i - 2, \quad 1 \leq i \leq n \\ f(w_iv_i) &= 3i - 1, \quad 1 \leq i \leq n \end{aligned}$$

∴ f is a Contra Harmonic mean labeling of G. The labeling pattern of $S(P_5 \odot K_1)$ is shown below.

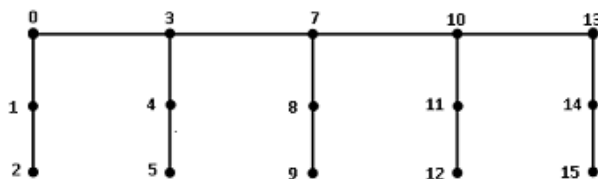


Figure 2.

Case (iii): G is obtained by subdividing all the edges of $P_n \odot K_1$. Let t_i and w_i be the vertices which subdivide the edges $u_i u_{i+1}$ and $u_i v_i$ respectively. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned}
 f(u_1) &= 0, f(u_2) = 4, f(u_i) = 4i - 3, 3 \leq i \leq n \\
 f(v_1) &= 2, f(v_2) = 6, f(v_i) = 4i - 1, 3 \leq i \leq n \\
 f(t_1) &= 3, f(t_i) = 4i, 2 \leq i \leq n - 1 \\
 f(w_1) &= 1, f(w_2) = 5, f(w_i) = 4i - 2, 3 \leq i \leq n
 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned}
 f(u_i t_i) &= 4i - 1, 1 \leq i \leq n - 1 \\
 f(t_i u_{i+1}) &= 4i, 1 \leq i \leq n - 1 \\
 f(u_i w_i) &= 4i - 3, 1 \leq i \leq n \\
 f(w_i v_i) &= 4i - 2, 1 \leq i \leq n
 \end{aligned}$$

∴ f is a Contra Harmonic mean labeling of G. The labeling pattern of $S(P_5 \odot K_1)$ is shown below.

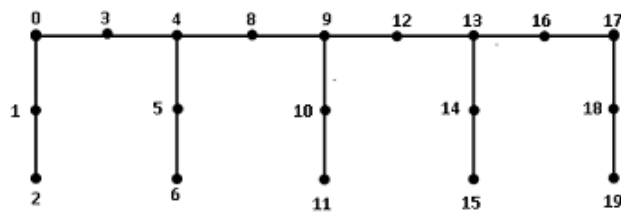


Figure 3.

From case (i) and (ii), it is clear that $S(P_n \odot K_1)$ is a Contra Harmonic mean graph. □

Theorem 2.2. $S(C_n \odot K_1)$ is a Contra Harmonic mean graph.

Proof. Let $u_1 u_2 \dots u_n$ be a cycle C_n of length n and let v_i be the pendant vertices adjacent to $u_i, 1 \leq i \leq n$. The resulting graph is $C_n \odot K_1$. Let $G = S(C_n \odot K_1)$ be the graph obtained by subdividing the edges. Here we consider the following cases.

Case (i): Let G be a graph obtained by subdividing each edge of the cycle $u_1u_2 \dots u_n$. Let $w_i, 1 \leq i \leq n$ be the vertices which subdivide the edges of the cycle $u_1u_2 \dots u_n$. Define a function $f : V(G) \rightarrow \{0, 1, 2, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 3i - 3, \quad 1 \leq i \leq n \\ f(v_i) &= 3i - 2, \quad 1 \leq i \leq n \\ f(w_i) &= 3i - 1, \quad 1 \leq i \leq n - 1 \quad \text{and} \quad f(w_n) = 3n \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_iv_i) &= 3i - 2, \quad 1 \leq i \leq n \\ f(u_iw_i) &= 3i - 1, \quad 1 \leq i \leq n \\ f(w_iu_{i+1}) &= 3i, \quad 1 \leq i \leq n \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(C_5 \odot K_1)$ is shown below.

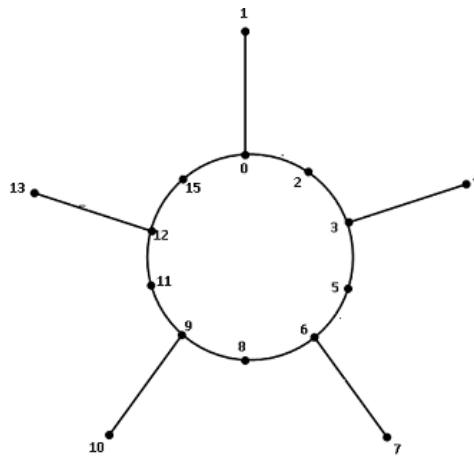


Figure 4.

Case (ii) : Let G be a graph obtained by subdividing the edge u_iv_i of $C_n \odot K_1$. Let $t_i, 1 \leq i \leq n$ be the vertices which subdivide u_i and v_i . Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned} f(u_1) &= 0, \quad f(u_2) = 3 \quad \text{and} \quad f(u_i) = 3i - 2, \quad 3 \leq i \leq n \\ f(v_1) &= 2, \quad f(v_i) = 3i, \quad 2 \leq i \leq n \\ f(t_1) &= 1, \quad f(t_i) = 3i - 1, \quad 2 \leq i \leq n \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_iu_{i+1}) &= 3i, \quad 1 \leq i \leq n - 1, \quad f(u_nu_1) = 3n - 1, \\ f(t_iv_i) &= 3i - 1, \quad 1 \leq i \leq n - 1, \quad f(t_nv_n) = 3n \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(C_5 \odot K_1)$ is shown below.

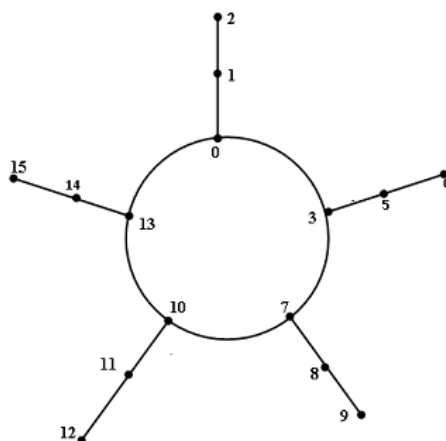


Figure 5.

Case (iii) : Let G be a graph obtained by subdividing all the edges of $C_n \odot K_1$. Let t_i, w_i be the vertices which subdivide the edges of the cycle $u_1 u_2 \dots u_n$ and the edges u_i and v_i for $1 \leq i \leq n$ respectively. Define a function $f : V(G) \rightarrow \{1, 2, \dots, q\}$ by

$$\begin{aligned}
 f(u_1) &= 0, \quad f(u_i) = 4i - 3, \quad 2 \leq i \leq n \\
 f(v_1) &= 2, \quad f(v_i) = 4i - 1, \quad 2 \leq i \leq n \\
 f(w_1) &= 1, \quad f(w_i) = 4i - 2, \quad 2 \leq i \leq n \\
 f(t_1) &= 3, \quad f(t_i) = 4i, \quad 2 \leq i \leq n
 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned}
 f(u_i w_i) &= 4i - 3, \quad 1 \leq i \leq n \\
 f(w_i v_i) &= 4i - 2, \quad 1 \leq i \leq n \\
 f(u_i t_i) &= 4i - 1, \quad 1 \leq i \leq n \\
 f(t_i u_{i+1}) &= 4i, \quad 1 \leq i \leq n
 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(C_5 \odot K_1)$ is shown below. From the above cases we

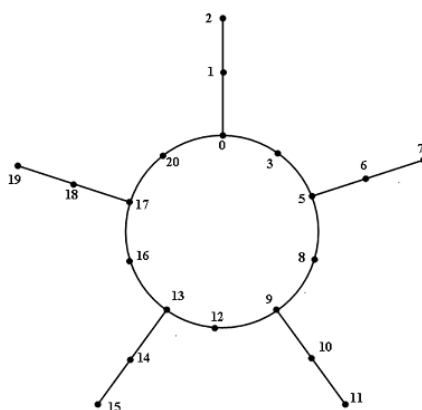


Figure 6.

conclude that $S(C_n \odot K_1)$ is a Contra Harmonic mean graph. □

Theorem 2.3. $S(T_n)$ is a Contra Harmonic mean graph.

Proof. Let $u_1u_2 \dots u_n$ be a path of length n . Let T_n be a triangular snake obtained by joining u_i and u_{i+1} to a new vertex v_i , $1 \leq i \leq n - 1$. Let us subdivide the edges of T_n . We consider the following cases.

Case (i) : G is obtained by subdividing each edge of the path. Let t_1, t_2, \dots, t_{n-1} be the vertices which subdivide the edge u_iu_{i+1} . Define a function $f : V(G) \rightarrow \{0, 1, 2, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 4i - 4, \quad 1 \leq i \leq n \\ f(v_i) &= 4i - 3, \quad 1 \leq i \leq n - 1 \\ f(t_i) &= 4i - 2, \quad 1 \leq i \leq n - 1 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_it_i) &= 4i - 2, \quad 1 \leq i \leq n - 1 \\ f(t_1u_2) &= 3, \quad f(t_iu_{i+1}) = 4i, \quad 2 \leq i \leq n - 1 \\ f(u_iv_i) &= 4i - 3, \quad 1 \leq i \leq n - 1 \\ f(v_1u_2) &= 4, \quad f(v_iu_{i+1}) = 4i - 1, \quad 2 \leq i \leq n - 1 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(T_6)$ is

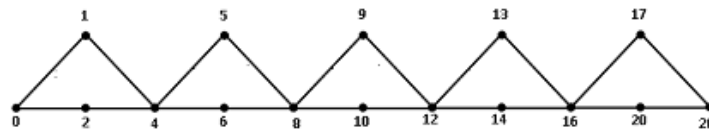


Figure 7.

Case (ii) : G is obtained by subdividing the edges u_iv_i and $u_{i+1}v_i$. Let t_i and s_i be the two vertices which subdivide the edges u_iv_i and $u_{i+1}v_i$, $1 \leq i \leq n - 1$ respectively. Define a function $f : V(G) \rightarrow \{0, 1, 2, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 5i - 5, \quad 1 \leq i \leq n \\ f(v_1) &= 2, \quad f(v_i) = 5i - 2, \quad 2 \leq i \leq n - 1 \\ f(t_1) &= 1, \quad f(t_i) = 5i - 3, \quad 2 \leq i \leq n - 1 \\ f(s_1) &= 3, \quad f(s_i) = 5i - 1, \quad 2 \leq i \leq n - 1 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_1u_2) &= 5, \quad f(u_iu_{i+1}) = 5i - 2, \quad 2 \leq i \leq n - 1 \\ f(u_it_i) &= 5i - 4, \quad 1 \leq i \leq n - 1 \\ f(t_iv_i) &= 5i - 3, \quad 1 \leq i \leq n - 1 \\ f(v_1s_1) &= 3, \quad f(v_is_i) = 5i - 1, \quad 2 \leq i \leq n - 1 \\ f(s_1u_2) &= 4, \quad f(s_iu_{i+1}) = 5i, \quad 2 \leq i \leq n - 1 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(T_5)$ is shown below.

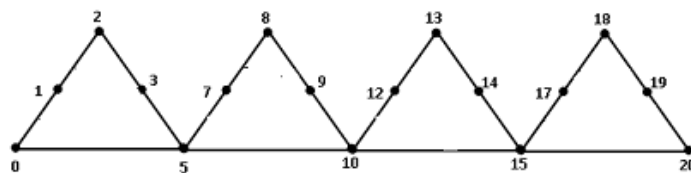


Figure 8.

Case (iii) : G is obtained by subdividing all the edges of T_n . Let x_i, y_i and t_i be the vertices which subdivide the edges $u_i v_i, v_i u_{i+1}$ and $u_i u_{i+1}$ respectively. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned}
 f(u_i) &= 6i - 6, \quad 1 \leq i \leq n \\
 f(v_1) &= 2, \quad f(v_i) = 6i - 3, \quad 2 \leq i \leq n - 1 \\
 f(t_1) &= 3, \quad f(t_i) = 6i - 2, \quad 2 \leq i \leq n - 1 \\
 f(x_1) &= 1, \quad f(x_i) = 6i - 4, \quad 2 \leq i \leq n - 1 \\
 f(y_1) &= 4, \quad f(y_i) = 6i - 1, \quad 2 \leq i \leq n - 1
 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned}
 f(u_i t_i) &= 6i - 3, \quad 1 \leq i \leq n - 1 \\
 f(t_i u_{i+1}) &= 6i - 1, \quad 1 \leq i \leq n - 1 \\
 f(u_i x_i) &= 6i - 5, \quad 1 \leq i \leq n - 1 \\
 f(x_i v_i) &= 6i - 4, \quad 1 \leq i \leq n - 1 \\
 f(v_i y_i) &= 6i - 2, \quad 1 \leq i \leq n - 1 \\
 f(y_i u_{i+1}) &= 6i, \quad 1 \leq i \leq n - 1
 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G. The labeling pattern of $S(T_5)$ is shown below.

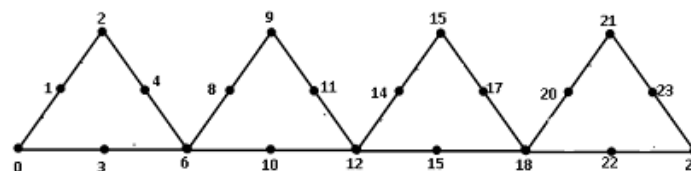


Figure 9.

From Case (i), Case (ii), Case (iii) it can be seen that $S(T_n)$ is a Contra Harmonic mean graph. □

Theorem 2.4. $S(Q_n)$ is a Contra Harmonic mean graph.

Proof. Let $u_1 u_2 \dots u_n$ be a path of length n. Join u_i and u_{i+1} to new vertices v_i and $w_i, 1 \leq i \leq n - 1$ respectively and then join v_i and w_i . The resulting graph is a Quadrilateral snake Q_n . Let G be the graph obtained by subdividing the edges of Q_n . Here we consider the following cases.

Case (i) : G is obtained by subdividing the edges of the path. Let $t_1 t_2 \dots t_{n-1}$ be the vertices which subdivide the edge $u_i u_{i+1}$, $1 \leq i \leq n - 1$. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 5i - 5, \quad 1 \leq i \leq n \\ f(v_1) &= 1, \quad f(v_i) = 5i - 3, \quad 2 \leq i \leq n - 1 \\ f(t_1) &= 3, \quad f(t_i) = 5i - 1, \quad 2 \leq i \leq n - 1 \\ f(w_1) &= 2, \quad f(w_i) = 5i - 2, \quad 2 \leq i \leq n - 1 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_i t_i) &= 5i - 2, \quad 1 \leq i \leq n - 1 \\ f(t_i u_{i+1}) &= 5i, \quad 1 \leq i \leq n - 1 \\ f(u_i v_i) &= 5i - 4, \quad 1 \leq i \leq n - 1 \\ f(w_i u_{i+1}) &= 5i - 1, \quad 1 \leq i \leq n - 1 \\ f(v_i w_i) &= 5i - 3, \quad 1 \leq i \leq n - 1 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(Q_5)$ is shown below.

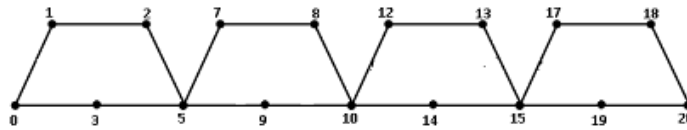


Figure 10.

Case (ii) : G is obtained by subdividing all the edges of Q_n . Let t_i, x_i, y_i, z_i be the vertices which subdivide the edges $u_i u_{i+1}, u_i v_i, v_i w_i$ and $w_i u_{i+1}$ respectively. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned} f(u_i) &= 8i - 8, \quad 1 \leq i \leq n \\ f(v_1) &= 2, \quad f(v_i) = 8i - 5, \quad 2 \leq i \leq n - 1 \\ f(w_1) &= 4, \quad f(w_i) = 8i - 3, \quad 2 \leq i \leq n - 1 \\ f(t_1) &= 5, \quad f(t_i) = 8i - 2, \quad 2 \leq i \leq n - 1 \\ f(x_1) &= 1, \quad f(x_i) = 8i - 6, \quad 2 \leq i \leq n - 1 \\ f(y_1) &= 3, \quad f(y_i) = 8i - 4, \quad 2 \leq i \leq n - 1 \\ f(z_1) &= 6, \quad f(z_i) = 8i - 1, \quad 2 \leq i \leq n - 1 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned} f(u_1 t_1) &= 5, \quad f(u_i t_i) = 8i - 4, \quad 2 \leq i \leq n - 1 \\ f(t_i u_{i+1}) &= 8i - 1, \quad 1 \leq i \leq n - 1 \\ f(u_i x_i) &= 8i - 7, \quad 1 \leq i \leq n - 1 \end{aligned}$$

$$\begin{aligned}
 f(x_i v_i) &= 8i - 6, \quad 1 \leq i \leq n - 1 \\
 f(v_i y_i) &= 8i - 5, \quad 1 \leq i \leq n - 1 \\
 f(y_1 w_1) &= 4, \quad f(y_i w_i) = 8i - 3, \quad 2 \leq i \leq n - 1 \\
 f(w_i z_i) &= 8i - 2, \quad 1 \leq i \leq n - 1 \\
 f(u_{i+1} z_i) &= 8i, \quad 1 \leq i \leq n - 1
 \end{aligned}$$

∴ f is a Contra Harmonic mean labeling of G. The labeling pattern of $S(Q_5)$ is shown below.

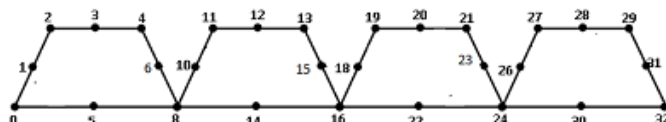


Figure 11.

From case (i) and case (ii), it is clear that $S(Q_n)$ is a Contra Harmonic mean graph.

□

Theorem 2.5. $S(P_n \odot K_3)$ is a Contra Harmonic mean graph.

Proof. Let $u_1 u_2 \dots u_n$ be the path of length n. Let v_i, w_i be the vertices of K_2 which are joined to the vertex u_i of the path $P_n, 1 \leq i \leq n$. The resultant graph is $P_n \odot K_3$. Let G be a graph obtained by subdividing all the edges of $P_n \odot K_3$. Here we consider the following cases.

Case (i) : Let G be a graph obtained by subdividing each edge of the path. Let $t_1 t_2 \dots t_{n-1}$ be the vertices which subdivide the edge $u_i u_{i+1}$. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned}
 f(u_1) &= 0, \quad f(u_i) = 5i - 4, \quad 2 \leq i \leq n \\
 f(v_1) &= 1, \quad f(v_i) = 5i - 3, \quad 2 \leq i \leq n \\
 f(w_1) &= 3, \quad f(w_i) = 5i - 1, \quad 2 \leq i \leq n \\
 f(t_1) &= 4, \quad f(t_i) = 5i, \quad 2 \leq i \leq n - 1
 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned}
 f(u_i t_i) &= 5i - 1, \quad 1 \leq i \leq n - 1 \\
 f(t_i t_{i+1}) &= 5i, \quad 1 \leq i \leq n - 1 \\
 f(u_i v_i) &= 5i - 4, \quad 1 \leq i \leq n \\
 f(u_1 w_1) &= 3, \quad f(u_i w_i) = 5i - 3, \quad 2 \leq i \leq n \\
 f(v_1 w_1) &= 2, \quad f(v_i w_i) = 5i - 2, \quad 2 \leq i \leq n
 \end{aligned}$$

Then f is a Contra Harmonic mean labeling of G. The labeling pattern of $S(P_4 \odot K_3)$ is

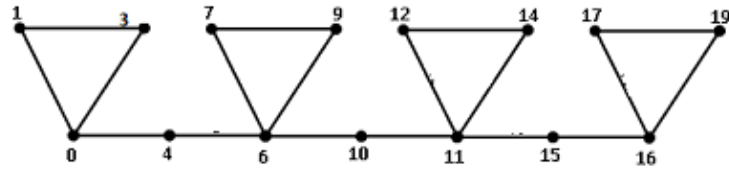


Figure 12.

Case (ii): Let G be a graph obtained by subdividing all the edges of $(P_n \odot K_3)$. Let t_i, x_i, y_i, s_i be the vertices which subdivide the edges $u_i u_{i+1}, u_i v_i, u_i w_i$ and $v_i w_i$ respectively. Define a function $f : V(G) \rightarrow \{0, 1, \dots, q\}$ by

$$\begin{aligned}
 f(u_1) &= 2, \quad f(u_i) = 8i - 5, \quad 2 \leq i \leq n \\
 f(v_1) &= 0, \quad f(v_i) = 8i - 9, \quad 2 \leq i \leq n \\
 f(s_1) &= 3, \quad f(s_i) = 8i - 6, \quad 2 \leq i \leq n \\
 f(w_i) &= 8i - 4, \quad 1 \leq i \leq n \\
 f(t_i) &= 8i + 1, \quad 1 \leq i \leq n - 1 \\
 f(x_1) &= 1, \quad f(x_i) = 8i - 8, \quad 2 \leq i \leq n \\
 f(y_i) &= 8i - 2, \quad 1 \leq i \leq n
 \end{aligned}$$

Then the distinct edge labels are

$$\begin{aligned}
 f(u_i t_i) &= 8i - 1, \quad 1 \leq i \leq n - 1 \\
 f(t_i u_{i+1}) &= 8i + 3, \quad 1 \leq i \leq n - 1 \\
 f(u_i x_i) &= 8i - 6, \quad 1 \leq i \leq n \\
 f(u_i y_i) &= 8i - 3, \quad 1 \leq i \leq n \\
 f(x_1 v_1) &= 1, \quad f(x_i v_i) = 8i - 8, \quad 2 \leq i \leq n \\
 f(y_i w_i) &= 8i - 2, \quad 1 \leq i \leq n \\
 f(v_1 s_1) &= 3, \quad f(v_i s_i) = 8i - 7, \quad 2 \leq i \leq n \\
 f(s_i w_i) &= 8i - 4, \quad 1 \leq i \leq n
 \end{aligned}$$

$\therefore f$ is a Contra Harmonic mean labeling of G . The labeling pattern of $S(P_4 \odot K_3)$ is shown below.

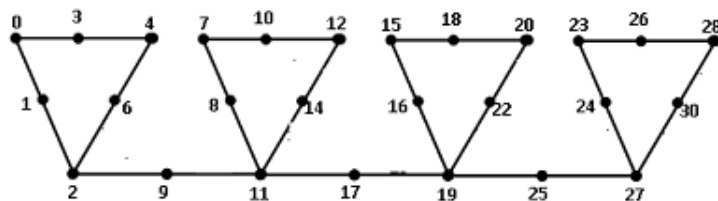


Figure 13.

From case (i), case (ii) it can be seen that $S(P_n \odot K_3)$ is a Contra Harmonic mean graph. □

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