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$\mathrm{T}_{(1,2)^{\star}\text{-}\psi}$-spaces

Research Article

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Abstract: In this paper, we introduce the notions called $\mathrm{T}_{(1,2)^{\star}\text{-}\psi}$-spaces, $\mathrm{gT}_{(1,2)^{\star}\text{-}\psi}$-spaces and $\alpha\mathrm{T}_{(1,2)^{\star}\text{-}\psi}$-spaces and obtain their properties and characterizations.

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1. Introduction

Levine [6] introduced the notion of $T_{\frac{1}{2}}$-spaces which properly lies between T_1-spaces and T_0-spaces. Many authors studied properties of $T_{\frac{1}{2}}$-spaces: Dunham [3], Arenas et al. [1] etc. In this paper, we introduce the notions called $T_{(1,2)^{\star}\text{-}\psi}$-spaces, $gT_{(1,2)^{\star}-\psi}$-spaces and $\alpha T_{(1,2)^{\star}-\psi}$-spaces and obtain their properties and characterizations.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space.

Definition 2.1. *Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [9] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.*

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 ([9])**.** *Let S be a subset of a bitopological space X. Then*

(1). the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup\{F : F \subseteq S$ and F is $\tau_{1,2}$-open$\}$.

(2). the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap\{F : S \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

Definition 2.3. *A subset A of a bitopological space X is called*

(1). $(1,2)^{\star}$-semi-open [10] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A));

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(2). $(1,2)^\star$-α-open [5] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)));

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.4. *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(1). $(1,2)^\star$-g-closed [15] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-g-closed set is called $(1,2)^\star$-g-open;

(2). $(1,2)^\star$-sg-closed [10] if $(1,2)^\star$-scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $(1,2)^\star$-semi-open in X. The complement of $(1,2)^\star$-sg-closed set is called $(1,2)^\star$-sg-open;

(3). $(1,2)^\star$-gs-closed [10] if $(1,2)^\star$-scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-gs-closed set is called $(1,2)^\star$-gs-open;

(4). $(1,2)^\star$-αg-closed [12] if $(1,2)^\star$-αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-αg-closed set is called $(1,2)^\star$-αg-open;

(5). $(1,2)^\star$-$\hat{g}$-closed [2] or $(1,2)^\star$-ω-closed [4] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $(1,2)^\star$-semi-open in X. The complement of $(1,2)^\star$-$\hat{g}$-closed (resp. $(1,2)^\star$-ω-closed) set is called $(1,2)^\star$-$\hat{g}$-open (resp. $(1,2)^\star$-ω-open);

(6). $(1,2)^\star$-$\ddot{g}_\alpha$-closed [7] if $(1,2)^\star$-αcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $(1,2)^\star$-sg-open in X. The complement of $(1,2)^\star$-$\ddot{g}_\alpha$-closed set is called $(1,2)^\star$-$\ddot{g}_\alpha$-open;

(7). $(1,2)^\star$-gsp-closed [14] if $(1,2)^\star$-spcl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-gsp-closed set is called $(1,2)^\star$-gsp-open;

(8). $(1,2)^\star$-ψ-closed [8] if $(1,2)^\star$scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $(1,2)^\star$-sg-open in X. The complement of $(1,2)^\star$-ψ-closed set is called $(1,2)^\star$-ψ-open.

Remark 2.5. *The collection of all $(1,2)^\star$-$\ddot{g}_\alpha$-closed (resp. $(1,2)^\star$-gsp-closed, $(1,2)^\star$-$\hat{g}$-closed, $(1,2)^\star$-g-closed, $(1,2)^\star$-gs-closed, $(1,2)^\star$-αg-closed, $(1,2)^\star$-sg-closed, $(1,2)^\star$-ψ-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-semi-closed) sets is denoted by $(1,2)^\star$-$\ddot{G}_\alpha C(X)$ (resp. $(1,2)^\star$-GSPC(X), $(1,2)^\star$-$\hat{G}C(X)$, $(1,2)^\star$-GC(X), $(1,2)^\star$-GSC(X), $(1,2)^\star$-αGC(X), $(1,2)^\star$-SGC(X), $(1,2)^\star$-$\psi C(X)$, $(1,2)^\star$-$\alpha C(X)$, $(1,2)^\star$-SC(X)). We denote the power set of X by P(X).*

Definition 2.6. *A subset A of a bitopological space X is called $(1,2)^\star$-preopen [13] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)). The complement of a $(1,2)^\star$-pre-open set is called $(1,2)^\star$-preclosed.*
The $(1,2)^\star$-preclosure [13] of a subset A of X, denoted by $(1,2)^\star$-pcl(A) is defined to be the intersection of all $(1,2)^\star$-preclosed sets of X containing A. It is known that $(1,2)^\star$-pcl(A) is a preclosed set. For any subset A of an arbitrarily chosen bitopological space, the $(1,2)^\star$-semi-interior [10] (resp. $(1,2)^\star$-α-interior [13], $(1,2)^\star$-preinterior [13]) of A, denoted by $(1,2)^\star$-sint(A) (resp. $(1,2)^\star$-αint(A), $(1,2)^\star$-pint(A)), is defined to be the union of all $(1,2)^\star$-semi-open (resp. $(1,2)^\star$-α-open, $(1,2)^\star$-preopen) sets of X contained in A.

Definition 2.7 ([11])**.** *A bitopological space X is called*

(1). $(1,2)^\star$-$T_{\frac{1}{2}}$-space if every $(1,2)^\star$-g-closed subset of X is $\tau_{1,2}$-closed in X.

(2). $(1,2)^\star$-T_b-space if every $(1,2)^\star$-gs-closed subset of X is $\tau_{1,2}$-closed in X.

Definition 2.8 ([12])**.** *Let X be a bitopological space and $A \subseteq X$. We define the $(1,2)^\star$-sg-closure of A (briefly $(1,2)^\star$-sg-cl(A)) to be the intersection of all $(1,2)^\star$-sg-closed sets containing A.*

Proposition 2.9 ([16]). *Every $\tau_{1,2}$-closed set is $(1,2)^\star$-ψ-closed.*

Proposition 2.10 ([16]). *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-$\hat{g}$-closed.*

Proposition 2.11 ([16]). *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-g-closed.*

Proposition 2.12 ([16]). *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-αg-closed.*

Proposition 2.13 ([16]). *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-gsp-closed.*

Proposition 2.14 ([16]). *Every $(1,2)^\star$-ψ-closed set is $(1,2)^\star$-gs-closed.*

3. Properties of $T_{(1,2)^\star\text{-}\psi}$-spaces

We introduce the following definitions.

Definition 3.1.

(1). A bitopological space X is called $(1,2)^\star$-semi generalized-R_0 (briefly $(1,2)^\star$-sg-R_0) if and only if for each $(1,2)^\star$-sg-open set G and $x \in G$ implies $(1,2)^\star$-sg-cl$(\{x\}) \subset G$.

(2). A subset A of a bitopological space X is called $(1,2)^\star$-g-preclosed (briefly $(1,2)^\star$-g*p-closed) if $(1,2)^\star$-pcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-g-open in X. The complement of a $(1,2)^\star$-g*p-closed set is called $(1,2)^\star$-g*p-open.*

*The family of all $(1,2)^\star$-g*p-closed sets of X is denoted by $(1,2)^\star$-G*PC(X).*

Definition 3.2. *A bitopological space X is called*

(1). $(1,2)^\star$-semi generalized-T_0 (briefly $(1,2)^\star$-sg-T_0) if and only if to each pair of distinct points x, y of X, there exists a $(1,2)^\star$-sg-open set containing one but not the other.

(2). $(1,2)^\star$-semi generalized-T_1 (briefly $(1,2)^\star$-sg-T_1) if and only if to each pair of distinct points x, y of X, there exist a pair of $(1,2)^\star$-sg-open sets, one containing x but not y, and the other containing y but not x.

Definition 3.3. *A bitopological space X is called*

(1). $(1,2)^\star$-αT_b-space if every $(1,2)^\star$-αg-closed subset of X is $\tau_{1,2}$-closed in X.

(2). $(1,2)^\star$-T_ω-space if every $(1,2)^\star$-ω-closed subset of X is $\tau_{1,2}$-closed in X.

*(3). $(1,2)^\star$-T_{p*}-space if every $(1,2)^\star$-g*p-closed subset of X is $\tau_{1,2}$-closed in X.*

*(4). $(1,2)^\star$-${}^\star{}_sT_p$-space if every $(1,2)^\star$-gsp-closed subset of X is $(1,2)^\star$-g*p-closed in X.*

(5). $(1,2)^\star$-${}_\alpha T_d$-space if every $(1,2)^\star$-αg-closed subset of X is $(1,2)^\star$-g-closed in X.

(6). $(1,2)^\star$-α-space if every $(1,2)^\star$-α-closed subset of X is $\tau_{1,2}$-closed in X.

Theorem 3.4. *For a bitopological space X, each of the following statement is equivalent:*

(1). X is $(1,2)^\star$-sg-T_1.

(2). Each one point set is $(1,2)^\star$-sg-closed set in X.

Definition 3.5. *A bitopological space X is called a $T_{(1,2)^\star\text{-}\psi}$-space if every $(1,2)^\star$-ψ-closed subset of X is $\tau_{1,2}$-closed in X.*

Example 3.6. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^\star$-$\psi C(X) = \{\emptyset, \{a\}, \{b, c\}, X\}$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space.*

Example 3.7. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^\star$-$\psi C(X) = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$. Thus X is not a $T_{(1,2)^\star\text{-}\psi}$-space.*

Proposition 3.8. *Every $(1,2)^\star$-$T_{\frac{1}{2}}$-space is $T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Then the sets in $\{\emptyset, X, \{c\}, \{a, b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{c\}, \{a, b\}\}$ are called $\tau_{1,2}$-closed. Then we have $(1,2)^\star$-$GC(X) = P(X)$ and $(1,2)^\star$-$\psi C(X)$ $= \{\emptyset, X, \{c\}, \{a, b\}\}$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space but not an $(1,2)^\star$-$T_{\frac{1}{2}}$-space.*

Proposition 3.10. *Every $(1,2)^\star$-T_ω-space is $T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. Then the sets in $\{\emptyset, X, \{b\}, \{a, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b\}, \{a, c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^\star$-$\hat{G}C(X) = P(X)$ and $(1,2)^\star$-$\psi C(X) =$ $\{\emptyset, \{b\}, \{a, c\}, X\}$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space but not an $(1,2)^\star$-T_ω-space.*

Proposition 3.12. *Every $(1,2)^\star$-αT_b-space is $T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13. *In Example 3.9, we have $(1,2)^\star$-$\alpha GC(X) = P(X)$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space but not an $(1,2)^\star$-αT_b-space.*

Proposition 3.14. *Every $(1,2)^\star$-${}^*sT_p$-space and $(1,2)^\star$-T_{p*}-space is $T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15. *In Example 3.9, we have $(1,2)^\star$-$GSPC(X) = P(X)$ and $(1,2)^\star$-$G^\star PC(X) = P(X)$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space but it is neither an $(1,2)^\star$-${}^*sT_p$-space nor an $(1,2)^\star$-T_{p*}-space.*

Proposition 3.16. *Every $(1,2)^\star$-T_b-space is $T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17. *In Example 3.9, we have $(1,2)^\star$-$GSC(X) = P(X)$. Thus X is a $T_{(1,2)^\star\text{-}\psi}$-space but not an $(1,2)^\star$-T_b-space.*

Remark 3.18. *Every $T_{(1,2)^\star\text{-}\psi}$-space is $(1,2)^\star$-α-space but not conversely.*

Example 3.19. *In Example 3.7, we have $(1,2)^\star$-$\alpha C(X) = \{\emptyset, \{b\}, X\}$. Thus X is an $(1,2)^\star$-α-space but not an $T_{(1,2)^\star\text{-}\psi}$-space.*

Theorem 3.20. *For a bitopological space X, we have (1) ⇒ (2) where*

(1). X is a $T_{(1,2)^\star\text{-}\psi}$-space.

(2). Every singleton subset of X is either $(1,2)^\star$-sg-closed or $\tau_{1,2}$-open.

Proof. (1) $\Rightarrow$ (2). Assume that for some x $\in$ X, the set {x} is not $(1,2)^{\star}$-sg-closed in X. Then the only $(1,2)^{\star}$-sg-open set containing $\{x\}^c$ is X and so $\{x\}^c$ is $(1,2)^{\star}$-ψ-closed in X. By assumption $\{x\}^c$ is $\tau_{1,2}$-closed in X or equivalently {x} is $\tau_{1,2}$-open. □

Theorem 3.21. *For a bitopological space X the following properties hold:*

(1). If X is $(1,2)^{\star}$-sg-T_1, then it is $T_{(1,2)^{\star}\text{-}\psi}$.

(2). If X is $T_{(1,2)^{\star}\text{-}\psi}$, then it is $(1,2)^{\star}$-sg-T_0 .

Proof.
(1). The proof is obvious from Theorem 3.4.
(2). Let x and y be two distinct elements of X. Since the space X is $T_{(1,2)^{\star}\text{-}\psi}$, we have that {x} is $(1,2)^{\star}$-sg-closed or $\tau_{1,2}$-open. Suppose that {x} is $\tau_{1,2}$-open. Then the singleton {x} is a $(1,2)^{\star}$-sg-open set such that x $\in$ {x}and y $\notin$ {x}. Also, if {x} is $(1,2)^{\star}$-sg-closed, then X $\setminus$ {x} is $(1,2)^{\star}$-sg-open such that y $\in$ X $\setminus$ {x} and x $\notin$ X $\setminus$ {x}. Thus, in the above two cases, there exists a $(1,2)^{\star}$-sg-open set U of X such that x $\in$ U and y $\notin$ U or x $\notin$ U and y $\in$ U. Thus, the space X is $(1,2)^{\star}$-sg-T_0. □

Theorem 3.22. *For a $(1,2)^{\star}$-sg-R_0 bitopological space X the following properties are equivalent:*

(1). X is $(1,2)^{\star}$-sg-T_0.

(2). X is $T_{(1,2)^{\star}\text{-}\psi}$.

(3). X is $(1,2)^{\star}$-sg-T_1.

Proof. It suffices to prove only (1) $\Rightarrow$ (3). Let x $\neq$ y and since X is $(1,2)^{\star}$-sg-T_0, we may assume that x $\in$ U $\subseteq$ X $\setminus$ {y} for some $(1,2)^{\star}$-sg-open set U. Then x $\in$ X $\setminus$ $(1,2)^{\star}$-sg-cl({y}) and X $\setminus$ $(1,2)^{\star}$-sg-cl({y}) is $(1,2)^{\star}$-sg-open. Since X is $(1,2)^{\star}$-sg-R_0, we have $(1,2)^{\star}$-sg-cl({x}) $\subseteq$ X $\setminus$ $(1,2)^{\star}$-sg-cl({y}) $\subseteq$ X $\setminus$ {y} and hence y $\notin$ $(1,2)^{\star}$-sg-cl({x}). There exists $(1,2)^{\star}$-sg-open set V such that y $\in$ V $\subseteq$ X $\setminus$ {x} and X is $(1,2)^{\star}$-sg-T_1. □

4. $gT_{(1,2)^{\star}\text{-}\psi}$-spaces

Definition 4.1. *A bitopological space X is called an $gT_{(1,2)^{\star}\text{-}\psi}$-space if every $(1,2)^{\star}$-g-closed set in it is $(1,2)^{\star}$-ψ-closed.*

Example 4.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_{1,2}$-closed. Then X is an $gT_{(1,2)^{\star}\text{-}\psi}$-space and the space X in the Example 3.6, is not an $gT_{(1,2)^{\star}\text{-}\psi}$-space.*

Proposition 4.3. *Every $T_{(1,2)^{\star}\text{-}\psi}$-space and $gT_{(1,2)^{\star}\text{-}\psi}$-space is $(1,2)^{\star}$-$T_{1/2}$-space but not conversely.*

The converse of Proposition 4.3 need not be true as seen from the following example.

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then X is a $gT_{(1,2)^{\star}\text{-}\psi}$-space but not an $(1,2)^{\star}$-$T_{\frac{1}{2}}$-space.*

Remark 4.5. *$T_{(1,2)^{\star}\text{-}\psi}$-spaces and $gT_{(1,2)^{\star}\text{-}\psi}$-spaces are independent.*

Example 4.6. *In Example 3.6, X is $gT_{(1,2)^{\star}\text{-}\psi}$-space but not an $T_{(1,2)^{\star}\text{-}\psi}$-space.*

Example 4.7. *In Example 3.6, X is an $T_{(1,2)^\star\text{-}\psi}$-space but not an $gT_{(1,2)^\star\text{-}\psi}$-space.*

Theorem 4.8. *If X is a $gT_{(1,2)^\star\text{-}\psi}$-space, then every singleton subset of X is either $(1,2)^\star$-g-closed or $(1,2)^\star$-ψ-open.*

Proof. Assume that for some x ∈ X, the set {x} is not a $(1,2)^\star$-g-closed set in X. Then {x} is not a $\tau_{1,2}$-closed set, since every $\tau_{1,2}$-closed set is a $(1,2)^\star$-g-closed set. So $\{x\}^c$ is not $\tau_{1,2}$-open and the only $\tau_{1,2}$-open set containing $\{x\}^c$ is X itself. Therefore $\{x\}^c$ is trivially a $(1,2)^\star$-g-closed set and by assumption, $\{x\}^c$ is an $(1,2)^\star$-ψ-closed set or equivalently {x} is $(1,2)^\star$-ψ-open. □

The converse of Theorem 4.8 need not be true as seen from the following example.

Example 4.9. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a, c\}\}$ are called $\tau_{1,2}$-closed, the sets $\{a\}$ and $\{c\}$ are $(1,2)^\star$-g-closed in X and the set $\{b\}$ is $(1,2)^\star$-ψ-open. But the space X is not an $gT_{(1,2)^\star\text{-}\psi}$-space.*

5. $\alpha\mathrm{T}_{(1,2)^\star\text{-}\psi}$-spaces

Definition 5.1. *A bitopological space X is called an $\alpha T_{(1,2)^\star\text{-}\psi}$-space if every $(1,2)^\star$-αg-closed subset of X is $(1,2)^\star$-ψ-closed in X.*

Example 5.2. *In Example 3.7, X is an $\alpha T_{(1,2)^\star\text{-}\psi}$-space and the space X in the Example 3.6, is not an $T_{(1,2)^\star\text{-}\psi}$-space.*

Proposition 5.3. *Every $(1,2)^\star$-αT_b-space is an $\alpha T_{(1,2)^\star\text{-}\psi}$-space but not conversely.*

The converse of Proposition 5.3 need not be true as seen from the following example.

Example 5.4. *In Example 3.7, X is an $\alpha T_{(1,2)^\star\text{-}\psi}$-space but not an $(1,2)^\star$-αT_b-space.*

Proposition 5.5. *Every $\alpha T_{(1,2)^\star\text{-}\psi}$-space is an $(1,2)^\star$-αT_d-space but not conversely.*

Proof. Let X be an $\alpha\mathrm{T}_{(1,2)^\star\text{-}\psi}$-space and let A be an $(1,2)^\star$-αg-closed set of X. Then A is a $(1,2)^\star$-ψ-closed subset of X and by Proposition 2.11, A is $(1,2)^\star$-g-closed. Therefore X is an $(1,2)^\star$-$\alpha\mathrm{T}_d$-space. □

The converse of Proposition 5.5 need not be true as seen from the following example.

Example 5.6. *In Example 4.9, X is an $(1,2)^\star$-αT_d-space but not an $\alpha T_{(1,2)^\star\text{-}\psi}$-space.*

Theorem 5.7. *If (X, τ) is an $\alpha T_{(1,2)^\star\text{-}\psi}$-space, then every singleton subset of X is either $(1,2)^\star$-αg-closed or $(1,2)^\star$-ψ-open.*

The converse of Theorem 5.7 need not be true as seen from the following example.

Example 5.8. *In Example 4.9, the sets $\{a\}$ and $\{c\}$ are $(1,2)^\star$-αg-closed in X and the set $\{b\}$ is $(1,2)^\star$-ψ-open. But the space X is not an $\alpha T_{(1,2)^\star\text{-}\psi}$-space.*

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