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Regularly Closed Sets via Hereditary Classes

Research Article

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Abstract: In this paper, we introduce and study the notion of regular $\mathcal{H}$-closed sets in hereditary generalized topological space.

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1. Introduction and Preliminaries

In 2002, Csaszar [1] introduced the notions of generalized topology and generalized continuity. A nonempty family $\mathcal{H}$ of subsets of X is said to be hereditary class [2], if $A \in \mathcal{H}$ and $B \subset A$, then $B \in \mathcal{H}$. A generalized topological space (X, μ) with a hereditary class $\mathcal{H}$ is called hereditary generalized topological space and is denoted by $(X, \mu, \mathcal{H})$. $\mathcal{H}$ is said to be μ-codense if $\mu \cap \mathcal{H} = \emptyset$. A generalized topological space (X, μ) with a hereditary class $\mathcal{H}$, for each $A \subset X$, $A^\star(\mathcal{H}, \mu) = \{x \in X : A \cap V \notin \mathcal{H}$ for every $V \in \mu$, such that $x \in V\}$ [2]. If $c^\star(A) = A \cup A^\star(\mathcal{H}, \mu)$ for every subset A of X, then $\mu^\star = \{A \subset X : X \backslash A = c^\star(X \backslash A)\}$ is a GT, $\mu^\star$ is finer than μ [2]. A subset A of $(X, \mu, \mathcal{H})$ is said to be $\star$-perfect [2] (resp. $\star$-closed [2], $\star$-dense-in-itself [2], $f_{\mathcal{H}}$-set [4]) if $A = A^\star$ (resp. $A^\star \subset A$, $A \subset A^\star$, $A \subset (i_\mu(A))^\star$). A subset A is said to be $\mathcal{H}$-locally closed if $A = B \cap C$, where B is μ-open and C is $\star$-perfect. A subset A of (X, μ) is said to be μ-semiopen [3] (resp. μ-regular closed [3]) if $A \subset ci(A)$ (resp. $A = ci(A)$). The collection of μ-semiopen (resp. μ-regular closed) subsets of (X, μ) is denoted by $\sigma(\mu)$ (resp. $rc(\mu)$). we introduce and study the notion of regular $\mathcal{H}$-closed sets in hereditary generalized topological space.

Lemma 1.1. *If A is a subset of $(X, \mu, \mathcal{H})$ such that $A \subset A^\star$, then $A^\star = c(A^\star) = c^\star(A) = c(A)$.*

Lemma 1.2. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and A, B be subsets of X. If $U \in \mu$, then $U \cap A^\star \subset (U \cap A)^\star$.*

2. Properties of Regular $\mathcal{H}$-closed Sets

Definition 2.1. *A subset A of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ is called a regular $\mathcal{H}$-closed set if $A = (i(A))^\star$.*

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Proposition 2.2. *Every regular $\mathcal{H}$-closed set is an $f_{\mathcal{H}}$-set.*

Proof. The proof is clear. □

Proposition 2.3. *Every regular $\mathcal{H}$-closed set is $\star$-perfect.*

Proof. Let $A \in rc(\mu)$. Then we have $A = (i(A))^\star$. Since $i(A) \subset A$, $i(A)^\star \subset A^\star$. Hence $A = i(A)^\star \subset A^\star$. By Lemma 1, $A = i(A)^\star \Rightarrow A^\star = i(A)^{\star\star} \subset i(A)^\star = A$. Therefore, $A = A^\star$; hence A is $\star$-perfect. □

Corollary 2.4. *Every regular $\mathcal{H}$-closed set is $\star$-closed and $\star$-dense-in-itself.*

Proof. The proof follows from Proposition 2.3. □

Proposition 2.5. $rc(\mathcal{H}, \mu) \subset rc(\mu)$.

Proof. Let $A \in rc(\mathcal{H}, \mu)$. Then we have $A = (i(A))^\star$. Thus we obtain that $c(A) = c(i(A))^\star = (i(A))^\star = A$. Also, $(i(A))^\star \subset ci(A)$; hence $A = (i(A))^\star \subset ci(A) \subset c(A) = A$. Then we have $A = ci(A)$. Consequently, $A \in rc(\mu)$. □

We will denote the family of all regular $\mathcal{H}$-closed sets in $(X, \mu, \mathcal{H})$ by $rc(\mathcal{H}, \mu)$. If the hereditary class $\mathcal{H}$ is not μ-codense, then X is μ-regular closed in $(X, \mu, \mathcal{H})$ but not regular $\mathcal{H}$-closed and so μ-regular closed sets need not be regular $\mathcal{H}$-closed. But every μ-regular $\mathcal{H}$-closed set is a μ-regular closed set. The easy proof of the following Theorems are omitted. Theorem 2.7 below gives a characterization of μ-codense ideals.

Theorem 2.6. *If $(X, \mu, \mathcal{H})$ is a hereditary generalized topological space, then $rc(\mathcal{H}, \mu) \cap \mathcal{H} = \{\emptyset\}$.*

Theorem 2.7. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then $\mathcal{H}$ is μ-codense if and only if X is regular $\mathcal{H}$-closed.*

Theorem 2.8. *If $(X, \mu, \mathcal{H})$ is a hereditary generalized topological space where $\mathcal{H}$ is μ-codense, then $rc(\mathcal{H}, \mu) = rc(\mu)$.*

Proof. $A \in rc(\mathcal{H}, \mu)$, $A = (i(A))^\star$, $A = ci(A)$, since $\mathcal{H}$ is μ-codense, $A \in rc(\mu)$. □

Corollary 2.9. *If $(X, \mu, \mathcal{H})$ is a hereditary generalized topological space where $\mathcal{H}$ is μ-codense, then the following are equivalent:*

(1). $A \in rc(\mu)$.

(2). $A \in rc(\mathcal{H}, \mu)$.

(3). $A \in f_{\mathcal{H}}$ and A is $\star$-closed.

(4). $A \in \sigma(\mu)$ and A is $\star$-closed.

Proof. Proof follows from their respective definitions. □

The following Theorem 2.10 gives some properties of regular $\mathcal{H}$-closed sets. Also, it is established that every regular $\mathcal{H}$-closed set is $\mathcal{H}$-locally closed.

Theorem 2.10. *If A is a regular $\mathcal{H}$-closed set of a hereditary generalized topological space $(X, \mu, \mathcal{H})$, then*

(1). A and $i(A)$ are $\star$-dense-in-itself.

(2). $A^\star = (i(A))^\star = ((i(A))^\star)^\star = A$.

(3). A is $\star$-perfect and $\mathcal{H}$-locally closed.

(4). $(i(A))^\star$ is $\star$-perfect and $\mathcal{H}$-locally closed.

Proof.

(1). Since $i(A) \subset A = (i(A))^\star \subset A^\star$, $i(A)$ and A are $\star$-dense in itself.

(2). Since $A = (i(A))^\star \subset A^\star$, $A^\star = (i(A))^{\star\star} \subset (i(A))^\star = A \subset A^\star$ and so $A^\star = (i(A))^{\star\star} = (i(A))^\star = A$.

(3). Since $A = A^\star$, A is $\star$-perfect and so is $\mathcal{H}$-locally closed.

(4). By (2), $(i(A))^\star$ is $\star$-perfect and so $\mathcal{H}$-locally closed. □

We end this section with the following characterization of regular $\mathcal{H}$-closed sets in terms of μ-open sets.

Theorem 2.11. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then A is a regular $\mathcal{H}$-closed subset of X if and only if there exists a μ-open set G such that $G \subset A = G^\star$.*

Proof. Suppose A is a regular $\mathcal{H}$-closed subset of X. Let $G = i(A)$. Then G is the required μ-open set such that $G \subset A = G^\star$. Conversely, suppose that there is a μ-open set G such that $G \subset A = G^\star$. Now $G \subset A)G \subset i(A))G^\star \subset (i(A))^\star)A \subset (i(A))^\star$ and $(i(A))^\star \subset A^\star = G^\star = A$. Therefore, A is regular $\mathcal{H}$-closed. □

Theorem 2.12. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then $\mathcal{H}$ is μ-codense if and only if X is regular $\mathcal{H}$-closed.*

Proof. Proof follows from their respective definitions. □

Theorem 2.13. *If $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space where $\mathcal{H}$ is μ-codense, then $rc(\mathcal{H}, \mu) = rc(\mu)$.*

Proof. $A \in rc(\mathcal{H}, \mu)$ if and only if $A = (i(A))^\star$ if and only if $A = ci(A)$, since $\mathcal{H}$ is μ-codense if and only if $A \in rc(\mu)$. □

Theorem 2.14. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then $\mathcal{H}$ is μ-codense if and only if $rc(\mathcal{H}, \mu) = rc(\mu)$.*

Proof. Suppose $\mathcal{H}$ is μ-codense. Then $A \in rc(\mathcal{H}, \mu)$ if and only if $A = (i(A))^\star$ if and only if $A = ci(A)$, if and only if $A \in rc(\mu)$. Conversely, suppose $rc(\mathcal{H}, \mu) = rc(\mu)$. Since X is μ-regular closed, X is regular $\mathcal{H}$-closed and so $X = (i(X))^\star = X^\star$ which implies that $\mathcal{H}$ is μ-codense. □

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