

International Journal of Mathematics And its Applications

On Relaxed Skolem Mean Labling for Five Star

Research Article

A.Manshath¹, V.Balaji²* and P.Sekar³

1 Department of Mathematics, Sriram Engineering College, Thiruvallur, Tamilnadu, India.

2 Department of Mathematics, Sacred Heart College, Tirupattur, Tamilnadu, India.

3 Department of Mathematics, C. Kandasamy Naidu College, Chennai, Tamilnadu, India.

Abstract:	In this paper, we prove $\ell \leq m < n$, the five star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a Relaxed skolem mean graph if $ m-n \leq 3\ell + 6$ for $\ell = 2, 4, 3, \ldots$; $m = 2, 3, 4 \ldots$; and $3\ell + m \leq n \leq 3\ell + m + 6$.
MSC:	05C78.
Keywords:	Graphs, Relaxed Skolem mean graph, star graphs.
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1. Introduction

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harry [10]. In [5], $\ell \leq m < n$, the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m - n| \leq 6 + \ell$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$; $\ell + m \leq n \leq \ell + m + 6$. Also if $\ell \leq m < n$ the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m - n| \leq 6 + 2\ell$ for $\ell = 2, 3, 4, \ldots$; $m = 2, 3, 4, \ldots$ and $2\ell + m \leq n \leq 2\ell + m + 6$. In [4], the necessary condition for a graph to be relaxed skolem mean is that $p \geq q$.

2. Relaxed Skolem Mean Labeling

Definition 2.1. The five star is the disjoint union of $K_{1,a}, K_{1,b}, K_{1,c}, K_{1,d}, K_{1,e}$. then it is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e}$.

Definition 2.2. A graph G = (V, E) with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, 3, ..., p + 1\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, ..., p + 1\}$ defined by

$$f^*(e = uv) = \begin{cases} \frac{f(u) + f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u) + f(v) + 1}{2}, & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

then the resulting edges get distinct labels from the set $\{2, 3, 4, \ldots, p+1\}$.

Note 2.3. In a Relaxed skolem mean graph, $p \ge q$.

^{*} E-mail: pulibala70@gmail.com

Theorem 2.4. If $\ell \le m < n$, five star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a Relaxed skolem mean graph if $|m - n| \le 3\ell + 6$ for $\ell = 2, 4, 3, ...; m = 2, 3, 4, ...;$ and $3\ell + m \le n \le 3\ell + m + 6$.

Proof. Let $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$. Without loss of generality assume that $\ell \le m < n$. Hence $|m - n| \le 3\ell + 6$ Implies $n - m \le 3\ell + 6$ and it means $3\ell + m \le n \le 3\ell + 6$. There are seven cases viz: $n = 3\ell + m + 6$, $n = 3\ell + m + 5$, $n = 3\ell + m + 4$, $n = 3\ell + m + 3$, $n = 3\ell + m + 2$, $n = 3\ell + m + 1$ and $n = 3\ell + m$. Let us prove in each of the cases the graph G is a relaxed skolem mean graph.

Case (a): When $n = 3\ell + m + 6$.

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 6$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\},$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} \text{ and}$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}.$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \ f(v) = 2; \ f(w) = 4; \ f(x) = 6; \ f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 6 \ \text{for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 6 \ \text{for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 6 \ \text{for } 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 6 \ \text{for } 1 \le h \le m$$

$$f(y_s) = 2s + 1 \ \text{for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 4 for $1 \le i \le \ell$; vv_j is $\ell + j + 4$ for $1 \le i \le \ell$; ww_k is $2\ell + k + 5$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 6$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 6}{2}$; $1 \le s \le n - 2$; $yy_n - 1$ is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph. Case (b): When $n = 3\ell + m + 5$.

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 5$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} \text{ and}$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \ f(v) = 2; \ f(w) = 3; \ f(x) = 5; \ f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 7 \ \text{for} \ 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 7 \ \text{for} \ 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 7 \ \text{for} \ 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 7 \text{ for } 1 \le h \le m$$
$$f(y_s) = 2s + 2 \text{ for } 1 \le s \le n - 2$$
$$f(y_{n-1}) = 3\ell + m + n + 4$$
$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 4 for $1 \le i \le \ell$; vv_j is $\ell + j + 5$ for $1 \le j \le \ell$; ww_k is $2\ell + k + 5$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 6$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 7}{2}$; $1 \le s \le n - 2$; yy_{n-1} is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph. **Case (c):** When $n = 3\ell + m + 4$

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 4$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} and$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \ f(v) = 2; \ f(w) = 3; \ f(x) = 4; \ f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 6 \ \text{for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 6 \ \text{for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 6 \ \text{for } 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 6 \ \text{for } 1 \le h \le m$$

$$f(y_s) = 2s + 3 \ \text{for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 4 for $1 \le i \le \ell$; vv_j is $\ell + j + 4$ for $1 \le j \le \ell$; ww_k is $2\ell + k + 5$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 5$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 8}{2}$; $1 \le s \le n - 2$; yy_{n-1} is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph. Case (d): When $n = 3\ell + m + 3$.

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 3$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} \text{ and}$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; f(v) = 2; f(w) = 3; f(x) = 5; f(y) = 3\ell + m + n + 5;$$

 $f(u_i) = 2i + 5 \text{ for } 1 \le i \le \ell$

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$$f(v_j) = 2\ell + 2j + 5 \text{ for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 5 \text{ for } 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 5 \text{ for } 1 \le h \le m$$

$$f(y_s) = 2s + 4 \text{ for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 3 for $1 \le i \le \ell$; vv_j is $\ell + j + 4$ for $1 \le j \le \ell$; ww_k is $2\ell + k + 4$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 5$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 9}{2}$; $1 \le s \le n - 2$; $yy_n - 1$ is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph.

Case (e): When $n = 3\ell + m + 2$

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 2$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} \text{ and}$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \ f(v) = 2; \ f(w) = 3; \ f(x) = 5; \ f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 4 \ \text{for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 4 \ \text{for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 4 \ \text{for } 1 \le k \le \ell$$

$$f(w_k) = 6\ell + 2h + 4 \ \text{for } 1 \le h \le m$$

$$f(y_s) = 2s + 5 \ \text{for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 3 for $1 \le i \le \ell$; vv_j is $\ell + j + 3$ for $1 \le j \le \ell$; ww_k is $\ell + k + 4$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 5$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 10}{2}$; $1 \le s \le n - 2$; yy_{n-1} is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph. Case (f): When $n = 3\ell + m + 1$.

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m + 1$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} \text{ and}$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \quad f(v) = 2; \quad f(w) = 4; \quad f(x) = 6; \quad f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 6 \quad \text{for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 6 \quad \text{for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 6 \quad \text{for } 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 6 \quad \text{for } 1 \le h \le m$$

$$f(y_s) = 2s + 6 \quad \text{for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 4 for $1 \le i \le \ell$; vv_j is $\ell + j + 4$ for $1 \le j \le \ell$; ww_k is $2\ell + k + 5$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 6$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 11}{2}$; $1 \le s \le n - 2$; yy_{n-1} is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph. **Case (g):** When $n = 3\ell + m$.

We've to prove that G is a relaxed skolem mean graph $n = 3\ell + m$.

$$V(G) = \{u, v, w, x, y\} \cup \{u_i : 1 \le i \le \ell\}, \{v\} \cup \{v_j : 1 \le j \le \ell\}, \{w\} \cup \{w_k : 1 \le k \le \ell\}$$
$$\{x\} \cup \{x_h : 1 \le h \le m\}, \{y\} \cup \{y_s : 1 \le s \le n\} and$$
$$E(G) = \{uu_i : 1 \le i \le \ell\} \cup \{vv_j : 1 \le j \le \ell\} \cup \{ww_k : 1 \le k \le \ell\} \cup \{xx_h : 1 \le h \le m\} \cup \{yy_s : 1 \le s \le n\}$$

Then G has $3\ell + m + n + 5$ nodes and $3\ell + m + n$ links. The required node labeling $f: V(G) \rightarrow \{1, 2, 3, 4, \dots, 3\ell + m + n + 5\}$ is defined as follows

$$f(u) = 1; \quad f(v) = 3; \quad f(w) = 5; \quad f(x) = 7; \quad f(y) = 3\ell + m + n + 5;$$

$$f(u_i) = 2i + 2 \text{ for } 1 \le i \le \ell$$

$$f(v_j) = 2\ell + 2j + 2 \text{ for } 1 \le j \le \ell$$

$$f(w_k) = 4\ell + 2k + 2 \text{ for } 1 \le k \le \ell$$

$$f(x_h) = 6\ell + 2h + 2 \text{ for } 1 \le h \le m$$

$$f(y_s) = 2s + 7 \text{ for } 1 \le s \le n - 2$$

$$f(y_{n-1}) = 3\ell + m + n + 4$$

$$f(y_n) = 3\ell + m + n + 6$$

The corresponding link labels are as follows: The link label of uu_i is i + 2 for $1 \le i \le \ell$; vv_j is $\ell + j + 3$ for $1 \le j \le \ell$; ww_k is $2\ell + k + 4$ for $1 \le k \le \ell$; xx_h is $3\ell + h + 5$ for $1 \le h \le m$; $yy_s \frac{3\ell + m + n + 2s + 12}{2}$; $1 \le s \le n - 2$; yy_{n-1} is $3\ell + m + n + 5$; yy_n is $3\ell + m + n + 6$. Hence the induced link labels are distinct. Hence the graph G is Relaxed skolem mean graph.

3. Application of Graph Labeling

The skolem mean labeling is applied on a graph (network), such as bus topology, mesh topology and star topology in order to solve the problems in establishing fastness, efficient communication and various issues in that area, in which the following will be taken into account.

- (1). A protocol, with secured communication can be achieved, provided the graph (network) is sufficiently connected.
- (2). To find an efficient way for safer transmissions in areas such as Cellular telephony, Wi-Fi, Security systems and many more.
- (3). Channel labeling can be used to determine the time at which sensor communicate.

4. Conclusion

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

Acknowledgement

One of the authors (Dr. V. Balaji) acknowledges University Grants Commission, SERO, Hyderabad , India for financial assistance (No. FMRP 5766 / 15 (SERO / UGC)).

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