



Applications of Graph Theory in Different Branches of Science

Review Article

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Abstract: Graph theory is a branch of mathematics which has wide application in other area of mathematics as well as in other branches of science. It has also significant role in our everyday life. In this article we have discussed the application of graph theory in our daily life and different branches of science such as computer science, operation research, chemistry, physics, engineering etc.

Keywords: Graph, Path, Eulerian Graph, Network.

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1. Introduction

In the first part of this article, we discuss the history and origin of Graph theory and some definitions which are frequently used in studying this subject. The main origin of graph theory was the problem of Konigsberg bridge [1]. The city Konigsberg was on the both sides of the river Pregel and included two large islands which were connected to each other or to the city by seven bridges. There was a question in the mind of residents of Konigsberg whether they could travel around the city, crossing each of the seven bridges once and only once. In 1736 Leonhard Euler (1707-1783) turned his mind to the problem of Konigsberg bridge and he came to the conclusion that it did not matter how a man walked around the land or where exactly bridges were, it was not possible to do this. This led to the concept of Eulerian graph. By solving this problem Euler introduced the new branch of mathematics namely Graph theory [1]. In 1840, A.F. Mobius presented the idea of complete graph and bipartite graph. In 1845, Gustav Kirchoff introduced the concept of tree and he applied the concept of tree in calculation of currents in electrical circuits. In 1852, Thomas Guthrie found the famous four color problem. In 1856, P. Kirkman and William R. Hamilton studied the cycles of polyhydra and invented the concept of Hamiltonian graph. Although the four color problem was invented it was not solved then. This problem was solved after a century by Kenneth Appel and Wolfgang Haken [3] in 1976. It was the first major theorem to be proved using computer. The term Graph was introduced by Sylvester in 1878. This is how the graph theory was developed.

2. Preliminaries

In this section, we have listed some important definitions frequently used in Graph theory.

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Definition 2.1. A graph G is defined by (V, E, γ) where the elements of the set V are called vertices of graph G and the elements of the set E are called edges of graph G . Here γ is a function which assigns each edge of the graph G with two vertices.

Definition 2.2. A walk on a graph is a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident to its preceding and following vertices. Length of a walk is given by the number of edges in the walk.

Definition 2.3. A trail is a walk in which all edges are distinct.

Definition 2.4. A path or simple path is an open walk (walk whose beginning and ending vertices are not same) in which no vertex appears twice or more.

Definition 2.5. A Euler path is a simple path in a graph which visits every edge of the graph exactly once.

Definition 2.6. A circuit is a path which ends at the same vertex where it begins.

Definition 2.7. A graph which contains either Euler path or Euler circuit is called Eulerian Graph.

Definition 2.8. A path which contains each vertex of graph exactly once is called Hamiltonian path.

Definition 2.9. A circuit that contains each vertex of graph exactly once except for the first and the last vertex is called Hamiltonian circuit.

Definition 2.10. A graph which contains either Hamiltonian circuit or Hamiltonian path is called Hamiltonian graph.

Definition 2.11. A Di-Graph or Directed Graph consists of a vertex set V and an edge set E such that each edge e in E is associated with an ordered pair of vertices. So, if γ is a function then it assigns every edge to a ordered pair of vertices.

The difference of directed graph and the graph is in directed graph we use arrow sign in the edges means it has direction.

3. Applications of Graph Theory

The ideas and concepts of Graph theory are widely used in various branches of science. In general, without knowing the concepts of graph we also use these in our day to day life. For example when we have to go to a place which is connecting with our starting point by different ways then we use the shortest road to arrive the destination soon. Here if we observe this problem from the point of view of graph theory the two places can be considered as vertices and roads are as edges. If we also consider the direction of travel, then the graph must be directed. Similarly, we can use these concepts of graph theory in various situations. A graph can be used to present almost any physical situation involving discrete and relationship among them [2]. Here we are now going to discuss the applications of graph theory in various branches of science.

3.1. Applications in Chemistry

Graph theory is used in chemistry for mathematical modelling of chemical phenomena. We can make natural model of a molecule where vertices represent atoms and edges represent bond. There is a branch of mathematical chemistry called Chemical graph theory (CGT) which deals with the non trivial applications of graph theory to solve molecular problems. The pioneers of the chemical graph theory are Alexander Balaban, Ante Graovac, Ivan Gutman, Haruo Hosoya, Milan Randić and Nenad Trinajstić and others. Graph theory is also used in computational biochemistry.

3.2. Applications in Physics

Graph theory is also used in the field of physics. Generally, graph theory concepts are used in different electrical circuits. The current, voltage and resistance on a circuit can be drawn by using graph theory concept. When we want to show the flow of current in circuits then we can use directed graphs. Also we can connect the different physical process with the help of graph theory concepts.

3.3. Applications in Biology

Graph theory is used in many areas of biology. Graph can be used in drug target identification, determining a protein's or gene's function. The concepts of graph theory can be also used in studying the structures of DNA and RNA. If we want to study the food chain of different animals in a ecological system, then we draw some arrow diagrams which represent the dependence of one animal upon another for their food. This diagram can be considered as graph where the animals are vertices of graph and they must be connected if any one of them depends on other for food.

3.4. Applications in Computer Science

There is a major role of graph theory in computer science. Graph theory concepts are used to develop the algorithm of different programmes. Using these algorithms and programmes we can solve different theoretical problems. There are some algorithms listed below.

- (1). Shortest path algorithm in a network.
- (2). Finding minimum spanning tree.
- (3). Finding graph planarity.
- (4). Algorithms to find adjacency matrices.
- (5). Algorithms to find the connectedness.
- (6). Algorithms to find the cycles in a graph etc.

There are many computer languages which helps to solve different problems using graph theory concepts. Some computer languages available are.

- (1). GTPL - Graph Theoretic Language.
- (2). GASP - Graph Algorithm Software Package.
- (3). HINT - Extension of LISP.
- (4). GRASPE - Another extension of LISP.
- (5). DIP - Directed Graph Processor.
- (6). An Interactive Graph Theory System - Extension of FORTRAN.
- (7). GEA - Graphic Extended ALGOL.
- (8). GIRL - Graph Information Retrieval Language.
- (9). FGRAAL - FORTRAN Extended Graph Algorithmic Language.
- (10). AMBIT/G - Extension of AMBIT [4].

3.5. Applications in Operation Research

Graph theory is a very useful tool in operation research. There are some OR problems that can be solved using graphs. In transportation problem, when we need to minimize the transportation cost or maximize the profit, then the graph theoretical approach is very useful. It is also used in different assignment problems such as assigning different peoples to different jobs, manage of time table for school, college , assigning office staffs etc.

3.6. Applications in Google map

Now a days, Google map is a very useful tool for travelling anywhere in the world. Using google map we can find all routes from any place to any other place and also can find the shortest route. In case of google map, we can consider the places as vertices of graph and the routes as the edges. Then the software of google map, when find the routes between two places it find all edges between these two places or vertices and also gives the shortest edge as the shortest path.

3.7. Applications in Internet

Internet is a very useful invention of modern science. In the working technique of internet the concepts of graph theory are used. In case of connectivity of internet, all the users are considered as vertices and the connection between them are edges. Then all internet users form a very complicated graph and data and information from one user to another user are shared through the shortest route in between them. Similarly, in case of social networking sites one friend is connected to all of his friend and his friends are also connected to others. If we consider the friends as vertices of graph and define an edge in between them if they are friend then it will be a graph.

4. Conclusion

The main objective of this article is to present the importance of graph theory in different branches of science and our everyday life . Here we have discussed only a few applications of graph theory. There are many application of graph theory in different branches like economics, logistics etc. Therefore graph theory has developed into a subject itself with variety of applications.

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