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$g^{\#}$b-Interior and $g^{\#}$b-Closure in Topological Spaces

Research Article

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Abstract: Interior and closure is most important concepts in topological spaces .$g^{\#}$b-closed sets $g^{\#}$b-open sets are introduced by S.Chandrasekar, A.Atkinswestley et.al,. In this paper we find some basic properties and applications of $g^{\#}$b-interior and $g^{\#}$b-closure in topological spaces.

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1. Introduction

Topology is one of the most important subjects in Mathematics. Closed sets and open sets has been studied extensively by many topologists. The investigation of generalized closed sets has led to several new and interesting concepts in topological spaces. In 1970, O. Njastad introducd α-open sets in Topological Spaces. Andrijevic [1] introduced one such new version called b-open sets in 1996. Maki, et al. [8] introduced generalized α-closed and α-generalized closed sets (briefly, gα-closed, αg-closed). M.K.R.S.Veera Kumar [18] introducd $g^{\#}$-closed sets in topological spaces. $g^{\#}$b-closed sets $g^{\#}$b-open sets are introduced by S.Chandrasekar, A.Atkinswestley et.al. In this paper, we introduce $g^{\#}$b-Interior and $g^{\#}$b-closure in topological spaces and its various properties are discussed.

2. Preliminaries

Throughout this paper (X, τ) (or simply X) represent topological spaces. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

Definition 2.1. *A subset A of a space (X, τ) is called a*

(1). α-closed set [8] if $cl(int(cl(A))) \subseteq A$.

(2). b-closed set [15] if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

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(3). A subset A of X is called an α-generalized closed (briefly αg-closed) set [8] if $\alpha cl(A) \subseteq H$ whenever $A \subseteq H$ and H is open in (X, τ).

(4). A subset A of a space (X, τ) is called a $g^{\#}b$-closed set if $bcl(A) \subseteq H$ whenever $A \subseteq H$ and H is a αg-open set in (X,τ). The complements of the above mentioned closed sets are their respective open sets.

3. $g^{\#}$b Interior in Topological Spaces

Definition 3.1. *Let (X, τ) be a topological space and let $x \in X$. A subset N of X is said to be $g^{\#}b$ neighbourhood of x if there exists a $g^{\#}b$ open set H such that $x \in H \subset N$.*

Definition 3.2. *Let A be a subset of (X, τ). A point $x \in A$ is said to be $g^{\#}b$-interior point of A is a $g^{\#}b$-neighbourhood of x. The set of all $g^{\#}b$-interior points of A is called the $g^{\#}b$-interior of A and is denoted by $g^{\#}b$-int(A).*

Theorem 3.3. *If A be a subset of X. Then $g^{\#}b\text{-}int(A) = \cup\{H : H$ is a $g^{\#}b$-open, $H \subset A\}$.*

Proof. Let A be a subset of (X,τ). $x \in g^{\#}b\text{-}int(A) \Leftrightarrow x$ is a $g^{\#}b$-interior point of A $\Leftrightarrow$ A is a $g^{\#}b$-nbhd of point x $\Leftrightarrow$ there exists $g^{\#}b$-open set H such that $x \in H \subset A \Leftrightarrow x \in \cup\{H :$ H is a $g^{\#}b$-open, $H \subset A\}$. Hence $g^{\#}b\text{-}int(A) = \cup\{H :$ H is a $g^{\#}b$-open, $H \subset A\}$. □

Theorem 3.4. *Let A and B be subsets of (X, τ). Then*

(1). $g^{\#}b\text{-}int(X) = X$ and $g^{\#}b\text{-}int(\phi) = \phi$

(2). $g^{\#}b\text{-}int(A) \subset A$.

(3). If B is any $g^{\#}b$-open set contained in A, then $B \subset g^{\#}b\text{-}int(A)$.

(4). If $A \subset B$, then $g^{\#}b\text{-}int(A) \subset g^{\#}b\text{-}int(B)$.

(5). $g^{\#}b\text{-}int(g^{\#}b\text{-}int(A)) = g^{\#}b\text{-}int(A)$.

Proof.

(1). Since X and ϕ are $g^{\#}$b-open sets, by Theorem 3.3 $g^{\#}b\text{-}int(X) = \cup\{\text{H : H is a g}^{\#}b\text{-open}, H \subset X\} = X \cup \{\text{A is a } g^{\#}\text{b-open sets}\} = X$ (ie) $g^{\#}b\text{-}int(X) = X$. Since ϕ is the only $g^{\#}b$-open set contained in ϕ, $g^{\#}b\text{-}int(\phi) = \phi$.

(2). Let x is a $g^{\#}b\text{-}int(A)$ interior point of A. Let $x \in g^{\#}b\text{-}int(A) \Rightarrow x$ is a interior point of A. $\Rightarrow$ A is a nbhd of x. $\Rightarrow x \in A$. Thus, $x \in g^{\#}b\text{-}int(A) \Rightarrow x \in A$. Hence $g^{\#}b\text{-}int(A) \in A$.

(3). Let B be any $g^{\#}b$-open sets such that $B \subset A$. Let $x \in B$. Since B is a $g^{\#}b$-open set contained in A. x is a $g^{\#}b$-interior point of A (ie) B is a $g^{\#}b\text{-}int(A)$. Hence $B \subset g^{\#}b\text{-}int(A)$.

(4). Let A and B be subsets of (X, τ) such that $A \subset B$. Let $x \in g^{\#}b\text{-}int(A)$. Then x is a $g^{\#}b$-interior point of A and so A is a $g^{\#}b$-nbhd of x. Since $B \supset A, B$ is also $g^{\#}b$-nbhd of x. $\Rightarrow x \in g^{\#}b\text{-}int(B)$. Thus we have shown that $x \in g^{\#}b\text{-}int(A) \Rightarrow x \in g^{\#}b\text{-}int(B)$.

(5). Let A be any subset of X. By definition of $g^{\#}b$-interior, $g^{\#}b\text{-}int(A)$ is $g^{\#}b$ open and hence $g^{\#}b\text{-}int(g^{\#}b\text{-}int(A)) = g^{\#}b\text{-}int(A)$.

□

Theorem 3.5. *If a subset A of space (X, τ) is $g^{\#}$b-open, then $g^{\#}$b-int(A)=A.*

Proof. Let A be $g^{\#}$b-open subset of (X, τ) We know that $g^{\#}b\text{-}int(A) \subset A$. Also, A is $g^{\#}$b-open set contained in A. From Theorem 3.4 $A \subset g^{\#}b\text{-}int(A)$. Hence $g^{\#}b\text{-}int(A) = A$. □

The converse of the above theorem need not be true, as seen from the following example.

Example 3.6. *Let $X = \{a, b, c, d\}$. Then $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. $g^{\#}$b-open sets are $g^{\#}b - O = \{X, \phi\{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. $g^{\#}b\text{-}int(\{a, c, d\}) = \{a\} \cup \{a, c\} \cup \{a, d\} \cup \phi = \{a, c, d\}$. But $\{a, c, d\}$ is not $g^{\#}$b-open set in X.*

Theorem 3.7. *If $A, B \subset$ of (X, τ), then $g^{\#}b\text{-}int(A) \cup g^{\#}b\text{-}int(B) \subset g^{\#}b\text{-}int(A \cup B)$.*

Proof. We know that $A \subset A \cup B$ and $B \subset A \cup B$. We have Theorem 3.4 (4), $g^{\#}b\text{-}int(A) \subset g^{\#}b\text{-}int(A \cup B)$, $g^{\#}b\text{-}int(B) \subset g^{\#}b\text{-}int(A \cup B)$. This implies that $g^{\#}b\text{-}int(A) \cup g^{\#}b\text{-}int(B) \subset g^{\#}b\text{-}int(A \cup B)$. □

Theorem 3.8. *If $A, B \subset (X, \tau)$, then $g^{\#}b\text{-}int(A \cap B) = g^{\#}b\text{-}int(A) \cap g^{\#}b\text{-}int(B)$.*

Proof. We know that $A \cap B \subset A$ and $A \cap B \subset B$. We have $g^{\#}b\text{-}int(A \cap B) \subset g^{\#}b\text{-}int(A)$ and $g^{\#}b\text{-}int(A \cap B) \subset g^{\#}b\text{-}int(B)$. This implies that

$$g^{\#}b\text{-}int(A \cap B) \subset g^{\#}b\text{-}int(A) \cap g^{\#}b\text{-}int(B) \tag{1}$$

Again let $x \in g^{\#}b\text{-}int(A) \cap g^{\#}b\text{-}int(B)$. Then $x \in g^{\#}b\text{-}int(A)$ and $x \in g^{\#}b\text{-}int(B)$. Hence x is a g$^{\#}$b-int point of each of sets A and B. It follows that A and B is g$^{\#}$b-nbhds of x, so that their intersection $A \cap B$ is also a $g^{\#}$b-nbhds of x. Hence $x \in g^{\#}b\text{-}int(A \cap B)$. Thus $x \in g^{\#}b\text{-}int(A)$ implies that $x \in g^{\#}b\text{-}int(A \cap B)$. Therefore

$$g^{\#}b\text{-}int(A) \cap g^{\#}b\text{-}int(B) \subset g^{\#}b\text{-}int(A \cap B) \tag{2}$$

From (1) and (2), We get $g^{\#}b\text{-}int(A \cap B) = g^{\#}b\text{-}int(A) \cap g^{\#}b\text{-}int(B)$. □

Theorem 3.9. *If A is a subset of X, then $int(A) \subset g^{\#}b\text{-}int(A)$.*

Proof. Let A be a subset of X. Let $x \in int(A) \Rightarrow x \cup \{H :$ H is open, $H \subset A\} \Rightarrow$ there exists an open set H such that $x \in H \subset A \Rightarrow$ there exist a $g^{\#}$b-open set H such that $x \in H \subset A$, as every open set is a $g^{\#}$b-open set in $X \Rightarrow x \in \cup\{H :$ H is $g^{\#}$b-open, $H \subset A\} \Rightarrow x \in g^{\#}b\text{-}int(A)$. Thus $x \in int(A) \Rightarrow x \in g^{\#}b\text{-}int(A)$. Hence $int(A) \subset g^{\#}b\text{-}int(A)$. □

Remark 3.10. *The relation of the above theorem $int(A) \subset g^{\#}b\text{-}int(A)$ is proved by the following example.*

Example 3.11. *Let X= {a,b,c,d,e}, τ= {X, ϕ, {a}, {a, b}}. $g^{\#}$b-open sets are $g^{\#}b - O$= {X, ϕ, {a}, {b}, {a,b}, {a,c}, {a,d}, {a,e}, {b,c}, {b,d},{b,e}, {a,b,c}, {a,b,d}, {a,b,e}, {a,c,d}, {a,c,e}, {a,d,e}, {b,c,d}, {b,c,e}, {b,d,e}, {a,b,c,d}, {a,b,c,e}, {a,b,d,e}, {a,c,d,e}, {b,c,d,e}}. Let $A = \{a, c, d\}$. Now $g^{\#}b\text{-}int(A) = \{a, c, d\}$ and $int(A) = \{a\}$. It follows that $int(A) \subset g^{\#}b\text{-}int(A)$ and $int(A) \neq g^{\#}b\text{-}int(A)$.*

Theorem 3.12. *If A is a subset of X, then $b\text{-}int(A) \subset g^{\#}b\text{-}int(A)$, where b-int(A) is given by $b\text{-}int(A) = \cup\{H$:H is b-open, $H \subset A\}$.*

Proof. Let A be a subset of (X, τ). Let $x \in b\text{-}int(A) \Rightarrow x \in \{H :$ H is b-open, $H \subset A\} \Rightarrow$ there exists a b-open set H such that $x \in H \subset A \Rightarrow$ there exists a $g^{\#}$b-open set H such that $x \in H \subset A$, as every b-open set is a $g^{\#}$b-open set in $X \Rightarrow x \in \cup\{H \subset X :$ H is g$^{\#}$b -open, $H \subset A\}$. $x \in g^{\#}b\text{-}int(A)$. Hence $b\text{-}int(A) \subset g^{\#}b\text{-}int(A)$. □

Remark 3.13. *The relation of the above theorem* $b\text{-}int(A) \subset g^{\#}b\text{-}int(A)$ *is proved by the following example.*

Example 3.14. *Let* $X = \{a, b, c\}$ *and* $\tau = \{X, \phi, \{b\}, \{b, c\}\}$. $g^{\#}$*b-open sets are* $g^{\#}b - O = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. *b-open sets are* $b - O = \{X, \phi, \{b\}, \{a, b\}, \{b, c\}\}$. *Let* $A = \{a\}$, $g^{\#}b\text{-}int(A) = \{a\}$ *and* $b - int(A) = \phi$. *It follows* $b - int(A) \subset g^{\#}b\text{-}int(A)$ *and* $b - int(A) \neq g^{\#}b\text{-}int(A)$.

4. g$^{\#}$b Closure in Topological Spaces

Definition 4.1. *Let A be a subset of a space X. We define the* $g^{\#}$*b-closure of A to be the intersection of all* $g^{\#}$*b-closed sets containing A. In symbols,* $g^{\#}b\text{-}cl(A) = \cap\{F : A \subset F \in g^{\#}b\text{-}C(X)\}$.

Theorem 4.2. *If A, B are subsets X. Then*

(1). $g^{\#}b\text{-}cl(X) = X$ *and* $g^{\#}b\text{-}cl(\phi) = \phi$.

(2). $A \subset g^{\#}b\text{-}cl(A)$.

(3). If B is any $g^{\#}$*b-closed set containing A, then* $g^{\#}b\text{-}cl(A) \subset B$.

(4). If $A \subset B$ *then* $g^{\#}b\text{-}cl(A) \subset g^{\#}b\text{-}cl(B)$.

(5). $g^{\#}b\text{-}cl(g^{\#}b\text{-}cl(A)) = g^{\#}b\text{-}cl(A)$.

Proof.

(1). By the definition of g$^{\#}$b-closure, x is the only g$^{\#}$b-closed set containing X. Therefore g$^{\#}$bcl(X)=Intersection of all the g$^{\#}$b-closed sets containing $X = \cap\{X\} = X$. That is $g^{\#}b\text{-}cl(X) = X$. By the definition of g$^{\#}$b-closure, $g^{\#}b\text{-}cl(\phi)$ =Intersection of all the $g^{\#}b$-closed sets containing $\phi = \phi \cap g^{\#}b$-closed sets containing $\phi = \phi$. That is $g^{\#}b\text{-}cl(\phi) = \phi$.

(2). By the definition of g$^{\#}$b-closure of A, it is obvious that $A \subset g^{\#}b\text{-}cl(A)$.

(3). Let B be any g$^{\#}$b-closed set containing A. Since $g^{\#}b\text{-}cl(A)$ is the intersection of all g$^{\#}$b-closed sets containing A, $g^{\#}b\text{-}cl(A)$ is contained in every g$^{\#}$b-closed set containing A. Hence in particular $g^{\#}b\text{-}cl(A) \subset B$.

(4). Let A and B be subsets of X such that $A \subset B$. By the definition $g^{\#}b\text{-}cl(B) = \cap\{F : B \subset F \in g^{\#}bC(X)\}$. If $B \subset F \in g^{\#}bC(X)$, then $g^{\#}b\text{-}cl(B) \subset F$. Since $A \subset B$, $A \subset B \subset F \in g^{\#}bC(X)$, we have $g^{\#}b\text{-}cl(A) \subset F$. Therefore $g^{\#}b\text{-}cl(A) \subset \cap\{F : B \subset F \in g^{\#}bC(X)\} = g^{\#}b\text{-}cl(B)$. (i.e) $g^{\#}b\text{-}cl(A) \subset g^{\#}b\text{-}cl(A)$.

(5). Let B be a g$^{\#}$b-closed set containing A. Then by definition $g^{\#}b\text{-}Cl(A) \subset B$. Since B is $g^{\#}$b-closed set and contains $g^{\#}b\text{-}Cl(A)$ which contained in every $g^{\#}b$-closed set containing A. It follows that $g^{\#}b\text{-}Cl(g^{\#}b\text{-}Cl(A)) \subset g^{\#}b\text{-}Cl(A)$. Therefore $g^{\#}b\text{-}Cl(g^{\#}b\text{-}Cl(A)) = g^{\#}b\text{-}Cl(A)$.

□

Theorem 4.3. *If* $A \subset (X, \tau)$ *is* $g^{\#}$*b-closed, then* $g^{\#}b\text{-}cl(A) = A$.

Proof. Let A be g$^{\#}$b-closed subset of X. We know that $A \subset g^{\#}b\text{-}cl(A)$. Also $A \subset A$ and A is g$^{\#}$b-closed. By Theorem 4.2 (3) $g^{\#}b\text{-}cl(A) \subset A$. Hence $g^{\#}b\text{-}cl(A) = A$. □

Remark 4.4. *The relation of the above theorem is proved by the following example.*

Example 4.5. *Let X={a,b,c,d} with topology* τ ={X, ϕ , {a},{b},{a, b},{b,c},{a,b,c}}. $g^{\#}b$-$cl(X)$= {X, ϕ, {a}, {c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, c, d}, {b, c, d}}. $g^{\#}b$-$cl(\{b\}) = \{b,c\} \cap \{b,d\} \cap \{b,c,d\} = \{b\}$. *But* $\{b\}$ *is not* $g^{\#}b$*-closed set in X.*

Theorem 4.6. *If A and B are subsets of a space* (X, τ) *then* $g^{\#}b$-$cl(A \cap B) \subset g^{\#}b$-$cl(A) \cap g^{\#}b$-$cl(B)$.

Proof. Let A and B be subsets of X. Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem 4.2 (4) $g^{\#}b$-$cl(A \cap B) \subset g^{\#}b$-$cl(A)$ and $g^{\#}b$-$cl(A \cap B) \subset g^{\#}b$-$cl(B)$. Hence $g^{\#}b$-$cl(A \cap B) \subset g^{\#}b$-$cl(A) \cap g^{\#}b$-$cl(B)$. □

Theorem 4.7. *If A and B are subsets of a space (X,*τ*) then* $g^{\#}b$-$cl(A \cup B) = g^{\#}b$-$cl(A) \cup g^{\#}b$-$cl(B)$.

Proof. Let A and B be subsets of X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. We have

$$g^{\#}b\text{-}cl(A) \cup g^{\#}b\text{-}cl(B) \subset g^{\#}b\text{-}cl(A \cup B) \tag{3}$$

Now to prove $g^{\#}b$-$cl(A \cup B) \subset g^{\#}b$-$cl(A) \cup g^{\#}b$-$cl(B)$. Let $x \in g^{\#}b$-$cl(A \cup B)$ and suppose $x \notin g^{\#}b$-$cl(A) \cup g^{\#}b$-$cl(B)$. Then there exists $g^{\#}$b-closed sets A_1 and B_1 with $A \subset A_1$, $B \subset B_1$ and $x \notin A_1 \cup B_1$. We have $A \cup B \subset A_1 \cup B_1$ and $A_1 \cup B_1$ is $g^{\#}$b-closed set by Theorem such that $x \notin A_1 \cup B_1$. Thus $x \notin g^{\#}b$-$cl(A \cup B)$ which is a contradiction to $x \in g^{\#}b$-$cl(A \cup B)$. Hence

$$g^{\#}b\text{-}cl(A \cup B) g^{\#}b\text{-}cl(A) \cup g^{\#}b\text{-}cl(B) \tag{4}$$

From (3) and (4), we have $g^{\#}b$-$cl(A \cup B) = g^{\#}b$-$cl(A) \cup g^{\#}b$-$cl(B)$. □

Theorem 4.8. *For an* $x \in X$, $x \in g^{\#}b$-$cl(A)$ *if and only if* $V \cap A \neq \phi$ *for every* $g^{\#}b$*-closed sets V containing x.*

Proof. Let $x \in X$ and $x \in g^{\#}b$-$cl(A)$. To prove $V \cap A \neq \phi$ for every $g^{\#}b$-open set V containing x. We prove the result by contradiction. Suppose there exists a $g^{\#}b$-open set V containing x such that $V \cap A \neq \phi$. Then $A \subset X - V$ and $X - V$ is $g^{\#}b$-closed. We have $g^{\#}b$-$cl(A) \subset X - V$. This shows that $x \notin g^{\#}b$-$cl(A)$, which is a contradiction. Hence $V \cap A \neq \phi$ for every $g^{\#}b$-open set V containing x.
Conversely, let $V \cap A \neq \phi$ for every $g^{\#}b$-open set V containing x. To prove $x \in g^{\#}b$-$cl(A)$. We prove the result by contradiction. Suppose $x \notin g^{\#}b$-$cl(A)$. Then $x \in X - F$ and $X - F$ is $g^{\#}b$-open. Also $(X - F) \cap A = \phi$, which is a contradiction. Hence $x \in g^{\#}b$-$cl(A)$. □

Theorem 4.9. *If A is a subset of a space (X,* τ*), then* $g^{\#}b$-$cl(A) \subset cl(A)$.

Proof. Let A be a subset of a space (X, τ). By the definition of closure, $cl(A) = \cap\{F \subset A : X \subset F \in C(X)\}$. If $A \subset F \in C(X)$, Then $A \subset F \in g^{\#}b(X)$, because every closed set is $g^{\#}b$-closed. That is $g^{\#}b$-$cl(A) \subset F$. Therefore $g^{\#}b$-$cl(A) \subset \cap\{F \subset X : A \subset X \in C(X)\} = cl(A)$. Hence $g^{\#}b$-$cl(A) \subset cl(A)$. □

Remark 4.10. *The relation of the above theorem* $g^{\#}b$-$cl(A) \subset cl(A)$ *is proved by the following example.*

Example 4.11. *Let X= {a,b,c,d,e} and topology* τ ={X, ϕ ,{a},{b}, {a, b}}. $g^{\#}b$*-closed sets are* $g^{\#}b-C$={X, ϕ, {a}, {b}, {c}, {d}, {e}, {a,c}, {a,d}, {a, e}, {b,c}, {b,d}, {b,e}, {c,d}, {c,e}, {d,e}, {a,c,d}, {a,c,e}, {a,d,e}, {b,c,d}, {b,c,e}, {b,d,e}, {c,d,e}, {a,c,d,e}, {b,c,d,e}} *closed sets are* C={X, ϕ, {c,d,e}, {a,c,d,e}, {b,c,d,e}}. *Let* $A = \{b\}$. *Now* $g^{\#}b$-$cl(A) = \{b\}$ *and* $cl(A) = \{b, c, d, e\}$. *It follows that* $g^{\#}bcl(A) \subset cl(A)$ *and* $g^{\#}b$-$cl(A) \neq cl(A)$.

Theorem 4.12. *If A is a subset of (X,*τ*) then* $g^{\#}b$-$cl(A) \subset b - cl(A)$*, where* $b - cl(A)$ *is given by* $b - cl(A) = \cap\{F \subset X : A \subset F$ *and F is a b-closed set in X*$\}$.

Proof. Let A be a subset of X. By definition of $b-cl(A) = \cap\{F \subset X : A \subset F$ and F is a b-closed set in X$\}$. If $A \subset F$ and F is b-closed subset of x, then $A \subset F \in g^{\#}bC(X)$, because every b closed is $g^{\#}b$-closed subset in X. That is $g^{\#}b\text{-}cl(A) \subset F$. Therefore $g^{\#}b\text{-}cl(A) \subset \cap\{F \subset X : A \subset F$ and F is a b-closed set in X$\} = b\text{-}cl(A)$. Hence $g^{\#}b\text{-}cl(A) \subset b\text{-}cl(A)$. □

Remark 4.13. *The relation of the above theorem $g^{\#}b\text{-}cl(A) \subset b - cl(A)$ is proved by the following example.*

Example 4.14. *Let X=$\{a, b, c\}$, $\tau = \{X, \phi, \{b\}, \{b, c\}\}$. $g^{\#}b$-open sets are $g^{\#}b-C$=$\{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ b-open sets are $b-C$=$\{X, \phi, \{a\}, \{c\}, \{a, c\}\}$. Let A=$\{b,c\}$, b-cl(A) = $\{X\}$ and $g^{\#}$b-cl(A)=$\{b,c\}$. It follows $g^{\#}b\text{-}cl(A) \subset b-cl(A)$ and $g^{\#}b\text{-}cl(A) \neq b\text{-}cl(A)$.*

Corollary 4.15. *Let A be any subset of (X, τ). Then*

(1). $(g^{\#}b\text{-}int(A))^C = g^{\#}b\text{-}cl(A^C)$

(2). $g^{\#}b\text{-}int(A) = (g^{\#}b\text{-}cl(A^C))^C$

(3). $g^{\#}b\text{-}cl(A) = (g^{\#}b\text{-}cl(A^C))$.

Proof. Let $x \in (g^{\#}b\text{-}int(A))^C$. Then $x \notin (g^{\#}b\text{-}int(A))^C$. That is every $g^{\#}b$-open set V containing x is such that $V \not\subset A$. That is every $g^{\#}b$-open set V containing x is such that $V \cap A^C \neq \phi$. By Theorem 4.8 $x \in g^{\#}b\text{-}int(A^C))$ and therefore $(g^{\#}b\text{-}int(A))^C \subset g^{\#}b\text{-}cl(A^C)$.

Conversely, let $x \notin g^{\#}b\text{-}cl(A^C)$. Then by theorem, every $g^{\#}b$-open set V containing x is such that $V \cap A^C \neq \phi$. That is every $g^{\#}b$-open set V containing x is such that $V \not\subset A$. This implies by definition of $g^{\#}b$-interior of A, $x \notin g^{\#}b\text{-}int(A)$. That is $x \in g^{\#}b\text{-}int(A))^C$ and $g^{\#}b\text{-}cl(A^C) \subset (g^{\#}b\text{-}int(A))^C$. Thus $(g^{\#}b\text{-}int(A))^C = g^{\#}b\text{-}cl(A^C)$. Proof of (2) is very easy just taking complements in (1). Proof of (3) simply by replacing A by A^C in (1). □

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