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# Super Lehmer-3 Mean Labeling of Tree Related Graphs

Research Article

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**Abstract:** Let $f : V(G) \to \{1, 2, \ldots, p+q\}$ be an injective function. The induced edge labeling $f^*(e = uv)$ is defined by

$$f^*(e) = \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \quad (or) \quad \left\lfloor \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rfloor,$$

then $f$ is called Super Lehmer-3 mean labeling, if $\{f(V(G))\}\, U\, \{f(e)/e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. A graph which admits Super Lehmer-3 Mean labeling is called Super Lehmer-3 Mean graph. In this paper we prove that $P_m \Theta K_{1,n}$, $(P_m, S_n)$.

**Keywords:** Super Lehmer-3 mean graph, $P_m \Theta K_{1,n}$, $(P_m, S_n)$.


## 1. Introduction

A graph considered here are finite, undirected and simple. The vertex set and the edge set of a graph is denoted by $V(G)$ and $E(G)$ respectively. Lehmer mean is another type of generalized mean. For standard terminology and notations, we follow [2] and for the detailed survey of graph labeling we follow [1]. [3, 4] introduced the concept of Harmonic Mean Labeling of Graph and its basic results was proved. We will provide a brief summary of other in formations which are necessary for our present investigation.

**Definition 1.1.** *A graph $G = (V, E)$ with $p$ vertices and $q$ edges is called Lehmer-3 mean graph. If it is possible to label vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, 3, \ldots, q+1$ in such a way that when each edge $e = uv$ is labeled with*

$$f^*(e) = \left\lceil \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rceil \quad (or) \quad \left\lfloor \frac{f(u)^3 + f(v)^3}{f(u)^2 + f(v)^2} \right\rfloor,$$

*then the edge labels are distinct. In this case $f$ is called Lehmer-3 mean labeling of $G$.*

## 2. Main Results

**Theorem 2.1.** *The graph $P_m \Theta K_{1,n}$ is a super lehmer 3-mean graph.*

*Proof.* Let $\{u_i,\ 1 \le i \le m,\ u_{ij},\ 1 \le i \le m,\ 1 \le j \le n\}$ be the vertices and $\{e_i,\ 1 \le i \le m-1,\ e_{ij},\ 1 \le i \le m,\ 1 \le j \le n\}$ be the edges which are denoted as in Figure 1

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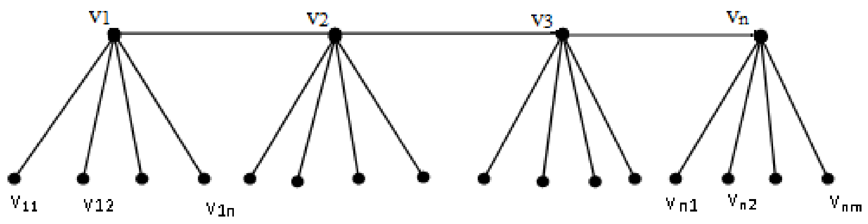


**Figure 1.** **Ordinary labeling of** $P_m \Theta K_{1,n}$

First we label the vertices as follows: Define $f : V \to \{1, 2, \ldots, p + q\}$ by

For $1 \leq i \leq m$, $f(u_i) = 2i - 1$; For $1 \leq i \leq m$, $1 \leq j \leq n$, $f(u_{ij}) = 2m + 2n(i-1) + 2j - 1$. Then the induced edge labels are:

For $1 \leq i \leq m-1$, $f^*(e_i) = 2i$; For $1 \leq i \leq m$, $1 \leq j \leq n$, $f^*(e_{ij}) = 2m + 2n(i-1) + 2(j-1)$. Therefore, the graph $P_m \Theta K_{1,n}$ is a super lehmer 3-mean graph. Super lehmer 3-mean labeling of $P_4 \Theta K_{1,4}$ is shown in Figure 2

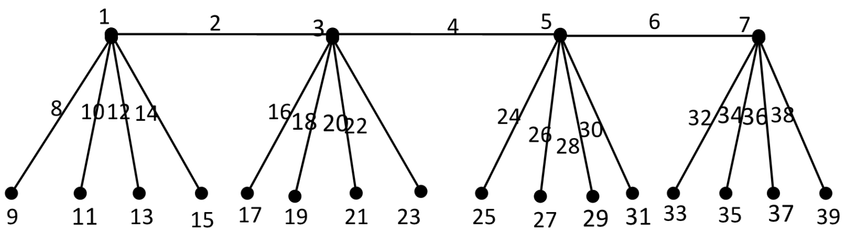


**Figure 2.** **Super Lehmer-3 mean graph** $P_4 \Theta K_{1,4}$

□

**Theorem 2.2.** *The graph* $(P_m, S_n)$ *is a super lehmer-3 mean graph.*

*Proof.* Let $\{u_i, v_i,\ 1 \leq i \leq m,\ v_{ij},\ 1 \leq i \leq m,\ 1 \leq j \leq n\}$ be the vertices and $\{a_i,\ 1 \leq i \leq m-1,\ \ b_i,\ 1 \leq i \leq m,\ e_{ij},\ 1 \leq i \leq m,\ 1 \leq j \leq n\}$ be the edges which are denoted as in Figure 3

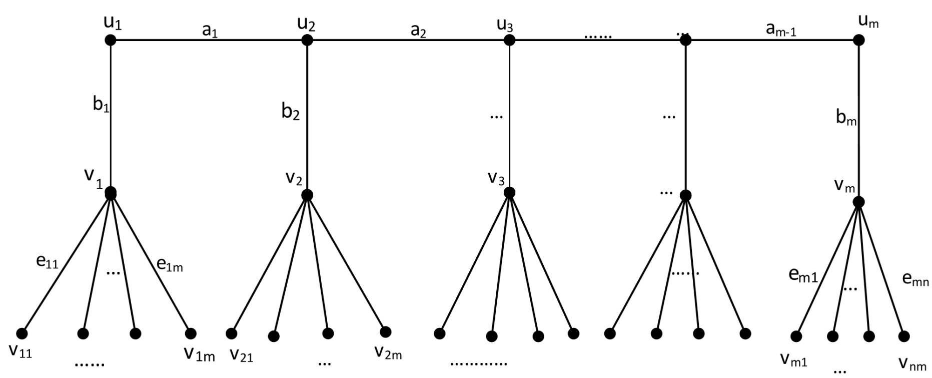


**Figure 3.** **Ordinary labeling of** $(P_m, S_n)$

First we label the vertices as follows: Define $f : V \to \{1, 2, \ldots, p + q\}$ by

For $1 \leq i \leq m$

$f(u_i) = 2i - 1$

$f(vi) = 2n + 2(m+1)(i-1) + 1$

For $1 \leq i \leq m$, $1 \leq j \leq n$, $f(v_{ij}) = 2n + 2(m+1)(i-1) + 2j + 1$. Then the induced edge labels are:

For $1 \leq i \leq m-1$, $f^*(a_i) = 2i$

For $1 \leq i \leq m$, $f^*(b_i) = 2n + 2(m+1)(i-1)$

For $1 \leq i \leq m$, $1 \leq i \leq n$, $f^*(e_{ij}) = 2n + 2(m+1)(i-1) + 2j$. Therefore, the graph $(P_m, S_n)$ is a super lehmer 3-mean graph. Super lehmer 3-mean labeling of $(P_4, S_4)$ is shown in Figure 4

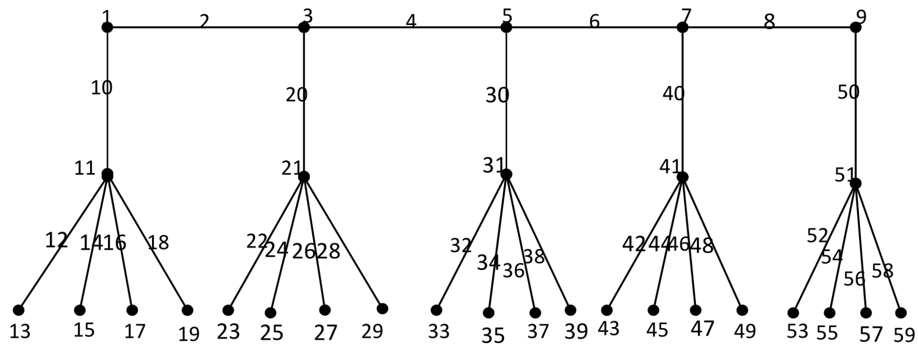


**Figure 4.** **Super Lehmer-3 mean graph** $(P_4, S_4)$

□

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