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Properties of (i, j)-β-compact Spaces

Research Article

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Abstract: A kind of new (i, j)-β-compactness axiom is introduced in L-bitopological spaces, where L is a fuzzy lattice. And its topological properties are systematically studied.

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Keywords: L-bitopology; (i, j)-β-open set, (i, j)-β-compact set.

1. Introduction

It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [3], various kinds of fuzzy compactness [3, 6, 10] have been established. However, these concepts of fuzzy compactness rely on the structure of L and L is required to be completely distributive. In [9], for a complete De Morgan algebra L, Shi introduced a new definition of fuzzy comactness in L-topological spaces using open L-sets and their inequality. This new definition does not depend on the structure of L. In this paper, A kind of new (i, j)-β-compactness axiom is introduced in L-bitopological spaces, where L is a fuzzy lattice. And its topological properties are systematically studied.

2. Preliminaries

Throughout this paper X and Y will be nonempty ordinary sets and $L = L(\leq, \vee, \wedge^{'})$ will denote a fuzzy lattice, that is, a completely distributive lattice with a smallest element 0 and largest element 1 $(0 \neq 1)$ and with an order reversion involution $a \to a^{'} (a \in L)$. We shall denote by L^X the lattice of all L-subsets of X and if $A \in X$ by χ_A the characteristic function of A. An L-topological space is a pair (X, τ), where τ is a subfamily of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. τ is called an L-topology on X. Each member of τ is called an open L-set and its quasi complementation is called a closed L-set. An L-bitopological space (or L-bts for short) is an ordered triple (X, τ_1, τ_2), where τ_1 and τ_2 are subfamilies of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. An L-bitopological space (or L-bts for short) is an ordered triple (X, τ_1, τ_2), where τ_1 and τ_2 are subfamilies of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. An element p of L is called prime if and only if $p \neq 1$ and whenever $a, b \in L$ with $a \wedge b \leq p$ then $a \leq p$ or $b \leq p$ [5, 6]. The set of all prime elements of L will be denoted by $pr(L)$. An element α of L is called union

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irreducible or coprime if and only if whenever $a, b \in L$ with $\alpha \leq a \vee b$ then $\alpha \leq a$ or $\alpha \leq b$ [5]. The set of all non zero union irreducible elements of L will be denoted by $M(L)$. It is obvious that $p \in pr(L)$ if and only if $p^{'} \in M(L)$. Warner [12] has determined the prime element of the fuzzy lattice L^X. We have $pr(L^X) = \{x_p : x \in X \text{ and } p \in pr(L)\}$, where for each $x \in X$ and each $p \in pr(L), x_p : X \to L$ is the L-subset defined by

$$x_p(y) = \begin{cases} p & \text{if y=x,} \\ 1 & \text{otherwise.} \end{cases}$$

These x_p are called the L-points of X and we say that x_p is a member of an L-subset f and write $x_p \in f$ if and only if $f(x) \nleq p$. Thus, the union irreducible elements of L^X are the function $x_\alpha : X \to L$ defined by

$$x_\alpha(y) = \begin{cases} \alpha & \text{if y=x,} \\ 0 & \text{otherwise,} \end{cases}$$

where $x \in X$ and $\alpha \in M(L)$. Hence, we have $M(L^X) = \{x_\alpha : x \in X \text{ and } \alpha \in M(L)\}$. As these x_α are identified with the L-points x_p of X, we shall refer to them as fuzzy points. When $x_\alpha \in M(L^X)$, we hall x and α the support of x_α $(x = Suppx_\alpha)$ and the height of $x_\alpha(\alpha = h(x_\alpha))$, respectively.

Definition 2.1 ([1]). *Let (X, τ_1, τ_2) be an L-bts, $A \in L^X$. Then A is called an (i,j)-β-open set if $A \leq j\operatorname{Cl}(i\operatorname{Int}(j\operatorname{Cl}(A)))$. The complement of an (i,j)-β-open set is called an (i,j)-β-closed set. Also, (i,j)-$\beta O(L^X)$ and (i,j)-$\beta C(L^X)$ will always denote the family of all (i,j)-β-open sets and (i,j)-β-closed sets respectively. Obviously, $A \in (i,j)$-$\beta O(L^X)$ if and only if $A' \in (i,j)$-$\beta C(L^X)$.*

Definition 2.2 ([1]). *Let (L^X, τ_1, τ_2) be an L-bitopological space, $A, B \in L^X$. Let (i,j)-$\beta\operatorname{Int}(A) = \vee\{B \in L^X | B \leq A, B \in (i,j)$-$\beta O(L^X)\}, (i,j)$-$\beta\operatorname{Cl}(A) = \wedge\{B \in L^X | A \leq B, B \in (i,j)$-$\beta C(L^X)\}$. Then (i,j)-$\beta\operatorname{Int}(A)$ and (i,j)-$\beta\operatorname{Cl}(A)$ are called the (i,j)-β-interior and (i,j)-β-closue of A respectively.*

Definition 2.3 ([11]). *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two L-bitopological spaces. A function $f(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called (i,j)-β-continuous if and only if $f^{-1}(g)$ is (i,j)-β-open in (X, τ_1, τ_2) for each $g \in \sigma_i$.*

Definition 2.4 ([2]). *Let $\alpha \in M(L)$ and $g \in L^X$. A collection η of L-subsets is said to form an α-level filter base in the L-subset g if and only if for any finite subcollection $\{f_1, ..., f_n\}$ of η, there exists $x \in X$ with $g(x) \geq \alpha$ such that $(\bigwedge_{i=1}^{n} f_i)(x) \geq \alpha$. When g is the whole space X, then η is an α-lever filter base if and only if for any finite subcollection $\{f_1, ..., f_n\}$ of η,t here exists $x \in X$ such that $(\bigwedge_{i=1}^{n} f_i)(x) \geq \alpha$.*

Lemma 2.5 ([9]). *Let (X, τ) be a topological space, f be an L-subset in the L-ts $(X, \omega(\tau))$ and $p \in pr(L)$. Then we have*

1. $(\operatorname{Cl}(f))^{-1}(\{t \in L : t \nleq p\}) \subset \operatorname{Cl}(f^{-1}(\{t \in L : t \nleq p\}))$.
2. $(\operatorname{Int}(f))^{-1}(\{t \in L : t \nleq p\}) \subset \operatorname{Int}(f^{-1}(\{t \in L : t \nleq p\}))$.

Lemma 2.6 ([9]). *Let (X, τ) be a topological space and $A \subset X$. Considering the L-ts $(X, \omega(\tau))$ and*

$$f(x) = \begin{cases} e \in L & \text{if } x \in A, \\ 0 & \text{otherwise,} \end{cases}$$

we have the following

$$\operatorname{Cl}(f)(x) = \begin{cases} e & \text{if } x \in \operatorname{Cl}(A), \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\operatorname{Int}(f)(x) = \begin{cases} e & \text{if } x \in \operatorname{Int}(A), \\ 0 & \text{otherwise}, \end{cases}$$

Definition 2.7 ([2]). *Let (X, τ) be an L-ts and $g \in L^X, r \in L$.*

1. *A collection $\mu = \{f_i\}_{i \in J}$ of L-subsets is called an r-level cover of g if and only if $(\bigvee_{i \in J} f_i)(x) \nleq r$ for all $x \in X$ with $g(x) \geq r'$. If each f_i is open then μ is called an r-level open cover of g. If g is the whole space X, then μ is called an r-level cover of X if and only if $(\bigvee_{i \in J} f_i)(x) \nleq r$ for all $x \in X$.*
2. *An r-level cover $\mu = \{f_i\}_{i \in J}$ of g is said to have a finite r-level subcover if there exists a finite subset F of J such that $(\bigvee_{i \in F} f_i)(x) \nleq r$ for all $x \in x$ with $g(x) \geq r'$.*

Definition 2.8. *Let (X, τ) be an L-ts and $g \in L^X$. Then g is said to be compact [7] if and ony if for every prime $p \in L$ and every collection $\{f_i\}_{i \in J}$ of open L-subsets with $(\bigvee_{i \in J} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p'$, there exists a finite subset F of J such that $(\bigvee_{i \in F} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p'$, that is, every p-level open cover of g has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then the L-ts (X, τ) is called compact.*

3. (i, j)-β-compactness and its Goodness

Definition 3.1. *Let (X, τ_1, τ_2) be an L-bts and $g \in L^X$. The g is called (i, j)-β-compact if and only if every p-level cover of g consisting of (i, j)-β-open L-subsets has a finite p-level subcover, where $p \in pr(L)$. If g is the whole space, then we say that the L-bts (X, τ_1, τ_2) is (i, j)-β-compact.*

Lemma 3.2. *Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. If A is (i, j)-β-open in (X, τ_1, τ_2), then χ_A is (i, j)-β-open in the L-bts $(X, \omega(\tau_1), \omega(\tau_2))$.*

Theorem 3.3. *Let (X, τ_1, τ_2) be a bitopological space. Then (X, τ_1, τ_2) is (i, j)-β-compact if and only if the L-bts $(X, \omega(\tau_1), \omega(\tau_2))$ is (i, j)-β-compact.*

Proof. Let $p \in pr(L)$ and $\{f_i\}_{i \in J}$ be a p-level (i, j)-β-open cover of $(X, \omega(\tau_1), \omega(\tau_2))$. Then $(\bigvee_{i \in J} f_i)(x) \nleq p$ for all $x \in X$. Hence for each $x \in X$ there is $i \in J$ such that $f_i(x) \nleq p$, that is, $x \in f_i^{-1}(\{t \in L : t \nleq p\})$. So, $X = \cup_{i \in J} f_i^{-1}(\{t \in L : t \nleq p\})$. Because f_i is (i, j)-β-open in $(X, \omega(\tau_1), \omega(\tau_2))$, there is an (i, j)-preopen L-subset g_i in $(X, \omega(\tau_1), \omega(\tau_2))$ such that $g_i \leq f_i \leq \operatorname{Cl}(g_i)$ for every $\in J$. Hence by Lemma 2.5, we get $g_i^{-1}(\{t \in L : t \nleq p\}) \subset f_i^{-1}(\{t \in L : t \nleq p\}) \subset (\operatorname{Cl}(g_i))^{-1}(\{t \in L : t \nleq p\}) \subset \operatorname{Cl}(g_i^{-1}(\{t \in L : t \nleq p\}))$, Which means that $f_i^{-1}(\{t \in L : t \nleq p\})$ is (i, j)-β-open in (X, τ_1, τ_2). Thus $\{f_i^{-1}(\{t \in L : t \nleq p\})\}_{i \in J}$ is an (i, j)-β-open cover of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is (i, j)-β-compact, there is a finite subset F of J such that $X = \cup_{i \in F} f_i^{-1}(\{t \in L : t \nleq p\})$, that is, $(\bigvee_{i \in F} f_i)(x) \nleq p$ for all $x \in X$. Hence, $(X, \omega(\tau_1), \omega(\tau_2))$ is (i, j)-β-compact.

Conversely let $\{A_i\}_{i \in J}$ be an (i, j)-β-open cover of (X, τ_1, τ_2). Then by Lemma 3.2$\{\chi_{A_i}\}_{i \in J}$ is a family of (i, j)-β-open L-subsets in $(X, \omega(\tau_1), \omega(\tau_2))$ such that $1 = (\bigvee_{i \in J} \chi_{A_i})(x) \nleq p$ for all $x \in X$ and for all $p \in pr(L)$, that is, $\{\chi_{A_i}\}_{i \in J}$ is a p-level (i, j)-β-open cover of $(X, \omega(\tau_1), \omega(\tau_2))$. Since $(X, \omega(\tau_1), \omega(\tau_2))$ is (i, j)-β-compact, there is a finite F of J such that $(\bigvee_{i \in F} \chi_{A_i})(x) \nleq p$ for all $x \in X$. Hence $(\bigvee_{i \in F} \chi_{A_i})(x) = 1$ for all $x \in X$, that is, $X = \cup_{i \in F} A_i$ and therefore (X, τ_1, τ_2) is (i, j)-β-compact. □

Theorem 3.4. *Let (X, τ_1, τ_2) be an L-bts. Then $g \in L^X$ is (i, j)-β-compact if and only if for every $\alpha \in M(L)$ and every collection $\{h_i\}_{i\in J}$ of (i, j)-β-closed L-subsets with $(\bigwedge_{i\in J} h_i)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, there is a finite subset F of J such that $(\bigwedge_{i\in F} h_i)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$.*

Proof. This follows immediately from Definition 3.1. □

Theorem 3.5. *Let (X, τ_1, τ_2) be an L-bts. Then $g \in L^X$ is (i, j)-β-compact if and only if for every $p \in pr(L)$ and every collection $\{f_i\}_{i\in J}$ of (i, j)-β-open L-subsets with $(\bigvee_{i\in J} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$.*

Proof. Let $p \in pr(L)$ and $\{f_i\}_{i\in J}$ be a collection of (i, j)-β-open L-subsets with $(\bigvee_{i\in J} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$. Then $(\bigvee_{i\in J} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p^{'}$. Since g is (i, j)-β-compact, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p^{'}$. Take an arbitrary $x \in X$. If $g^{'}(x) \leq p$, then $g^{'}(x) \vee (\bigvee_{i\in F} f_i)(x) = (\bigvee_{i\in F} f_i \vee g^{'})(x) \not\leq p$ because $(\bigvee_{i\in F} f_i)(x) \not\leq p$. If $g^{'}(x) \not\leq p$, then we have $g^{'}(x) \vee (\bigvee_{i\in F} f_i)(x) = (\bigvee_{i\in F} f_i \vee g^{'})(x) \not\leq p$. Thus, we have $(\bigvee_{i\in F} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$.

Conversely, let $p \in pr(L)$ and $\{f\}_{i\in J}$ be a p-level (i, j)-β-open cover of g. Then $(\bigvee_{i\in J} f_i)(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p^{'}$. Hence $(\bigvee_{i\in J} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$. From the hypothesis, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i \vee g^{'})(x) \not\leq p$ for all $x \in X$. Then $(\bigvee_{i\in F} f_i)(x) \not\leq p$ for all $x \in X$ with $g^{'}(x) \leq p$. Thus g is (i, j)-β-compact. □

Definition 3.6. *Let (X, τ_1, τ_2) be an L-bts, x_α be an L-point in $M(L^X)$ and $S = (S_m)_{m\in D}$ be a net. x_α is called (i, j)-β-cluster point of S if and only if for each (i, j)-β-closed L-subset f with $f(x) \not\geq \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geq n$ and $S_m \not\leq f$, that is, $h(S_m) \not\leq f$ ($SuppS_m$).*

Theorem 3.7. *Let (X, τ_1, τ_2) be an L-bts. Then $g \in L^X$ is (i, j)-β-compact if and only if every constant α-net in g, where $\alpha \in M(L)$, has an (i, j)-β-cluster point in g with height α.*

Proof. Let $\alpha \in M(L)$ and $S = (S_m)_{m\in D}$ be a constant α-net in g without any (i, j)-β-cluster point with height α in g. Then for each $x \in X$ with $g(x) \geq \alpha, x_\alpha$ is not an (i, j)-β-cluster point of S, that is, there are $n_x \in D$ and an (i, j)-β-closed L-subset f_x with $f_x(x) \not\geq \alpha$ and $S_m \leq f_x$ for each $m \geq n_x$. Let $x^1, ..., x^k$ be elements of X with $g(x^i) \geq \alpha$ for each $i \in \{1, ..., k\}$. Then there are $n_{x_1}, ..., n_{x_k} \in D$ and (i, j)-β-closed L-subset f_{x_i} with $f_{x_i}(x^i) \not\geq \alpha$ and $S_m \leq f_{x_i}$ for each $m \geq n_{x_i}$ and for each $i \in \{1, ..., k\}$. Since D is a directed set, there is $n_0 \in D$ such that $n_0 \geq n_{x_i}$ for each $i \in \{1, ..., k\}$ and $S_m \leq f_{x_i}$ for $i \in \{1, ..., k\}$ and each $m \geq n_0$. Now, consider the family $\mu = \{f_x\}_{x\in X}$ with $g(x) \geq \alpha$. Then $(\bigwedge_{f_x\in\mu} f_x)(y) \not\geq \alpha$ for all $y \in X$ with $g(y) \geq \alpha$ because $f_y(y) \not\geq \alpha$. We also have that for any finite subfamily $\upsilon = \{f_{x_1}, ..., f_{x_k}\}$ of μ, there is $y \in X$ with $g(y) \geq \alpha$ and $(\bigwedge_{i=1}^{k} f_{x_i})(y) \geq \alpha$ since $S_m \leq \bigwedge_{i=1}^{k} f_{x_i}$ for each $m \geq n_0$ because $S_m \leq f_{x_i}$ for each $i \in \{1, ..., k\}$ and for each $m \geq n_0$. Hence, by Theorem 3.5, g is not (i, j)-β-compact.

Conversely, suppose that g is not (i, j)-β-compact. Then by Theorem 3.5, there exist $\alpha \in M(L)$ and a collection $\mu = \{f_i\}_{i\in J}$ of (i, j)-β-closed L-subsets with $(\bigwedge_{i\in J} f_i)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, but for any finite subfamily υ of μ there is $x \in X$ with $g(x) \geq \alpha$ and $(\bigwedge_{f\in\upsilon} f_i)(x) \geq \alpha$. Consider the family of all finite subsets of μ, $2^{(\mu)}$, with the order $\upsilon_1 \leq \upsilon_2$ if and only if $\upsilon_1 \subset \upsilon_2$. Then $2^{(\mu)}$ is a directed set. So, writing x_α as S_υ for every $\upsilon \in 2^{(\mu)}, (X_\upsilon)_{\upsilon\in 2}(\mu)$ is a constant α-net in g because the height of S_υ for all $\upsilon \in 2^{(\mu)}$ is α and $S_\upsilon \leq g$ for all $\upsilon \in 2^{(\mu)}$, that is, $g(x) \geq \alpha$. $(S_\upsilon)_{\upsilon\in 2}(\mu)$ also satisfies the condition that for each (i, j)-β-closed L-subset $f_i \in \upsilon$ we have $x_\alpha = S_\upsilon \leq f_i$. Let $y \in X$ with $g(y) \geq \alpha$. Then $(\bigwedge_{i\in J} f_i)(y) \not\geq \alpha$, that is, there exists $j \in J$ with $f_j(y) \ngeq \alpha$. Let $\upsilon_0 = \{f_i\}$. So, for any $\upsilon \geq \upsilon_0, S_\upsilon \leq \bigwedge_{f_i \in \upsilon} f_i \leq \bigwedge_{f_i \in \upsilon_0} f_i = f_j$. Thus, we get an (i,j)-β-closed L-subset f_j with $f_j(y) \geq \alpha$ and $\upsilon_0 \in 2^{(\mu)}$ such that for any $\upsilon \geq \upsilon_0, S_\upsilon \leq f_j$. That means that $y_\alpha \in M(L^X)$ is not an (i,j)-β-cluster point $(X_\upsilon)_{\upsilon \in 2}(\mu)$ for all $y \in X$ with $g(y) \geq \alpha$. Hence, the constant α-net $(S_\upsilon)_{\upsilon \in 2}(\mu)$ has no (i,j)-β-cluster point in g with height α. □

Corollary 3.8. *An L-bts (X, τ_1, τ_2) is (i,j)-β-compact if and only if every constant α-net in (X, τ_1, τ_2) has an (i,j)-β-cluster point with height α, where $\alpha \in M(L)$.*

Definition 3.9. *Let (X, τ_1, τ_2) be an L-bts and η an α-level filter base, where $\alpha \in M(L)$. An L-point $x_r \in M(L^X)$ is called an (i,j)-β-cluster point of η, if $\bigwedge_{f \in \eta} (i,j)\text{-}\beta\,\mathrm{Cl}(f)(x) \geq r$.*

Theorem 3.10. *Let (X, τ_1, τ_2) be an L-bts. Then $g \in L^X$ is (i,j)-β-compact if and only if every α-filter base in g, where $\alpha \in M(L)$, has an (i,j)-β-cluster point x_α in g with height α.*

Proof. Assume that η is an α-level filter base in g with no (i,j)-β-cluster point in g with height α, where $\alpha \in M(L)$. Then for each $x \in X$ with $g(x) \geq \alpha, x_\alpha$ is not an (i,j)-β-cluster point of η, that is, there is $f_x \in \eta$ with $(i,j)\text{-}\beta\,\mathrm{Cl}(f_x)(x) \ngeq \alpha$. Hence $(i,j)\text{-}\beta\,\mathrm{Cl}(f_x)^{'}(x) \nleq \alpha^{'} = p \in pr(L)$. This means that the collection $\{(i,j)\text{-}\beta\,\mathrm{Cl}(f_x)^{'}\}_{x \in X}$ with $g(x) \geq \alpha$ is a p-level (i,j)-β-open cover of g. Since g is (i,j)-β-compact, there are $(i,j)\text{-}\beta\,\mathrm{Cl}(f_{x_1}), ..., (i,j)\text{-}\beta\,\mathrm{Cl}(f_{x_n})$ such that $(\bigvee_{i=1}^{n} (i,j)\text{-}\beta\,\mathrm{Cl}(f_{x_i})^{'})(x) \nleq p$ for all $x \in X$ with $g(x) \geq p^{'} = \alpha$. Hence $\bigwedge_{i=1}^{n} (i,j)\text{-}\beta\,\mathrm{Cl}(f_{x_i})(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$ which implies that $(\bigwedge_{i=1}^{n} f_{x_i})(x) \ngeq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$. This is a contradiction.

Conversely, suppose that g is not (i,j)-β-compact. Then there is a p-level (i,j)-β-open cover μ of g with no finite p-level subcover, where $p \in pr(L)$. Hence for each finite subcollection $\{h_1, ..., h_n\}$ of μ, there exists $x \in X$ with $g(x) \geq p^{'}$ such that $(\bigvee_{i=1}^{n} h_i)(x) \leq p$, that is, $(\bigvee_{i=1}^{n} h_i^{'})(x) \geq p^{'} = \alpha \in M(L)$. Thus, $\eta = \{h^{'} : h \in \mu\}$ forms an α-level filter base in g. By the hypothesis, μ has an (i,j)-β-cluster point $y_\alpha \in M(L^X)$ in g with height α, that is, $g(y) \geq \alpha$ and $\bigwedge_{h \in \mu} (i,j)\text{-}\beta\,\mathrm{Cl}(h^{'})(y) = (\bigwedge_{h \in \mu} h^{'})(y) \geq \alpha$. Then $(\bigwedge_{h \in \mu} h^{'})(y) \leq p$, which yileds a contradiction. □

Corollary 3.11. *An L-bts (X, τ_1, τ_2) is (i,j)-β-compact if and only if every α-filter base has an (i,j)-β-cluster point with height α, where $\alpha \in M(L)$.*

Theorem 3.12. *Let (X, τ_1, τ_2) be an L-bts and $g, h \in L^X$. If g and h are (i,j)-β-compact, then $g \vee h$ is (i,j)-β-compact.*

Proof. Let $p \in pr(L)$ and $\{f_i\}_{i \in J}$ be a collection of (i,j)-β-open L-subsets with $(\bigvee_{i \in J} f_i)(x) \nleq p$ for all $x \in X$ with $(g \vee h)(x) \geq p^{'}$. Since p is prime, we have $(g \vee h)(x) \geq p^{'}$ if and only if $g(x) \geq p^{'}$ or $h(x) \geq p^{'}$. So, by the (i,j)-β-compactness of g and h, there are finite subsets E, F of J such that $(\bigvee_{i \in E} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p^{'}$ and $(\bigvee_{i \in F} f_i)(x) \nleq p$ for all $x \in X$ with $h(x) \geq p^{'}$. Then $(\bigvee_{i \in E \cup F} f_i)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p^{'}$ or $h(x) \geq p^{'}$, that is, $(\bigvee_{i \in E \cup F} f_i)(x) \nleq P$ for all $x \in X$ with $(f \vee h)(x) \geq p^{'}$. Thus, $g \vee h$ is (i,j)-β-compact. □

Theorem 3.13. *Let (X, τ_1, τ_2) be an L-bts and $g, h \in L^X$. If f is (i,j)-β-compact and h is (i,j)-β-closed, then $g \wedge h$ is (i,j)-β-compact.*

Proof. Let $p \in pr(L)$ and $\{f_i\}_{i \in J}$ be a collection of (i,j)-β-open L-subsets with $(\bigvee_{i \in J} f_i)(x) \nleq p$ for all $x \in X$ with $(g \wedge h)(x) \geq p^{'}$. Thus $\mu = \{f_i\}_{i \in J} \cup \{h^{'}\}$ is a family of (i,j)-β-open L-subsets with $(\bigvee_{k \in \mu} k)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p^{'}$. In fact, for each $x \in X$ with $g(x) \geq p^{'}$, if $h(x) \geq p^{'}$, then $(g \wedge h)(x) \geq p^{'}$ which implies that $(\bigvee_{i \in J} f_i)(x) \nleq p$, thus $(\bigvee_{k \in \mu} k)(x) \nleq p$. If $h(x) \ngeq p^{'}$, then $h^{'}(x) \nleq p$ which implies $(\bigvee_{k \in \mu} k)(x) \nleq p$. From the (i,j)-β-compactness of g there is a finite subfamily υ of μ, say $\upsilon = \{f_1, ..., f_n, h^{'}\}$ with $(\bigvee_{k\in\upsilon} k)(x) \nleq p$ for all $x \in X$ with $g(x) \geq p^{'}$. Then $(\bigvee_{i=1}^{n} f_i)(x) \nleq p$ for all $x \in X$ with $(g \wedge h)(x) \geq p^{'}$. Hence $g \wedge h$ is (i,j)-β-compact. □

Corollary 3.14. *Let (X, τ_1, τ_2) be an (i,j)-β-compact space and g be an (i,j)-β-closed L-subset. Then g is (i,j)-β-compact.*

Theorem 3.15. *Let (X, τ_1, τ_2) be an L-bts where X is a finite set. Then (X, τ_1, τ_2) is (i,j)-β-compact.*

Proof. Let $\{f_i\}_{i\in J}$ be a p-level (i,j)-β-open cover of (X, τ_1, τ_2), where $p \in pr(L)$. Then $(\bigvee_{i\in J} f_i)(x) \nleq p$ for all $x \in X$. Hence, for each $x \in X$ there is $i \in J$ such that $x \in f_i^{-1}(\{t \in T : t \nleq p\})$. Since X is finite subset F of J such that $X = \underset{i\in F}{\cup} f_i^{-1}(\{t \in T : t \nleq p\})$, that is, $(\bigvee_{i\in F} f_i)(x) \nleq p$ for each $x \in X$. Hence (X, τ_1, τ_2) is (i,j)-β-compact. □

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