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Further Odd-Even Sum Labeling Graphs

Research Article

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Abstract: A (p, q) graph $G = (V, E)$ is said to be an odd-even sum graph if there exists an injective function $f : V(G) \to \{\pm 1, \pm 3 \pm 5, ..., \pm(2p-1)\}$ such that the induced mapping $f^* : E(G) \to \{2, 4, 6, ..., 2q\}$ defined by $f^*(uv) = f(u) + f(v)\ \forall\ uv \in E(G)$ is bijective. The function f is called an odd-even sum labeling of G. In this paper, odd-even sum labeling of subdivision of star $K_{1,n} (n \geq 1)$,Subdivision of bistar $B_{m,n}$,and H-graphs are studied.

MSC: 05C78.

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1. Introduction

Graphs considered in this paper are finite, undirected and without loops or multiple edges. Let $G = (V, E)$ be a graph with p vertices and q edges. Terms not defined here are used in the sense of Harary [3]. For number theoretic terminology [1] is followed.A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions.

If the domain of the mapping is the set of vertices(edges/both) then the labeling is called a vertex(edge/total) labeling. There are several types of graph labeling and a detailed survey is found in [6]. Harary [4] introduced the notion of a sum graph. A graph $G = (V, E)$ is called a sum graph if there is an bijection f from V to a set of $+ve$ integers S such that $xy \in E$ if and only if $(f(x) + f(y)) \in S$. In 1991 Harary [5] defined a real sum graph. An injective function $f : V(G) \to \{0, 1, 2, ..., q\}$ is an odd sum labeling [2] if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)\ \forall\ uv \in E(G)$ is bijective and $f^* : E(G) \to \{1, 3, 5, ..., 2q - 1\}$.

A graph is said to be an odd sum graph if it admits an odd sum labeling. D.Ramya et al. introduced Skolem Even-Vertex-odd difference mean labeling in [10]. Ponraj et al. [9] defined pair sum labeling. An injective function $f : V(G) \to \{\pm 1, \pm 3 \pm 5, ..., \pm(2p-1)\}$ is an odd-even sum labeling [7] if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)\ \forall\ uv \in E(G)$ is bijective and $f^* : E(G) \to \{2, 4, 6, ..., 2q\}$. A graph is said to be an odd-even sum graph if it admits an odd-even sum labeling. In this paper, odd-even sum labeling of some other graphs are studied. The following definitions are used in the subsequent section.

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Definition 1.1 ([3]). *A complete bipartite graph $K_{1,n}(n \geq 1)$ is called a star and it has $n+1$ vertices and n edges.*

Definition 1.2 ([3]). *Bistar $B_{m,n}$ is the graph obtained from a copy of star $K_{1,m}$ and a copy of star $K_{1,n}$ by joining the vertices of maximum degree by an edge.*

Definition 1.3 ([11]). *A subdivision of a graph G is a graph that can be obtained from G by sequence of edge subdivisons.*

Definition 1.4 ([8]). *The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd $v_{\frac{n}{2}}$ and $u_{\frac{n}{2}+1}$ if n is even.*

2. Main Results

Theorem 2.1. *Subdivision of Star $K_{1,n}(n \geq 1)$ is an odd-even sum graph.*

Proof. Let $S(K_{1,n})$ is a subdivision of star $K_{1,n}$. Let $v, v_1, v_2, ..., v_n$ be the vertices of $K_{1,n}$. Let $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the vertices obtained by subdivision of every edge exactly once. Let $V(S(K_{1,n})) = \left\{u, v_i, v_i^{'}/1 \leq i \leq n\right\}$.
Let $E(S(K_{1,n})) = \left\{uv_i^{'}, v_i^{'}v_i/1 \leq i \leq n\right\}$. Then $|V(S(K_{1,n}))| = 2n+1, |E(S(K_{1,n}))| = 2n$. Let $f : V(S(K_{1,n})) \rightarrow \{\pm 1, \pm 3 \pm 5, ..., \pm(4n+1)\}$ be defined as follows.

$$f(v) = 4n+1$$
$$f(v_{i+1}) = 4i+3; \ \ 0 \leq i \leq n-1$$
$$f(v_{i+1}^{'}) = -1-2i; \ \ 0 \leq i \leq n-1$$

Let f^* be the induced edge labeling of f. Then

$$f^*(uv_{i+1}^{'}) = 4n-2i, \ \ 0 \leq i \leq n-1$$
$$f^*(v_{i+1}^{'}v_i) = 2+2i, \ \ 0 \leq i \leq n-1$$

The induced edge labels are $2, 4, 6, ..., 4n$ which are all distinct. Hence $S(K_{1,n})$ is an odd-even sum graph. □

Theorem 2.2. *The graph obtained by subdiving central edge of Bistar $B_{m,n}$ is an odd-even sum graph.*

Proof. Let G be the graph obtained by subdiving central edge of $B_{m,n}$. Let $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ and u be the vertices of G. Let $V(G) = \{v, w, u, v_i, w_j/1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(G) = \{vv_i, ww_j, vu, uw/1 \leq i \leq m, 1 \leq j \leq n\}$. Then $|V(G)| = m+n+3, |E(G)| = m+n+2$. Let $f : V(G) \rightarrow \{\pm 1, \pm 3 \pm 5, ..., \pm(2m+2n+5)\}$ be defined as follows.

$$f(v) = 2m+2n+5$$
$$f(w) = 2m+2n+3$$
$$f(v_i) = -1-2i; \ \ 0 \leq i \leq m-1$$
$$f(u) = f(v_n) - 2$$
$$f(w_j) = f(u) - 2j; \ \ 1 \leq j \leq n$$

Let f^* be the induced edge labeling of f. Then

$$f^*(vv_i) = 2m + 2n + 4 - 2i; \;\; 0 \le i \le m-1$$
$$f^*(vu) = 2n + 4$$
$$f^*(uw) = 2n + 2$$
$$f^*(ww_j) = 2n - 2j; \;\; 0 \le j \le n-1$$

The induced edge labels are $2, 4, 6, ..., 2m + 2n + 4$ which are all distinct. Hence G is an odd-even sum graph. □

Theorem 2.3. *The graph obtained by subdiving all the edges except central edge of Bistar $B_{m,n}$ is an odd-even sum graph.*

Proof. Let G be the graph obtained by subdiving all the edges except central edge of Bistar $B_{m,n}$. Let $v, w, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ and $v_1^{'}, v_2^{'}, ..., v_n^{'}, w_1^{'}, w_2^{'}, ..., w_n^{'}$ be the vertices of G.
Let $V(G) = \left\{v, w, v_i, w_j, v_i^{'}, w_j^{'}/1 \le i \le m, 1 \le j \le n\right\}$ and $E(G) = \left\{vv_i^{'}, v_i^{'}v_i, ww_j^{'}, w_j^{'}w_j, vw/1 \le i \le m, 1 \le j \le n\right\}$. Then $|V(G)| = 2m + 2n + 2, |E(G)| = 2m + 2n + 1$. Let $f : V(G) \to \{\pm 1, \pm 3 \pm 5, ..., \pm(4m + 4n + 3)\}$ be defined as follows.

$$f(v) = 4m + 4n + 3$$
$$f(v_{i+1}^{'}) = -1 - 2i; \;\; 0 \le i \le m-1$$
$$f(w) = f(v_n^{'}) - 2$$
$$f(v_{i+1}) = 3 + 4i; \;\; 0 \le i \le m-1$$
$$f(w_j) = f(w) - 2j; \;\; 1 \le j \le n$$
$$f(w_{j+1}^{'}) = 4m + 3 + 4j; \;\; 0 \le j \le n-1$$

Let f^* be the induced edge labeling of f. Then

$$f^*(vv_i^{'}) = 4m + 4n + 2 - 2i; \;\; 0 \le i \le m-1$$
$$f^*(v_{i+1}^{'}v_{i+1}) = 2 + 2i; \;\; 0 \le i \le m-1$$
$$f^*(vw) = 2m + 4n + 2$$
$$f^*(ww_{j+1}^{'}) = 2m + 4n - 2j; \;\; 0 \le j \le n-1$$
$$f^*(w_{j+1}^{'}w_{j+1}) = 2m + 2j, \;\; 1 \le j \le n$$

The induced edge labels are $2, 4, 6, ..., 4m + 4n + 2$ which are all distinct. Hence G is an odd-even sum graph. □

Theorem 2.4. *The graph obtained by subdiving all the edges of Bistar $B_{m,n}$ is an odd-even sum graph.*

Proof. Let G be the graph obtained by subdiving all the edges of Bistar $B_{m,n}$. Let $v, w, u, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n$ and $v_1^{'}, v_2^{'}, ..., v_n^{'}, w_1^{'}, w_2^{'}, ..., w_n^{'}$ be the vertices of G.
Let $V(G) = \left\{v, u, w, v_i, w_j, v_i^{'}, w_j^{'}/1 \le i \le m, 1 \le j \le n\right\}$ and $E(G) = \left\{vv_i^{'}, v_i^{'}v_i, ww_j^{'}, w_j^{'}w_j, vu, uw/1 \le i \le m, 1 \le j \le n\right\}$.
Then $|V(G)| = 2m + 2n + 3, |E(G)| = 2m + 2n + 2$. Let $f : V(G) \to \{\pm 1, \pm 3 \pm 5, ..., \pm(4m + 4n + 5)\}$ be defined as follows.

$$f(v) = 4m + 4n + 5$$
$$f(v_{i+1}^{'}) = -1 - 2i; \;\; 0 \le i \le m-1$$
$$f(v_{i+1}) = 3 + 4i; \;\; 0 \le i \le m-1$$

$$f(u) = f(v_n^{'}) - 2$$
$$f(w) = 4m + 4n + 3$$
$$f(w_j^{'}) = f(u) - 2j; \;\; 1 \leq j \leq n$$
$$f(w_{j+1}) = 4m + 5 + 4j; \;\; 0 \leq j \leq n - 1$$

Let f^* be the induced edge labeling of f. Then

$$f^*(vv_i^{'}) = 4m + 4n + 4 - 2i; \;\; 0 \leq i \leq m - 1$$
$$f^*(v_{i+1}^{'}v_{i+1}) = 2 + 2i; 0 \leq i \leq m - 1$$
$$f^*(vu) = 2m + 4n + 4$$
$$f^*(vu) = 2m + 4n + 2$$
$$f^*(ww_{j+1}^{'}) = 2m + 4n - 2j; \;\; 0 \leq j \leq n - 1$$
$$f^*(w_{j+1}^{'}w_{j+1}) = 2m + 2j, \;\; 1 \leq j \leq n$$

The induced edge labels are $2, 4, 6, ..., 4m + 4n + 4$ which are all distinct. Hence G is an odd-even sum graph. □

Theorem 2.5. *The H-graph of a path P_n is an odd-even sum graph.*

Proof. Let G be the H-graph. Let $u_1, u_2, ..., u_n, v_1, v_2, ..., v_n$ be the vertices of G. Let $V(G) = \{v_i, u_i/1 \leq i \leq n\}$ and $E(G) = \{v_iv_{i+1}, u_iu_{i+1}/1 \leq i \leq n - 1\} \cup \{u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$ if n is odd $u_{\frac{n}{2}}v_{\frac{n}{2}+1}$ if n is even $\}$. Then $|V(G)| = 2n, |E(G)| = 2n - 1$. Let $f : V(G) \to \{\pm 1, \pm 3 \pm 5, ..., \pm(4n - 1)\}$ be defined as follows.

Case(i): if n is odd

$$f(u_{2i+1}) = (4n - 1) - 2i; \;\; 0 \leq i \leq \frac{n-1}{2}$$
$$f(u_{2i+2}) = -1 - 2i; \;\; 0 \leq i < \frac{n-1}{2}$$
$$f(v_{2i-1}) = f(v_{n-1}) - 2i; \;\; 1 \leq i \leq \frac{n+1}{2}$$
$$f(v_{2i}) = f(v_n) - 2i; \;\; 1 \leq i \leq \frac{n-1}{2}$$

Case(ii): if n is even

$$f(u_{2i+1}) = (4n - 1) - 2i; \;\; 0 \leq i < \frac{n}{2}$$
$$f(u_{2i+2}) = -1 - 2i; \;\; 0 \leq i < \frac{n}{2}$$
$$f(v_{2i-1}) = f(v_{n-1}) - 2i; \;\; 1 \leq i \leq \frac{n}{2}$$
$$f(v_{2i}) = f(v_n) - 2i; \;\; 1 \leq i \leq \frac{n}{2}$$

Let f^* be the induced edge labeling of f. Then, if n is odd

$$f^*(u_iu_{i+1}) = 4n - 2i; \;\; 1 \leq i \leq n - 1$$
$$f^*(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}) = 2n$$
$$f^*(v_iv_{i+1}) = 2n - 2i; \;\; 1 \leq i \leq n - 1$$

if n is even

$$f^*(u_iu_{i+1}) = 4n - 2i;\ \ 1 \leq i \leq n-1$$
$$f^*(v_{\frac{n}{2}}u_{\frac{n}{2}+1}) = 2n$$
$$f^*(v_iv_{i+1}) = 2n - 2i;\ \ 1 \leq i \leq n-1$$

The induced edge labels are $2, 4, 6, ..., 4n-2$ which are all distinct. Hence G is an odd-even sum graph. □

References

[1] M.Apostal, *Introduction to Analytic Number Theory*, Narosa Publishing House, Second Edition, (1991).

[2] S.Arockiaraj, P.Mahalakshmi and P.Namasivayam, *Odd Sum Labeling Of Some Subdivision Graphs*, Kragujevac Journal of Mathematics, 38(1)(2014), 203-222.

[3] Frank Harary, *Graph Theory*, Narosa Publishing House, New Delhi, (2001).

[4] F.Harary, *Sum Graphs and Difference graphs*, Congr.Numer., 72(1990), 101-108.

[5] F.Harary, *Sum Graphs over all the integers*, Discrete Math., 124(1994), 99-105.

[6] Joseph A.Gallian, *A Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinatorics, (2015), #DS6.

[7] K.Monika and K.Murugan, *Odd-Even Sum labeling of Some Graphs*, International Journal of Mathematics and Soft Computing, 7(1)(2017), 57-63.

[8] K.Murugan and A.Subramanian, *Labeling of Subdivided Graphs*, American Journal of Mathematics and Sciences, 4(1)(2014), 129-137.

[9] R.Ponraj and J.V.X.Parthipan, *Pair Sum Labeling of Graphs*, J.Indian Acad.Math., 32(2)(2010), 587-595.

[10] D.Ramya, R.Kalaiyarasi and P.Jeyanthi, *Skolem Odd-Difference Mean Graphs*, Journal of Algorithms and Computation, 45(2014), 1-12.

[11] T.Tharmaraj and P.B.Sarasija, *Square Graceful Graphs*, International Journal of Mathematics and Soft Computing, 1(1)(2015), 119-127.