International Journal of Mathematics And its Applications
Volume 5, Issue 3–B (2017), 113–119.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Some Aspects on 2-fuzzy 2-anti Continuous and Bounded Functions

Research Article

Thangaraj Beaula[1]* and Beulah Mariya[2]

1 Department of Mathematics, T.B.M.L.College, Porayar, Tamilnadu, India.

2 Department of Mathematics, Bharathidasan University, Tiruchirapalli, Tamil Nadu, India.

Abstract: In this paper various types of 2-fuzzy 2-anti continuity and 2-fuzzy 2- anti boundedness are defined. Relationships between the types of 2-fuzzy 2-anti continuities are developed. Further the inter and intra relationship between strongly, weakly boundedness are studied.

MSC: 46S4, 03E72.

Keywords: 2-fuzzy 2-anti continuous, strongly weakly, sequentially 2-fuzzy 2-anti continuous, strongly and weakly 2-fuzzy 2-anti boundedness.

1. Introduction

The theory of fuzzy set was introduced by L.A Zadeh [12] in 1965. The idea of fuzzy norm was initiated by Katsaras [6] in 1984. A satisfactory theory of 2-norm on a linear space has been introduced and developed by Gahler [5] in 1964. The notion of 2-fuzzy 2-normed space was introduced by R.M. Somasundaram and Thangaraj Beaula [11]. In 2011 Reddy [7, 8] studied fuzzy anti 2-norm and some results are established in fuzzy anti 2-normed linear space. Sinha, Mishra, Lal [9] (2011) and Sinha, Mishra [10] (2012) introduced the concept of fuzzy anti 2-continuous linear operator and fuzzy anti 2-bounded linear operator on fuzzy anti 2-normed linear space. In 2003 Bag and Samanta [1–3] modified the definition of fuzzy norm of Cheng-Moderson [4] and established the concept of continuity and boundedness of linear operator. In this paper strong 2-fuzzy 2-anti continuity, weak 2-fuzzy 2-anti continuity, sequentially 2-fuzzy 2-anti continuity, strongly 2-fuzzy 2-anti boundedness, weakly 2-fuzzy 2-anti boundedness are defined. Further the relationships among the newly defined concepts are established.

2. Preliminaries

Definition 2.1. *A binary operation* $* : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-norm if it satisfies the following conditions*

(1). $*$ *is commutative and associative*

(2). $*$ *is continuous*

* *E-mail: edwinbeaula@yahoo.co.in*

(3). $a * 1 = a$, *for all* $a \in [0, 1]$

(4). $a * b = c * d$ *whenever* ac *and* bd *and* $a, b, c, d \in [0, 1]$.

Definition 2.2. *A binary operation* $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ *is a continuous t-conorm if it satisfies the following conditions*

(1). $\Diamond$ *is commutative and associative*

(2). $\Diamond$ *is continuous*

(3). $a \Diamond 0 = a$, *for all* $a \in [0, 1]$

(4). $a \Diamond b = c \Diamond d$ *whenever* $a = c$ *and* $b = d$ *and* $a, b, c, d \in [0, 1]$.

Definition 2.3. *Let* X *be the linear space over the real field* K. *A fuzzy subset* N *of* $F(X) \times F(X) \times R$. *(R, the set of all real numbers) is called a 2-fuzzy 2-norm on X. (or fuzzy 2-norm on F(X)) if and only if*

(N1) For all $t \in R$ *with* $t \leq 0$, $N(f_1, f_2, t) = 0$.

(N2) For all $t \in R$ *with* $t \geq 0$, $N(f_1, f_2, t) = 1$ *if and only if* f_1 *and* f_2 *are linearly dependent.*

(N3) $N(f_1, f_2, t)$ *is invariant under any permutation of* f_1, f_2.

(N4) For all $t \in R$ *with* $t \leq 0$, $N(f_1, cf_2, t) = N\left(f_1, f_2, \frac{t}{|c|}\right)$ *if* $c \neq 0$, $c \in K$ *(field).*

(N5) For all $s, t \in R$, $N(f_1, f_2 + f_3, s + t) \geq \min\{N(f_1, f_2, s), N(f_1, f_3, t)\}$.

(N6) $N(f_1, f_2, \cdot) : (0, \infty) \to [0, 1]$ *is continuous.*

(N7) $\lim_{t \to \infty} N(f_1, f_2, t) = 1$.

Then (F(X), N) is a fuzzy 2-normed linear space or (X, N) is a 2-fuzzy 2-normed linear space.

3. Fuzzy 2-anti Contiuity and Boundedness

Definition 3.1. *A mapping* $T : (A, N_1^*) \to (B, N_2^*)$ *where* A *and* B *are subsets of* $F(X)$ *is said to be 2-fuzzy 2-anti continuous at* $f_0 \in A$ *if for any given* $\varepsilon > 0$, $\alpha \in (0, 1)$ *there exists* $\delta = \delta(\alpha, \varepsilon) > 0$, $\beta = \beta(\alpha, \varepsilon) \in (0, 1)$ *such that for all* $f \in A$,

$$N_1^*(f - f_0, g, \delta) < \beta \ \text{ which implies } N_2^*(Tf - Tf_0, g, \varepsilon) < \alpha$$

Definition 3.2. *A mapping* $T : (A, N_1^*) \to (B, N_2^*)$ *is said to be sequentially 2-fuzzy 2-anti continuous at* $f_0 \in A$ *if for any sequence* $\{f_n\}$ *in* A *for all* n *with* $f_n \to f_0$ *implies* $T(f_n) \to T(f_0)$ *that is for all* $t > 0$ $\lim_{n \to \infty} N_1^*(f_n - f_0, g, t) = 0$ *implies* $\lim_{n \to \infty} N_2{}^*(T(f_n) - T(f_0), g, t) = 0$.

Definition 3.3. *If* (A, N_1^*) *and* (B, N_2^*) *be 2-fuzzy 2-anti normed linear spaces. A mapping* T *from* (A, N_1^*) *and* (B, N_2^*) *is said to be strongly 2-fuzzy 2-anti continuous at* $f_0 \in A$ *given* $\varepsilon > 0$, *there exist a* $\delta > 0$ *such that for all* $f \in A$,

$$N_2^*(Tf - Tf_0, g, \varepsilon) \leq N_1^*(f - f_0, g, \delta)$$

If T *is strongly 2-fuzzy 2-anti continuous at each point of* A *then it is said to be strongly 2-fuzzy 2-anti continuous on* A.

Definition 3.4. *T is a mapping from (A, N_1^*) and (B, N_2^*) is said to be weakly 2-fuzzy 2-anti continuous at $f_0 \in A$, given $\varepsilon > 0$, there exist a $\delta(\alpha, \varepsilon) > 0$ such that $f \in A$.*

$$N_1^*(f - f_0, g, \delta) < \alpha \text{ which implies } N_2^*(Tf - Tf_0, g, \varepsilon) < \alpha$$

If T is weakly 2-fuzzy 2-anti continuous at each point of A then it is said to be weakly 2-fuzzy 2-anti continuous on A.

Definition 3.5. *A mapping $T : (A, N_1^*) \to (B, N_2^*)$ is said to be strongly 2-fuzzy 2-anti bounded on A if and only if there exist a positive real number M such that for all $f \in A$ and $t \in R$,*

$$N_2^*(Tf, g, t) \leq N_1^*\left(f, g, \frac{t}{M}\right).$$

Definition 3.6. *Let T is a mapping from (A, N_1^*) and (B, N_2^*), T is said to be weakly 2-fuzzy 2-anti bounded if and only if there exist a positive real number M for all $\alpha \in (0, 1)$ such that for all $f \in A$ and $t \in R$,*

$$N_1^*\left(f, g, \frac{t}{M}\right) \leq \alpha \text{ implies } N_2^*(Tf, g, t) \leq \alpha.$$

Theorem 3.7. *Let $T : (A, N_1^*) \to (B, N_2^*)$ be a linear mapping then*

(1). T is strongly 2-fuzzy 2-anti continuous on A if T is strongly 2-fuzzy 2-anti continuous at a point $f_0 \in A$.

(2). T is strongly 2-fuzzy 2-anti continuous if and only if T is strongly 2-fuzzy 2-anti bounded.

Proof.

(1). By hypothesis, T is strongly 2 fuzzy 2-anti continuous at $f_0 \in A$, so for each $\varepsilon > 0$ there exist $\delta > 0$ such that

$$N_2^*(Tf - Tf_0, g, \varepsilon) = N_1^*(f - f_0, g, \delta)$$

Taking $h \in A$ and replacing $f = f + f_0 - h$ it follows that

$$\begin{aligned}
N_2^*(Tf - Tf_0, g, \varepsilon) &= N_1^*(f - f_0, g, \delta) \\
\Rightarrow N_2^*(T(f + f_0 - h) - Tf_0, g, \varepsilon) &= N_1^*((f + f_0 - h) - f_0, g, \delta) \\
\Rightarrow N_2^*(Tf + Tf_0 - Th - Tf_0, g, \varepsilon) &= N_1^*(f + f_0 - h - f_0, g, \delta) \\
\Rightarrow N_2^*(Tf - Th, g, \varepsilon) &= N_1^*(f - h, g, \delta)
\end{aligned}$$

Since h is arbitrary , T is strongly 2- fuzzy 2-anti continuous on A.

(2). Suppose that T is strongly 2-fuzzy 2-anti bounded there exist a positive real number M such that for all $f \in A$ and $\varepsilon \in R$.

$$\begin{aligned}
N_2^*(Tf, g, \varepsilon) &= N_1^*\left(f, g, \frac{\varepsilon}{M}\right) \\
N_2^*(Tf - T\theta, g, \varepsilon) &= N_1^*\left(f - \theta, g, \frac{\varepsilon}{M}\right) \\
N_2^*(Tf - T\theta, g, \varepsilon) &= N_1^*\left(f - \theta, g, \delta\right)
\end{aligned}$$

where $d = \frac{\varepsilon}{M}$. Thus T is strongly 2-fuzzy 2-anti continuous at θ and so T is strongly 2-fuzzy 2-anti continuous on A.

To prove the converse, Suppose T is strongly 2 fuzzy 2-anti continuous on A. Using 2-fuzzy 2-anti continuity of T at $x = \theta$ for $\varepsilon = 1$ there exist $\delta > 0$ such that for all $f \in A$.

$$N_2^*(Tf - T\theta, g, 1) = N_1^*(f - \theta, g, \delta)$$

If $f \neq \theta$ and $t > 0$ putting $f = ht$

$$\begin{aligned} N_2^*(Tf, g, t) &= N_2^*(t(Th), g, t) \\ &= N_2^*(Th, g, 1) \\ &= N_1^*(h, g, \delta) \\ &= N_1^*\left(\frac{f}{t}, g, \delta\right) \\ &= N_1^*\left(f, g, \frac{t}{M}\right), \quad \text{where } M = \frac{1}{\delta}, \text{ so} \\ N_2^*(Tf, g, t) &= N_1^*\left(f, g, \frac{t}{M}\right). \text{ If } f \neq 0 \text{ and } t = 0 \text{ then} \\ N_2^*(Tf, g, t) &= N_1^*\left(f, g, \frac{t}{M}\right) = 1 \end{aligned}$$

If $f = 0$ and $t \in R$. Therefore $N_2^*(Tf, g, t) = N_1^*\left(f, g, \frac{t}{M}\right)$. ∎

Theorem 3.8. *Let (X, N_1^*) and (Y, N_2^*) be 2-fuzzy 2-anti normed linear spaces, a linear mapping $T : (X, N_1^*) \to (Y, N_2^*)$ is sequentially 2-fuzzy 2-anti continuous if and only if it is 2-fuzzy 2-anti continuous.*

Proof. Let T be 2 fuzzy 2-anti continuous at $f_0 \in A$. Suppose $\{f_n\}$ is a sequence in f such that $f_n \to f_0$. Let $\varepsilon > 0$ be given assume that $\alpha \in (0, 1)$. Since T is 2-fuzzy 2-anti continuous at f_0 there exist a $\delta > 0$ and $\beta \in (0, 1)$ such that $f \in F(X)$. $N_1^*(f - f_0, g, \delta) < \beta$ implies $N_2^*(Tf - Tf_0, g, \varepsilon) < \alpha$ since $f_n \to f_0$ there exist a positive integer N such that $N_1^*(f_n - f_0, g, \delta) < \beta$ for all $n \geq N$ then $N_2^*(Tf_n - Tf_0, g, \varepsilon) < \alpha$ for all $n \geq N$. Hence $\lim_{n\to\infty} N_2^*(T(f_n) - T(f_0), g, \varepsilon) = 0$ implies T is sequentially 2-fuzzy 2-anti continuous.

To prove the converse, Suppose T is sequentially 2-fuzzy 2-anti continuous, assume that T is not fuzzy 2-anti continuous at f_0 there exist a $\varepsilon > 0$, $\delta > 0$ and $\alpha, \beta \in (0, 1)$ there exists h such that $N_1^*(f_0 - h, g, \delta) > \beta$ implies $N_2^*(Tf_0 - Th, g, \varepsilon) > \alpha$ Thus $\beta = 1 - \frac{1}{n+1}$ and $\delta = \frac{1}{n+1}$ there exist h_n such that $N_1^*(h_n - f_0, g, \delta) = \beta$ but $N_2^*(Th_n - Tf_0, g, \varepsilon) = \alpha$ so there is a sequence $\{h_{nk}\}$ such that $N_2^*(Th_{nk} - Tf_0, g, \varepsilon) = \alpha$ for all $k \geq 1$. Hence $\lim_{k\to\infty} N_2^*(T(f_{nk}) - T(f_0), g, \varepsilon) \neq 0$. Since $h_n \to f_0$ it follows that $h_{nk} \to f_0$ which is contradiction to our assumption. Thus T is 2-fuzzy 2-anti continuous at f_0. ∎

Theorem 3.9. *Let $T : (A, N_1^*) \to (B, N_2^*)$ be a linear mapping, where (A, N_1^*) and (B, N_2^*) are 2-fuzzy 2-anti normed linear spaces. If T is strongly 2-fuzzy 2-anti bounded then it is weakly 2-fuzzy 2-anti bounded.*

Proof. If T is strongly 2-fuzzy 2-anti bounded then there exists a positive integer $M > 0$ such that for $f \in A$ and $t \in R$; $N_2^*(Tf, g, t) = N_1^*\left(f, g, \frac{t}{M}\right)$. For any $\alpha \in (0, 1)$ there exist a $M > \alpha$ such that $N_1^*\left(f, g, \frac{t}{M}\right) = \alpha$ which implies $N_2^*(Tf, g, t) = \alpha$ for all $f \in A$ and $t \in R$ This implies that T is weakly 2-fuzzy 2-anti bounded. ∎

Theorem 3.10. *Let $T : (X, N_1^*) \to (Y, N_2^*)$ be a linear mapping. Then*

(1). T is weakly 2 fuzzy 2-anti continuous on A if T is weakly 2-fuzzy 2-anti continuous at a point $f_0 \in A$.

(2). T is weakly 2 fuzzy 2-anti continuous if and only if T is weakly 2-fuzzy 2-anti bounded.

Proof.

(1). Suppose that T is weakly 2-fuzzy 2-anti continuous at f_0 for $\varepsilon > 0$ and $\alpha \in (0,1)$ there exist $\delta = \delta(a,t) > 0$ such that for all $f \in A$.

$$N_1^*(f - f_0, g, \delta) < \alpha \Rightarrow N_2 * (Tf - Tf_0, g, \varepsilon) < \alpha$$

Taking $h \in A$ and replacing $f = f + f_0 - h$,

$$N_1^*((f + f_0 - h) - f_0, g, \delta) < \alpha \Rightarrow N_2^*(T(f + f_0 - h) - Tf_0, g, \varepsilon) < \alpha$$
$$N_1^*(f + f_0 - h - f_0, g, \delta) < \alpha \Rightarrow N_2^*(Tf + Tf_0 - Th - Tf_0, g, \varepsilon) < \alpha$$
$$N_1^*(f - h, g, \delta) < \alpha \Rightarrow N_2^*(Tf - Th, g, \varepsilon) < \alpha$$

since h is arbitrary it follows that T is weakly 2-fuzzy 2-anti continuous on A.

(2). Suppose that T is 2-fuzzy 2-anti bounded, for any $\alpha \in (0,1)$ there exist $M > 0$ such that for all $t \in R$ and $f \in A$

$$N_1^*\left(f, g, \frac{t}{M}\right) \leq \alpha \Rightarrow N_2^*(Tf, g, t) \leq \alpha$$

Therefore

$$N_1^*\left(f - \theta, g, \frac{t}{M}\right) \leq \alpha \Rightarrow N_2^*(Tf - T\theta, g, t) \leq \alpha$$
$$N_1^*\left(f - \theta, g, \frac{e}{M_a}\right) \leq \alpha \Rightarrow N_2^*(Tf - T\theta, g, \varepsilon) \leq \alpha$$
$$N_1^*(f - \theta, g, \delta) \leq \alpha \Rightarrow N_2^*(Tf - T\theta, g, \varepsilon) \leq \alpha, \quad \text{where } \delta = \frac{\varepsilon}{M_\alpha}.$$

Thus T is weakly 2-fuzzy 2-anti continuous at f_0 and so from (1) weakly 2-fuzzy 2-anti continuous on A.

Conversely, suppose T is weakly 2 fuzzy 2-anti continuous on A. Using continuity of T at θ and taking $\varepsilon = 1$, for all $\alpha \in (0,1)$ there exist $\delta > (\alpha, 1) > 0$ such that for all $f \in A$.

$$N_1^*(f - \theta, g, \delta) \leq \alpha \Rightarrow N_2^*(Tf - T\theta, g, 1) \leq C$$
$$N_1^*(f, g, \delta) \leq \alpha \Rightarrow N_2^*(Tf, g, 1) \leq \alpha$$

if $f = 0$, $t > 0$ putting $f = \frac{h}{t}$,

$$N_1^*\left(\frac{h}{t}, g, \delta\right) \leq \alpha \Rightarrow N_2^*\left(T\left(\frac{h}{t}\right), g, 1\right) \leq \alpha$$
$$N_1^*(h, g, t\delta) \leq \alpha \Rightarrow N_2^*(T(h), g, t) \leq \alpha$$
$$N_1^*\left(h, g, \frac{t}{M}\right) \leq \alpha \Rightarrow N_2^*\left(T\left(\frac{h}{t}\right), g, 1\right) \leq \alpha$$

where $M_\alpha = \frac{1}{\delta(\alpha,1)}$ if $f \neq 0$ and $t \leq 0$,

$$N_1^*\left(f, g, \frac{t}{M_\alpha}\right) \leq N_2^*(Tf, g, t) = 1 \quad \text{for any } M_\alpha > 0 \text{ if } f = \theta \text{ then for } M_\alpha > 0$$
$$N_1^*\left(f, g, \frac{t}{M_\alpha}\right) = N_2^*(Tf, g, t) = 0 \text{ if } t > 0$$
$$N_1^*\left(f, g, \frac{t}{M_\alpha}\right) = N_2^*(Tf, g, t) = 1 \text{ if } t \leq 0$$

Hence T is weakly 2-fuzzy 2-anti bounded. □

Theorem 3.11. *Let T be a mapping from (A, N_1^*) to (B, N_2^*). If T is strongly 2-fuzzy 2-anti continuous it is sequentially 2-fuzzy 2-anti continuous, but not converse.*

Proof. Let $T : (A, N_1^*) \to (B, N_2^*)$ be strongly 2-fuzzy 2-anti continuous on A and $f_0 \in A$ then for each $\varepsilon > 0$, there exist a $\delta = \delta(f_0, \varepsilon) > 0$ such that for all $f \in U$

$$N_2^*(Tf - Tf_0, g, \varepsilon) \leq N_1^*(f - f_0, g, \delta)$$

Let $\{f_n\}$ be a sequence in A such that $f_n \to f_0$ that is for all $t > 0$.

$$\lim_{t\to\infty} N_1^*(f_n - f_0, g, t) = 0$$

Thus $N_2^*(Tf_n - Tf_0, g, \varepsilon) \leq N_1^*(f_n - f_0, g, \delta)$ which implies $\lim_{n\to\infty} N_2^*(Tf_n - Tf_0, g, \varepsilon) = 0$. That is $T(f_n) \to T(f_0)$ in (B, N_2^*). □

Example 3.12. *Let $(U, \|.\ .\|)$ be a 2-fuzzy 2-anti normed linear spaces. Where $\|f\| = |f|$ for all $f \in U$. Define $a * b = \min\{a, b\}$ and $a \Diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$ also define, $N_1^*, N_2^* : F(X) \times F(X) \to [0, 1]$ by $N_1^*(f, g, t) = \frac{|f,g|}{t+|f,g|}$ and $N_2^*(f, g, t) = \frac{k|f,g|}{t+k|f,g|}$ with $k > 0$. Let*

$$A = \{(f, g, t), N_1^*) : (f, g, t) \in F(X) \times F(X) \times R^*\} \text{ and}$$
$$B = \{(f, g, t), N_2^*) : (f, g, t) \in F(X) \times F(X) \times R^*\}$$

Then $(F(X), N_1^*)$ and $(F(X), N_2^*)$ are 2-fuzzy 2-anti norm linear space. Define a mapping $T : A \to B$ as $T(f) = \frac{f^4}{1+f^4}$ for all $f \in X$. Let $f_0 \in X$ and $\{f_n\}$ be a sequence in $F(X)$ such that $f_n \to f_0$ in $(F(X), N_1^*)$ that is for all $t > 0$.

$$\lim_{n\to\infty} N_1^*(f_n - f_0, g, t) = 0$$

That is

$$\lim_{n\to\infty} N_1^* \left(\frac{|f_n - f_0, g|}{t + |f_n - f_0, g|} \right) = 0$$

Now for all $t > 0$

$$\begin{aligned}
N_2^*\left(T(f_n) - T(f_0), g, t\right) &= \frac{k\left|\frac{f_n^4}{1+f_n^4} - \frac{f_0^4}{1+f_0^2}, g\right|}{t + k\left|\frac{f_n^4}{1+f_n^2} - \frac{f_0^4}{1+f_0^2}, g\right|} \\
&= \frac{\frac{k\left|f_n^4\left(1+f_0^2\right)-f_0^4\left(1+f_n^2\right),g\right|}{\left(1+f_n^2\right)\left(1+f_0^2\right)}}{\frac{t\left(1+f_n^2\right)\left(1+f_0^2\right)+k\left|f_n^4\left(1+f_0^2\right)-f_0^4\left(1+f_n^2\right),g\right|}{\left(1+f_n^2\right)\left(1+f_0^2\right)}} \\
&= \frac{\frac{k\left|f_n^4\left(1+f_0^2\right)-f_0^4\left(1+f_n^2\right),g\right|}{\left(1+f_n^2\right)\left(1+f_0^2\right)}}{\frac{t\left(1+f_n^2\right)\left(1+f_0^2\right)+k\left|f_n^4\left(1+f_0^2\right)-f_0^4\left(1+f_n^2\right),g\right|}{\left(1+f_n^2\right)\left(1+f_0^2\right)}} \\
&= \frac{k\left|f_n^4\left(1+f_0^2\right) - f_0^4\left(1+f_n^2\right), g\right|}{t\left(1+f_n^2\right)\left(1+f_0^2\right) + k\left|f_n^4\left(1+f_0^2\right) - f_0^4\left(1+f_n^2\right), g\right|} \\
&= \frac{k\left|f_n^4 + f_0^2 f_n^4 - f_0^4 - f_n^2 f_0^4, g\right|}{t\left(1+f_n^2\right)\left(1+f_0^2\right) + k\left|f_n^4 + f_0^2 f_n^4 - f_0^4 - f_n^2 f_0^4, g\right|} \\
&= \frac{k\left|\left(f_n^2+f_0^2\right)\left(f_n^2-f_0^2\right) + f_n^2 f_0^2\left(f_n^2-f_0^2\right), g\right|}{t\left(1+f_n^2\right)\left(1+f_0^2\right) + k\left|\left(f_n^2+f_0^2\right)\left(f_n^2-f_0^2\right) + f_n^2 f_0^2\left(f_n^2-f_0^2\right), g\right|}
\end{aligned}$$

$$\lim_{n\to\infty} N_2^*\left(T(f_n) - T(f_0), g, t\right) = 0$$

Thus f is sequentially continuous on X follows that T is not strongly 2-fuzzy 2-anti continuous.

Theorem 3.13. *Let T be a mapping from the 2-fuzzy 2-anti normed linear space $(F(X), N_1^*)$ to $(F(Y), N_2^*)$ and A be the compact subset of F(X). If T is fuzzy 2-anti continuous on F(X) then T(A) is a compact subset of F(Y).*

Proof. Let $\{g_n\}$ be a sequence in $T(A)$. Then for each n, there exist $f_n \in A$ such that $T(f_n) = g_n$. Since A is compact there exist $\{f_{nk}\}$ a subsequence of $\{f_n\}$ and $f_0 \in A$ such that $f_{nk} \to f_0$ in $(F(X), N_1^*)$. Since T is 2 fuzzy 2-anti continuous at f_0 if for any given $\varepsilon > 0$, $\alpha \in (0,1)$, there exist $\delta = \delta(\alpha, \varepsilon) > 0$ and $\beta = \beta(\alpha, \varepsilon) \in (0,1)$ such that for all $f \in F(X)$.

$$N_1^*(f - f_0, g, \delta) < \beta \Rightarrow N_2^*(Tf - Tf_0, g, \varepsilon) < \alpha$$

Now $f_{nk} \to f_0$ in $(F(X), N_1^*)$ implies that there exists $n_0 \in N$ such that for all $k \geq n_0$

$$N_1^*(f_{nk} - f_0, g, \delta) < \beta \Rightarrow N_2^*(T(f_{nk}) - T(f_0), g, \varepsilon) < \alpha$$

$N_2^*(T(f_{nk}) - T(f_0), g, \varepsilon) < \alpha$ for all $k \geq n_0$ which implies $T(A)$ is a compact subset of $F(Y)$. □

References

[1] T.Bag and S.K.Samanta, *Finite dimensional fuzzy normed linear spaces*, J. Fuzzy Math., 11(3)(2003), 687-705.

[2] T.Bag and S.K.Samanta, *Fuzzy bounded linear operator*, Fuzzy Sets and Systems, 115(2005), 513-547.

[3] T.Bag and S.K.Samanta, *A comparative study of fuzzy norms on a linear space*, J. Fuzzy sets and systems, 153(2008), 670-684.

[4] S.C.Cheng J.N.Mordeson, *Fuzzy linear operator & Fuzzy normed linear spaces*, Bull. Cal. Math. Soc., 86(1994), 429-436.

[5] S.Gahler, *Lineare 2-normierte Raume*, Math Nachr., 28(1964), 1-43.

[6] A.K.Katsaras, *Fuzzy topological vector spaces II*, Fuzzy Sets and Systems, 12(1984), 143-154.

[7] B.S.Reddy, *Fuzzy anti 2-normed linear spaces*, Journal of Mathematics and Technology, 2(1)(2011), 14-26.

[8] B.S.Reddy, *Fuzzy anti 2-normed linear spaces*, Journal of Mathematics Research, 3(2)(2011), 137-144.

[9] P.Sinha, D.Mishra and G.Lal, *Fuzzy anti 2-continuous linear operator*, Journal of Mathematics and Technology, 2(4)(2011), 9-19.

[10] P.Sinha, D.Mishra and G.Lal, *Fuzzy anti 2-bounded linear operator*, International Journal of Mathematics Research, 4(1)(2011), 71-84.

[11] R.M.Somasundaram and Thangaraj Beaula, *Some aspect of Fuzzy 2- Normed linear space*, Bull. Malays Math. Sci. Soc., (2)32(2)(2009), 211-221.

[12] L.A Zadeh, *Fuzzy Sets*, Information and Control, 8(1965), 338-353.