International Journal of Mathematics And its Applications
Volume 2017, Issue 3–B (2017), 185–187.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

General Results on Odd-Even Sum Labeling of Graphs

Research Article

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Abstract: This paper is aimed to focus on some general results on odd-even sum labeling of graphs. We proved that every graceful graph with α-labeling is an odd-even sum graph. Odd-even sum labeling is defined by Monika and Murugan [3].

MSC: 05C78.

Keywords: Odd-even sum labeling, graceful labeling, α-labeling.

1. Introduction

In 1966, Rosa [4] defined α-labeling and graceful labeling. α-labeling is a graceful labeling with an additional property that there is an integer k $(0 \leq k < q)$ such that for every edge $e = uv$, $\min\{f(u), f(v)\} \leq k < \max\{f(u), f(v)\}$. A graph which admits an α-graceful labeling is necessarily bipartite graph with partition $V_1 = \{w \in V(G)/f(w) \leq k\}$ and $V_2 = \{w \in V(G)/f(w) > k\}$. Monika and Murgun [3] have introduced a new concept odd-even sum labeling, which is an injective labeling function $f : V(G) \to \{\pm 1, \pm 3, \ldots, \pm(2p-1)\}$ such that the edge induced labeling function $f^* : E(G) \to \{2, 4, \ldots, 2q\}$ defined by $f^*(uv) = f(u) + f(v)$, $\forall$ $uv \in E(G)$ is bijective function. In same paper, they have proved that every path $P_n (n \geq 2)$, star $K_{1,n}$, bistar $B_{m,n}$, coconut tree $CT(m, n)$, spliting graph of star $S(K_{1,n})$ and caterpillar $S(X_1, X_2, \ldots, X_n)$ are odd-even sum graphs.

2. Main Results

Theorem 2.1. *Cycle $C_n (n \equiv 0 \pmod 4)$ is an odd-even sum graph.*

Proof. Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$ and $E(C_n) = \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_1 v_n\}$. It is obvious that $p = q = n$ in C_n. Define $f : V(C_n) \to \{\pm 1, \pm 3, \ldots, \pm(2n-1)\}$ as follows

$$\begin{aligned} f(v_i) &= 2 - i, && \forall\ i = 1, 3, \ldots, n-1; \\ &= 2n + 1 - i, && \forall\ i = 2, 4, \ldots, \frac{n}{2}; \\ &= 2n - 1 - i, && \forall\ i = \frac{n}{2} + 2, \frac{n}{2} + 4, \ldots, n. \end{aligned}$$

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Note that the induced edge labeling function f^* is bijective, $f^*(v_1v_n) = n$ and

$$\begin{aligned} f^*(v_iv_{i+1}) &= f(v_i) + f(v_{i+1}) \\ &= 2(n+1-i), \quad \text{when } 1 \le i \le \frac{n}{2} \\ &= 2(n-i), \qquad \text{when } \frac{n}{2} \le i. \end{aligned}$$

Hence, cycle $C_n (n \equiv 0\ (mod\ \ 4))$ is an odd-even sum graph. □

Theorem 2.2. *Every α-graceful tree T is an odd-even sum graph.*

Proof. Let $V(T) = \{v_1, v_2, \ldots, v_p\}$. It is obvious that $q = |E(T)| = p-1$. Let $f : V(T) \to \{0, 1, \ldots, p-1\}$ is an α-graceful labeling for T. Since, T is α-graceful, $\exists$ an integer $k\,(0 \le k < q) \ni$ for any $uv \in E(T), \min\{f(u), f(v)\} \le k < \max\{f(u), f(v)\}$. Without loss of generality, we assume here the vertex v is not a pendent vertex in T, where $f(v) = 0$. Otherwise $q - f$ is also an α-graceful labeling for T and the vertices w, v (with $f(w) = q, f(v) = 0$) both can't be pendent vertices in T unless $|V(T)| = 2$. Let $V_1 = \{w \in V(T)/\ f(w) \le k\}$ and $V_2 = \{w \in V(T)/\ f(w) > k\}$. Define $g : V(T) \to \{\pm1, \pm3, \ldots, \pm2(p-1)\}$ as follows

$$\begin{aligned} g(v) &= 1, & &\text{where } f(v) = 0. \\ g(u) &= -2f(u) + 1, & &\text{if } u \in V_1 - \{v\} \\ &= 2f(u) - 1, & &\text{if } u \in V_2. \end{aligned}$$

Note that g is an injective function and its induced edge labeling function g^* is bijective with $g^*(uv) = -2f(u)+1+2f(v)-1$ assuming

$$\begin{aligned} f(u) &< f(v) \\ &= 2(f(u) - f(v)) \\ &= 2f^*(uv). \end{aligned}$$

Thus, above defined labeling pattern give rise odd-even sum labeling to the tree T and so, T is an odd-even sum graph. For a tree T, $p = |V(T)| = q + 1 = |E(T)| + 1\ (q < p)$. But in multicycle graph G, $q >> p$. So, we revise odd-even sum labeling as follows: $A(p, q)$ graph G is said to be an odd-even sum graph if it admits an injective vertex labeling function $f : V(G) \to \{\pm1, \pm3, \ldots, \pm(2q-1)\}$ with its induced edge labeling function $f^* : E(G) \to \{2, 4, \ldots, 2q\}$ defined by $f^*(uv) = f(u) + f(v), \forall\ uv \in E(G)$ is bijective. □

Proof. Every α-graceful connected graph G is also an odd-even sum graph. □

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ and $f : V(T) \to \{0, 1, 2, \ldots, q\}$ is an α-graceful labeling for T. We assume here $p \le q$. Otherwise G is a connected graph with $p > q$ and in this case G is a minimal connected graph with $q = p-1$. Particularly G is a tree in this case. Since, G is an α-graceful graph, $\exists$ an integer $k(0 \le k < q) \ni$ for any $uv \in E(G)$, either $f(u) \le k < f(v)$ or $f(v) \le k < f(u)$. Let $V_1 = \{w \in V(G)/\ f(w) \le k\}$ and $V_2 = V(G) - V_1$. Define $g : V(G) \to \{\pm1, \pm3, \ldots, \pm(2q-1)\}$ as follows

$$\begin{aligned} g(v) &= 1, & &\text{where } f(v) = 0. \\ g(u) &= -2f(u) + 1, & &\text{when } u \in V_1 - \{v\} \\ &= 2f(u) - 1, & &\text{when } u \in V_2. \end{aligned}$$

Note that g is an injective function and its induced edge labeling function g^* is bijective with

$$\begin{aligned} g^*(uv) &= -2f(u) + 1 + 2f(v) - 1, \quad \text{assuming } u \in V_1 \\ &= 2(f(u) - f(v)) \\ &= 2f^*(uv). \end{aligned}$$

Thus, the above labeling pattern give rise an odd-even sum labeling to the α-graceful graph G and so, it is an odd-even sum graph. □

Acknowledgment

The authors are very much thankful to the anonymous referee and editorial board of this journal for the valuable comments, suggestions or any kind of support.

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