International Journal of Mathematics And its Applications
Volume 5, Issue 3–B (2017), 145–150.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Edge-Odd Graceful Labeling on Circulant Graphs With Different Generating Sets

Research Article

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Abstract: Let $G=(V,E)$ be a simple, finite, undirected and connected graph. A graph $G=(V,E)$ with p vertices and q edges is said to have an edge - odd graceful labeling if there exists a bijection f from E to $\{1,3,5,\ldots.2q-1\}$ so that the induced mapping f^{+} from V to $\{0,1,2,\ldots 2q-1\}$ given by $f^{+}(x)=\sum\{f(xy/xy\in E\}\ \ mod\ |2q|)$. In this paper, we have constructed an edge odd graceful labeling on circulant graphs $C_n(1,2,3,4,5,6)$ and $C_n(1,2,3,4,5,6,7)$ for odd n, $n\in I$. Here $(1,2,3,4,5,6)$ and $(1,2,3,4,5,6,7)$ are the generating sets.

MSC: 05c78.

Keywords: Labeling, Graceful labeling, Odd-graceful labeling, Edge graceful labeling, Edge-odd graceful labeling, Circulant graph.

1. Introduction

Throughout this paper, $G=(V,E)$ denotes a simple, finite connected and undirected graph. Let p and q denote the order and size respectively of the graph G. A graph labeling is an assignment of integers to the vertices or edge or both, subject to certain conditions. In 1967, Rosa introduced a lebeling of G called graceful labeling which is an injective function f from $V(G)$ to the set $\{0,1,2\ldots.q\}$ such that each edge xy is assigned with the label $|f(x)-f(y)|$. S.Lo introduced the concept of edge-graceful labeling in 1985. A.Solairaju and K.Chithra introduced a new type of labeling of a graph G with q edges called an edge - odd graceful labeling if there is a bijection f from the edges of the graph to the set $\{1,3,5\ldots.2q-1\}$ such that each vertex is assigned, the sum of all the edges incident to it $(mod\ 2q)$ and the resulting vertex labels are distinct for basic definitions, we can refer [5] and [11].

2. Edge-odd Graceful Labeling on Circulant Graph with Generating Set (1,2,3,4,5,6)

Theorem 2.1. *For odd $n\geq 13$, the circulant graph $G=C_n(1,2,3,4,5,6)$ admits edge - odd graceful labeling. Here $(1,2,3,4,5,6)$ are the generators of G.*

Proof. Let $G=C_n(1,2,3,4,5,6)$ be the 12-regular graph with $n\geq 13$. Let $V(G)=\{V_i\ /i=0,1,\ldots,n-1\}$. Here $q=6n$. Define the function $f:E(G)\{1,3,5,\ldots,2q-1\}$ by

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$$f\left(V_iV_{i+1}\right)=\begin{cases} i+1, & \text{for } i=0,2,4,\ldots,n-1; \\ n+1+i, & \text{for } i=1,3,5,\ldots,n-2. \end{cases}$$

$$f\left(V_iV_{i+2}\right)=4n-1-2i \;\text{ for } i=0,1.$$

$$f\left(V_iV_{i+3}\right)=5n+2i+10 \;\text{ for } i=0,1.$$

$$f\left(V_iV_{i+3}\right)=3n+2i+10 \;\text{ for } i=2,3,\ldots,n-1.$$

$$f\left(V_iV_{i+4}\right)=6n+1+2i \;\text{ for } i=0.$$

$$f\left(V_iV_{i+4}\right)=8n-2i+1 \;\text{ for } i=1,2,\ldots,n-1.$$

$$f\left(V_iV_{i+5}\right)=9n+i+12 \;\text{ for } i=0$$

$$f\left(V_iV_{i+5}\right)=8n+2i-1 \;\text{ for } i=1,2,\ldots,n-1.$$

$$f\left(V_iV_{i+6}\right)=11n-i+12 \;\text{ for } i=0,1,\ldots,n-1.$$

It can be verified that the edge label under the labeling f is a bijection from the set $E\left(C_n\left(1,2,3,4,5,6\right)\right)$ onto the set $\{1,3,5\ldots,2\left(6n\right)-1\}$. For every vertex $v\in V\left(G\right)$, the vertex weight $f^{+}\left(v\right)$ of $C_n\left(1,2,3,4,5,6\right)$ are defined as follows,

Case (i) For $i=0,1,2,3,4$.

For $i=0$

$$\begin{aligned}\sum_{e\in N(V_0)} f(e) &= 0+1+4n-1-0+5n+0+10+6n+1+0+9n+0+12+11n-0+12+11n-(-n-2)+12\\ &\quad +9n+(n-1)+12+6n+1+2(n)+5n+2(-(n+2))+10+4n-1-2(n+1)-2n-1+1.\\ &= 68n+64.\end{aligned}$$

For $i=1$

$$\begin{aligned}\sum_{e\in N(V_1)} f(e) &= n+1+1+4n-1-2+5n+2+10+8n-2+1+8n+2-1+11n-1+12+11n-(n-6)+1+12+8n\\ &\quad +2(-(n-5))-1+8n-2(n-4)+1+5n+2(-(n-3))+10+4n-1-2(n-2)+n+1+2n-3.\\ &= 67n+75.\end{aligned}$$

For $i=2$

$$\begin{aligned}\sum_{e\in N(V_2)} f(e) &= 2+1+4n-1-4+3n+4+10+8n-4+1+8n+4-1+11n-2+12+11n-(n-4)+12\\ &\quad +8n+2(n-3)-1+8n-2(n-2)+1+3n+2(n-1)+10+4n-1-2(n)+(2n+3)+1.\\ &= 69n+47.\end{aligned}$$

For $i=3$

$$\begin{aligned}\sum_{e\in N(V_3)} f(e) &= n+1+3+4n-1-6+3n+6+10+8n-6+1+8n+6-1+11n-3+12+11n-(n+2)+12+8n\\ &\quad +2(n+1)-1+8n-2(n)+1+3n+2(n-2)+10+4n-1-2(n-1)+n+1+3.\\ &= 69n+45.\end{aligned}$$

For $i = 4$

$$\sum_{e\in N(V_4)} f(e) = 4+1+4n-1-8+3n+8+10+8n-8+1+8n-1+11n-4+12+11n-(n+2)+12+8n$$

$$+2(n+1)-1+8n-2(n)+1+3n+2(n-2)+10+4n-1-2(n-1)+2n+1+1.$$

$$= 69n+43.$$

Case (ii) : For $i = 5, 7, 9, \ldots, n-2$.

$$\sum_{e\in N(V_i)} f(e) = n+1+i+4n-1-2i+3n+2i+10+8n-2i+1+8n+2i-1+11n-i+12+11n-(i-4)+12+8n$$

$$+2(i-3)-1+8n-2(i-2)+1+3n+2(i-1)+10+4n-1-2(i)+n+1-i-6.$$

$$= 70n-2i+38.$$

Case (iii) : For $i = 6, 8, 10, \ldots, n-1$.

$$\sum_{e\in N(V_i)} f(e) = i+1+4n-1-2i+3n+2i+10+8n-2i+1+8n+2i-1+11n-i+12+11n-(i-4)+12+8n$$

$$+2(i-3)-1+8n-2(i-2)+1+5n+2(i-1)+10+4n-1-2(i)-i-6+1.$$

$$= 70n-2i+38.$$

Thus, we obtain that the sum of values - assigned to all edges incident to a given vertex $v \in V$ are $68n+64$, $67n+75$, $69n+47$, $69n+45$, $69n+43$. The above integers are distinct.Hence the taking modulo $6n$ for the integers, we have the vertex- weights induced f^+ from V to $\{0, 1, 2 \ldots, 6n-1\}$. □

3. Edge-odd Graceful Labeling on Circulant Graph with Generating Set (1,2,3,4,5,6,7)

Theorem 3.1. *For odd $n \geq 15$, the circulant graph $G = C_n(1,2,3,4,5,6,7)$ admits edge-odd graceful labeling. Here (1,2,3,4,5,6,7) are the generators of G.*

Proof. Let $G = C_n(1,2,3,4,5,6,7)$ be the 14-regular graph with $n \geq 13$. Let $V(G) = \{V_i / i = 0, 1, \ldots, n-1\}$. Here $q = 7n$. Define the function $f : E(G) \to \{1, 3, 5 \ldots, 2q-1\}$ by

$$f(V_iV_{i+1}) = \begin{cases} i+1, & \text{for } i = 0, 2, 4, \ldots, n-1; \\ n+1+i, & \text{for } i = 1, 3, 5, \ldots, n-2. \end{cases}$$

$$f(V_iV_{i+2}) = 4n-1-2i \text{ for } i = 0, 1.$$

$$f(V_iV_{i+3}) = 5n+2i+12 \text{ for } i = 0, 1.$$

$$f(V_iV_{i+3}) = 3n+2i+12 \text{ for } i = 2, 3, \ldots, n-1.$$

$$f(V_iV_{i+4}) = 6n+1+2i \text{ for } i = 0.$$

$$f(V_iV_{i+4}) = 8n-2i+1 \text{ for } i = 1, 2, \ldots, n-1.$$

$$f(V_iV_{i+5}) = 9n + i + 14 \text{ for } i = 0.$$

$$f(V_iV_{i+5}) = 8n + 2i - 1 \text{ for } i = 1, 2, \ldots, n-1.$$

$$f(V_iV_{i+6}) = 11n - 2i + 14 \text{ for } i = 0, 1, \ldots, n-1.$$

$$f(V_iV_{i+7}) = 12n + 2i + 1 \text{ for } i = 0, 1, 2, 3, 4, 5.$$

It can be verified that the edge label under the labeling f is a bijection from the set $E(C_n(1,2,3,4,5,6,7))$ onto the set $\{1, 3, 5, \ldots, 2(7n) - 1\}$. For every vertex $v \in V(G)$, the vertex-weight $f^+(v)$ of $C_n(1,2,3,4,5,6,7)$ are defined as follows,

Case(i): For $i = 0, 1, 2, 3, 4, 5, 6$.

For i = 0

$$\sum_{e \in N(V_0)} f(e) = 1 + 4n - 1 + 5n + 12 + 6n + 1 + 9n + 14 + 11n + 14 + 12n + 1 + 12n + 2(n-5) + 1 + 11n - 2(n+4) + 14$$
$$+ 9n + (n-3) + 14 + 6n + 1 + 2(n+2) + 5n + 2(n-1) + 12 + 4n - 1 - 2(n) + (-2n - 17).$$
$$= 95n + 47.$$

For $i = 1$

$$\sum_{e \in N(V_1)} f(e) = n + 1 + 1 + 4n - 1 - 2 + 5n + 2 + 12 + 8n - 2 + 1 + 9n + 1 + 14 + 11n - 2 + 14 + 12 + 2 + 1 + 12n + 2(n-7)$$
$$+ 1 + 11n - 2(n+6) + 14 + 9n + (n-5) + 14 + 8n - 2(n+4) + 1 + 5n + 2(n-3) + 12 + 4n - 1 - 2(n+2)$$
$$+ n + 1 + (8n + 2).$$
$$= 95n + 49.$$

For $i = 2$

$$\sum_{e \in N(V_2)} f(e) = 2 + 1 + 4n - 1 - 4n + 3n + 4 + 12 + 8n - 4 + 1 + 8n + 4 - 1 + 11n - 4 + 14 + 12n + 4 + 1 + 12n + 2(n-7)$$
$$+ 1 + 11n - 2(n-6) + 14 + 8n + 2(n-5) - 1 + 8n - 2(n-4) + 1 + 3n + 2(n-3) + 12 + 4n - 1 - 2(n-2)$$
$$+ n + 1 + 2n + 1$$
$$= 95n + 51.$$

For $i = 3$

$$\sum_{e \in N(V_3)} f(e) = n + 1 + 3 + 4n - 1 - 6 + 3n + 6 + 12 + 8n - 6 + 1 + 8n + 6 - 1 + 11n - 6 + 14 + 12n + 6 + 1 + 12n + 2(n-6)$$
$$+ 1 + 11n - 2(n-5) + 14 + 8n + 2(n-4) - 1 + 8n - 2(n-3) + 1 + 3n + 2(n-2) + 12 + 4n - 1 - 2(n-1)$$
$$+ n + 1 + (n + 2)$$
$$= 95n + 53.$$

For $i = 4$

$$\sum_{e \in N(V_4)} f(e) = 4+1+4n-1-8+3n+8+12+8n-8+1+8n+8-1+11n-8+14+12n+8+1+12n+2(n-5)$$
$$+1+11n-2(n-4)+14+8n+2(n-3)-1+8n-2(n-2)+1+5n+2(n-1)+12+4n-1$$
$$-2(n)+(n+3)+1.$$
$$= 95n+55.$$

For $i = 5$

$$\sum_{e \in N(V_5)} f(e) = n+1+5+4n-1-10+3n+10+12+8n-10+1+8n-10-1+11n-10+14+12n+10+1+12n$$
$$+2(n-4)+1+11n-(n-3)+14+9n+(n-2)+14+8n-2(n-1)+1+3n+2(n)+12+4n-1$$
$$-2(n+1)+n+1-(n-13).$$
$$= 95n+57.$$

For $i = 6$

$$\sum_{e \in N(V_6)} f(e) = 6+1+4n-1-12+3n+12+12+8n-12+1+8n+12-1+11n-12+14+12n+12+1+12n$$
$$+2(n-5)+1+11n-2(n-4)+14+9n+(n-3)+14+8n-2(n-2)+1+5n+2(n-1)+12$$
$$+4n-1-2(n)+(n-13)+1.$$
$$= 95n+59.$$

Case(ii) For $i = Odd$

$$\sum_{e \in N(V_i)} f(e) = n+1+i+4n-1-2i+3n+2i+12+8n-2i+1+8n+2i-1+11n-2i+14+12n+2i+1+12n$$
$$+2(i-5)+1+11n-2(i+4)+14+8n+2(i-3)-1+8n-2(i+2)+1+5n+2(i-1)+12$$
$$+4n-1-2(i)+(i-7)+1.$$
$$= 95n+2i+17.$$

Case(iii): For $i = Even$

$$\sum_{e \in N(V_6)} f(e) = i+1+4n-1-2i+3n+2i+12+8n-2i+1+8n+2i-1+11n-2i+14+12n+2i+1+12n$$
$$+2(i-5)+1+11n-2(i+4)+14+8n+2(i-3)-1+8n-2(i+2)+1+5n+2(i-1)+12$$
$$+4n-1-2(i)+(i-7)+1+n.$$
$$= 95n+2i+17.$$

Thus, we obtain that the sum of valuesassigned to all edges incident to a given vertex $v \in V$ are $95n+47$, $95n+49$, $96n+32$, $95n+51$, $95n+53$, $95n+55$, $95n+57$, $95n+59$. The above integers are distinct. Hence the taking modulo 7n for the integers, we have the vertex- weight induced f^+ from V to $\{0, 1, 2 \ldots, 7n-1\}$. □

4. Conclusion

We concluded that the circulant graph with generating sets $(1, 2, 3, 4, 5, 6)$ and $(1, 2, 3, 4, 5, 6, 7)$ admits edge - odd graceful labeling.

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