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Connectedness and Compactness via Semi-Star-Regular Open Sets

Research Article

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Abstract: In this paper, we introduce new concepts namely, semi*r-connectedness and semi*r-compactness using semi*regular open sets. We investigate their basic properties. We also discuss their relationships with already existing concepts of connectedness and compactness.

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Keywords: Semi*regular closed, semi*regular closure, semi*regular compact, semi*regular connected, semi*regular open.

1. Introduction

In 1974, Das [2] defined the concept of semi-connectedness in topological spaces and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett [5] introduced and studied the concept of semi-compact spaces. Since then, Hanna and Dorsett [6], Ganster [5] investigated the properties of semi-compact spaces. PasunkiliPandian.S [12] introduced semi*-pre-compact spaces and investigated their properties. Robert, A. and Pious Missier, S. recently introduced and studied semi*-connectedness and semi*-compactness [16] in topological spaces. The authors have defined semi*regular open sets [13] and semi*regular closed sets [13] and investigated their properties. In this paper, we introduce the concept of semi*regular connected spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts namely connectedness, semi-connectedness, semi-pre connectedness and semi*α-connectedness. Further we define semi*regular compact spaces and investigate their properties. We also show the relationship of semi*r-compactness with each of the concepts of compactness, semi-compactness semi*-compactness and semi*-pre compactness.

2. Preliminaries

Throughout this paper X will always denote a topological space. If A is a subset of the space X, $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively.

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Definition 2.1. *Let A subset A of a topological space* (X, τ) *is called*

(1). generalized closed (briefly g-closed) [7] if $Cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in X.*

(2). generalized open (briefly g-open) [7] if X A is gclosed in X.

Definition 2.2. *A be a subset of X. The generalized closure [4] of A is defined as the intersection of all g-closed sets containing A and is denoted by Cl*(A).*

Definition 2.3. *A subset A of a topological space* (X, τ) *is called*

(1). semi-open [8] (resp. α*-open [9], semi* α*-open [10] semi-preopen [1] semi*-open [17], semi*regular open [13], semi*-preopen [12]) if* $A \subseteq Cl(Int(A))$ *(resp.* $A \subseteq Int(Cl(Int(A)))$*,* $A \subseteq Cl(Int(Cl(Int(A)))$*,* $A \subseteq Cl(Int(Cl(A)))$*,* $A \subseteq Cl*(Int(A))$*,* $A \subseteq Cl*(rInt(A))$*,* $A \subseteq Cl*(pInt(A))$*).*

(2). semi-closed [2] (resp. semi α*-closed [11], semipreclosed [1], semi*-closed [18], semi*regular closed [13], semi*-preclosed [12]) if* $Int(Cl(A)) \subseteq A$ *(resp.* $Int(Cl(Int(Cl(A)))) \subseteq A$*,* $Int(Cl(Int(A))) \subseteq A$*,* $Int{*}(Cl(A)) \subseteq A$*),* $Int{*}(rCl(A)) \subseteq A$*,* $Int*(pCl(A)) \subseteq A$*.*

Definition 2.4. *Let A be a subset of X. Then the semi*regular closure [13] of A is defined as the intersection of all semi*regular closed sets in X containing A and is denoted by s*rCl(A).*

Definition 2.5 ([13])**.**

*(1). Every Semi*regular open set is Semi**α*-open.*

*(2). Every Semi*regular open set is Semi*pre-open.*

*(3). Every Semi*regular open set is Semi*open.*

(4). Every Semi regular open set is Semi open.*

*(5). Every Semi*regular open set is Semi* α*-open.*

*(6). Every Semi*regular open set is Semi pre-open.*

*(7). Every Semi*regular open set is regular generalized open set.*

*(8). Every Semi*regular open set is generalized pre regular open set.*

Definition 2.6 ([15])**.** *If A is a subset of X, the semi*r-Frontier of A is defined by s*rFr(A)=s*rCl(A)\s*rInt(A).*

Result 2.7 ([15])**.** *If A is a subset of X, then s*rFr(A)=s*rCl(A)∩s*rCl(X\A). Let A be a subset of a space X. Then A is semi*r-regular if and only if* $s*rFr(A) = \emptyset$*.*

Theorem 2.8 ([13])**.** *If A is a subset of X, then*

*(1). s*rCl(X\A) = X\s*rInt(A).*

*(2). s*rInt(X\A) = X\s*rCl(A).*

*(3). A is semi*regular closed if and only if s*rCl(A) = A.*

Definition 2.9. *A topological space X is said to be connected [19] (resp. semi-connected [2], α-connected, semi*-connected [16], semi*-pre-connected [12]) if X cannot be expressed as the union of two disjoint nonempty open (resp. semi-open, α-open, semi*-open, semi*-preopen) sets in X.*

Definition 2.10 ([19]). *A subset A of a topological space (X, τ) is called clopen if it is both open and closed in X.*

Theorem 2.11 ([19]). *A topological space X is connected if and only if the only clopen subsets of X are ϕ and X.*

Definition 2.12. *A collection B of open (resp. semi-open) sets in X is called an open (resp. semi-open) cover of $A \subseteq X$ if $A \subseteq \cup\{U\alpha : U\alpha \in B\}$ holds.*

Definition 2.13. *A space X is said to be compact [19] (resp. semi compact [3]) if every open (resp. semi-open) cover of X has a finite subcover.*

Definition 2.14 ([15]). *A function $f : X \longrightarrow Y$ is said to be*

*(1). semi*r-continuous if $f^{-1}(V)$ is semi*regular open in X for every open set V in Y.*

*(2). semi*r-irresolute if $f^{-1}(V)$ is semi*regular open in X for every semi*regular open set V in Y.*

*(3). semi*regular open if f(V) is semi*regular open in Y for every open set V in X.*

*(4). semi*regular closed if f(V) is semi*regular closed in Y for every closed set V in X.*

*(5). pre-semi*regular open if f(V) is semi*regular open in Y for every semi*regular open set V in X.*

*(6). pre-semi*regular closed if f(V) is semi*regular closed in Y for every semi*regular closed set V in X.*

*(7). semi*r-totally continuous if $f^{-1}(V)$ is clopen in X for every semi*regular open set V in Y.*

*(8). contra-semi*r-continuous if $f^{-1}(V)$ is semi*regular closed in X for every open set V in Y.*

*(9). contra-semi*r-irresolute if $f^{-1}(V)$ is semi*regular closed in X for every semi*regular open set V in Y.*

Theorem 2.15 ([15]). *Let $f : X \to Y$ be a function. Then*

*(1). f is semi*r-continuous if and only if $f^{-1}(F)$ is semi*regular-closed in X for every closed set F in Y.*

*(2). f is semi*r-irresolute if and only if f^{-1}(F) is semi*regular closed in X for every semi*regular closed set F in Y.*

*(3). f is contra-semi*r-continuous if and only if $f^{-1}(F)$ is semi*regular open in X for every closed set F in Y.*

*(4). f is contra-semi*r-irresolute if and only if $f^{-1}(F)$ is semi*regular open in X for every semi*regular closed set F in Y.*

*(5). f is semi*r-totally continuous if $f^{-1}(F)$ is clopen in X for every semi*regular closed set F in Y.*

3. Semi*regular Connected Spaces

In this section we introduce semi*regular connected spaces. We give characterizations for semi*regular connected spaces and also investigate their basic properties.

Definition 3.1. *A topological space X is said to be semi*regular connected if X cannot be expressed as the union of two disjoint nonempty semi*regular open sets in X.*

Example 3.2. *Let $X = \{a, b, c, d\}$ and the topology $\tau = \{X, \emptyset, \{a\}, \{d\}, \{a, d\}, \{a, c, d\}\}$. S*RO(X)={$\emptyset$,{a, b}, {b, d} {a, b, d},X}. Then the space X is semi*regular connected.*

Theorem 3.3.

(1). Every semi-connected space is semi*regular connected.*

*(2). Every semi*pre-connected space is semi*regular connected.*

(3). Every semiα-connected space is semi*regular connected.*

*(4). Every semi connected space is semi*regular connected.*

*(5). Every semi α-connected space is semi*regular connected.*

*(6). Every semi pre connected space is semi*regular connected.*

Proof. (1). Let X be semi*connected space. Suppose X is not semi*regular connected. Then there exists a proper non empty subset B of X which is both semi*regular open and semi*regular closed in X. Since every semi*regular closed (open)set is semi*closed(open)set then X is not semi*connected. This proves (1). In the similar manner we can prove (2), (3), (4), (5) and (6). □

Remark 3.4. *It can be seen from the following example that the converse of each of the statements in Theorem 3.3 is not true.*

Example 3.5. *Let $X = \{a, b, c, d\}$ and the topology $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$*

$$S^*O(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$
$$S^*\alpha O(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$
$$S^*PO(X) = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$$
$$S^*RO(X) = \{X, \emptyset, \{b, d\}, \{a, c, d\}\}.$$

*Then X is semi*regular connected space but not semi*connected , not semi*α-connected and not semi*pre connected spaces.*

Definition 3.6. *The sets A and B in a topological space X are said to be semi*regular separated if $A \cap s^*rCl(B) = s^*rCl(A) \cap B = \emptyset$.*

Theorem 3.7. *For a topological space X, the following statements are equivalent:*

*(1). X is semi*regular connected.*

*(2). X cannot be expressed as the union of two disjoint nonempty semi*regular closed sets in X.*

*(3). The only semi*r-regular subsets of X are $\emptyset$ and X itself.*

*(4). Every semi*regular continuous function of X into a discrete space Y is constant.*

*(5). Every nonempty proper subset of X has nonempty semi*r-frontier.*

*(6). X cannot be expressed as the union of two non-empty semi*regular separated sets.*

Proof. (1) ⇒ (2) Let X be a semi*regular connected space. Suppose $X = A \cup B$ where A and B are disjoint nonempty semi*regular closed sets. Then $A = XnB$ and $B = XnA$ are disjoint non-empty semi*regular open sets in X. This is a contradiction to the fact that X is semi*regular connected. This proves (2).

(2)⇒(1) Assume that X cannot be expressed as the union of two disjoint nonempty semi*regular closed sets in X. Suppose $X = A \cup B$ where A and B are disjoint nonempty semi*regular open sets. Then $A = XnB$ and $B = XnA$ are disjoint non-empty semi*regular closed sets in X. This is a contradiction to (2).

(1)⇒(3) Suppose X is a semi*regular connected space. Let A be non-empty proper subset of X that is A is semi*r-regular set. Then XnA is a non-empty semi*regular open(or semi*regular closed) and $X = A \cup (XnA)$. This is a contradiction to X is semi*regular connected.

(3)⇒(1) Suppose $X = A \cup B$ where A and B are disjoint non-empty semi*regular open sets. Then $A = XnB$ is semi*regular closed. Thus A is a non-empty proper subset that is semi*r-regular.This is a contradiction to (3).

(3)⇒(4) Let f be a semi*regular continuous function of the semi*regular connected space X into the discrete space Y. Then for each $y \in Y$, $f^{-1}(\{y\})$ is a semi*r-regular set of X. Since X is semi* regular connected, $f^{-1}(\{y\}) = \emptyset$ or X. If $f^{-1}(\{y\}) = \emptyset$ for all $y \in Y$, then f fails to be a function. Therefore $f^{-1}(\{y_0\}) = X$ for a unique $y_0 \in Y$. This implies $f(X) = \{y_0\}$ and hence f is a constant function.

(4)⇒(3) Let U be a semi*r-regular set in X. Suppose $U \neq \emptyset$. We claim that $U = X$. Otherwise, choose two fixed points y_1 and y_2 in Y. Define $f : X \to Y$ by

$$f(x) = \begin{cases} y_1, & if x \in U; \\ y_2, & \text{otherwise.} \end{cases}$$

Then for any open set V in Y,

$$f^{-1}(V) = \begin{cases} U, & \text{if V contains } y_1 \text{ but not } y_2 \\ X \backslash U, & \text{if V contains } y_2 \text{ but not } y_1 \\ X, & \text{if V contains both } y_1 \text{ and } y_2 \\ \emptyset, & \text{otherwise.} \end{cases}$$

In all the cases $f^{-1}(V)$ is semi*regular open in X. Hence f is a non-constant semi*regular continuous function of X into Y. This is a contradiction to our assumption. This proves that the only semi* regular subsets of X are $\emptyset$ and X.

(3)⇒(5) Suppose that a space X is semi*regular connected. If possible, let A be a non-empty proper subset of X. We claim that $s^*rFr(A) \neq \emptyset$. If possible, let $s^*rFr(A) = \emptyset$. Then by the Result 2.7, A is semi*r-regular. This is a contradiction.

(5)⇒(3) Suppose that every non-empty proper subset of X has a non-empty semi*regular frontier. The only semi*r-regular subsets of X are $\emptyset$ and X itself. On the contrary, suppose that X has a non-empty proper subset A which is semi*r-regular. By the Result 2.7, $s^*rFr(A) = \emptyset$. This contradiction proves (3).

(1)⇒(6) Suppose $X = A \cup B$ where A and B are disjoint non-empty semi*r-separated sets in X. Since $A \cap s^*rCl(B) = \emptyset$, $s^*rCl(B) \subseteq XnA = B$ and hence $s^*rCl(B) = B$ and so by Theorem 2.8 (3), B is semi*regular closed. Therefore A is semi*regular open. Similarly, B is semi*regular open.Hence X is not semi*r-connected. This is contradiction to (1).

(6)⇒(1) Suppose X is not semi*r-connected. Then X can be written as $X = A \cup B$ where A and B are disjoint non-empty semi*regular open sets. Now $A = XnB$ is semi*regular closed and hence by Theorem 2.8 (3), $s^*rCl(A) = A$ and so $s^*rCl(A) \cap B = \emptyset$. Similarly $A \cap s^*rCl(B) = \emptyset$. Thus A and B are nonempty semi*regular separated sets. This is a contradiction to (6). □

Theorem 3.8. *Let $f : X \to Y$ be a semi*regular continuous bijection and X be semi*regular connected. Then Y is connected.*

Proof. Let $f : X \to Y$ be semi*regular continuous surjection and X be semi*regular connected. Let V be a clopen subset of Y. By Definition 2.14 (1) $f^{-1}(V)$ is semi*regular open and by Theorem 2.15 (1), $f^{-1}(V)$ is semi*regular closed and hence $f^{-1}(V)$ is semi*r-regular in X. Since X is semi*regular connected, by Theorem 3.7 $f^{-1}(V) = \emptyset$ or X. Hence $V = \emptyset$ or Y. This proves that Y is connected. □

Theorem 3.9. *Let $f : X \to Y$ be a semi*r-irresolute bijection. If X is semi*regular connected, so is Y.*

Proof. Let $f : X \to Y$ be a semi*r-irresolute surjection and let X be semi*regular connected. Let V be a subset of Y that is semi*r-regular in Y. By Definition 2.14 (2) and by Theorem 2.15 (2), $f^{-1}(V)$ is semi*r-regular in X. Since X is semi*regular connected, $f^{-1}(V) = \emptyset$ or X. Hence $V = \emptyset$ or Y. This proves that Y is semi*regular connected. □

Theorem 3.10. *Let $f : X \to Y$ be a pre-semi*regular open and pre-semi*regular closed bijection. If Y is semi*regular connected, so is X.*

Proof. Let A be subset of X that is semi*r-regular in X. Since f is both pre-semi*regular open and pre-semi*regular closed, $f(A)$ is semi*r-regular in Y. Since Y is semi*regular connected, $f(A) = \emptyset$ or Y. Hence $A = \emptyset$ or X. Therefore by Theorem 3.7, X is semi*regular connected. □

Theorem 3.11. *If $f : X \to Y$ is a semi*regular open and semi*regular closed bijection and Y is semi*regular connected, then X is connected.*

Proof. Let A be a clopen subset of X. Since f is semi*regular open, $f(A)$ is semi*regular open in Y. Since f is a semi*regular closed map, $f(A)$ is semi*regular closed in Y. Hence $f(A)$ is semi*r-regular in Y. Since Y is semi*regular connected, by Theorem 3.7, $f(A) = \emptyset$ or Y. Hence $A = \emptyset$ or X. By Theorem 2.11, X is connected. □

Theorem 3.12. *If there is a semi*r-totally continuous function from a connected space X onto Y, then the only semi*regular open sets in Y are $\emptyset$ and Y.*

Proof. Let f be a semi*r-totally continuous function from a connected space X onto Y. Let V be any open set in Y. Then by Theorem 2.5 (2), V is semi*regular open in Y. Since f is semi*r-totally continuous, $f^{-1}(V)$ is clopen in X. Since X is connected, by Theorem 2.11, $f^{-1}(V) = \emptyset$ or X. This implies $V = \emptyset$ or Y. □

Theorem 3.13. *If $f : X \to Y$ is a strongly semi*r-continuous bijection and Y is a space with at least two points, then X is not semi*regular connected.*

Proof. Let $y \in Y$. Then $f^{-1}(\{y\})$ is a non-empty proper subset that is semi*r-regular in X. Hence by Theorem 3.7, X is not semi*regular connected. □

Theorem 3.14. *Let $f : X \to Y$ be a contra-semi*regular continuous surjection and X be semi*regular connected. Then Y is connected.*

Proof. Let $f : X \to Y$ be a contra-semi*regular continuous surjection and X be semi*regular connected. Let V be a clopen subset of Y. By Definition 2.14 (8) and by Theorem 2.15 (3), $f^{-1}(V)$ is semi*r-regular in X. Since X is semi*regular connected, $f^{-1}(V) = \emptyset$ or X. Hence $V = \emptyset$ or Y. This proves that Y is connected. □

Theorem 3.15. *Let $f : X \to Y$ be a semi*r-irresolute bijection. If X is semi*regular connected, so is Y.*

Proof. Let $f : X \to Y$ be a semi*r-irresolute bijection and let X be semi* regular connected. Let V be a subset of Y that is semi*r-regular in Y. By Definition 2.14 (2) and by Theorem 2.15 (2), $f^{-1}(V)$ is semi*r-regular in X. Since X is semi*regular connected, $f^{-1}(V) = \emptyset$ or X. Hence $V = \emptyset$ or Y. This proves that Y is semi*regular connected. □

Theorem 3.16. *Every contra-semi*r-continuous function from a semi*regular connected space into a T_1 space is necessarily constant.*

Proof. Let $f : X \to Y$ be a contra-semi*r-continuous function and X be semi*regular connected and Y be T_1. Since Y is T_1, for each $y \in Y$, $\{y\}$ is closed in Y. Since f is contra-semi*r-continuous, by Theorem 2.15 (3), $f^{-1}(\{y\})$ is semi*regular open in X. Therefore $\{f^{-1}(\{y\}) : y \in Y\}$ is a collection of pair wise disjoint semi*regular open sets in X. Since X is semi*regular connected, $f^{-1}(\{y_0\}) = X$ for some fixed $y_0 \in Y$. Hence $f(X) = y_0$. Thus f is a constant function. □

Theorem 3.17. *Every contra-semi*r-irresolute function from a semi*regular connected space into a semi*r-T_1 space is necessarily constant.*

Proof. Let $f : X \to Y$ be a contra-semi*r-irresolute function and X be semi*regular connected and Y be semi*r-T_1. Since Y is semi*r-T_1, for each $y \in Y$, $\{y\}$ is semi*regular closed in Y. Since f is contra-semi*r-continuous, $f^{-1}(\{y\})$ is semi*regular open in X. Therefore $\{f^{-1}(\{y\}) : y \in Y\}$ is a collection of pair wise disjoint semi*regular open sets in X. Since X is semi*regular connected, $f^{-1}(\{y_0\}) = X$ for some fixed $y_0 \in Y$. Hence $f(X) = y_0$. Thus f is constant. □

4. Semi*r-Compact Spaces

In this section we introduce semi*r-compact spaces and study their properties. We also give characterizations for these spaces.

Definition 4.1. *A collection C of semi*regular open sets in X is called a semi*regular open cover of a subset B of X if $B \subseteq \cup\{U\alpha : U\alpha \in \mathbb{C}\}$ holds.*

Definition 4.2. *A space X is said to be semi*r-compact if every semi*regular open cover of X has a finite subcover.*

Definition 4.3. *A subset B of X is said to be semi*r-compact relative to X if for every semi*regular open cover $\mathbb{C}$ of B, there is a finite subcollection of $\mathbb{C}$ that covers B.*

Remark 4.4. *Every finite topological space is semi*regular compact.*

Theorem 4.5.

1. *Every semi-compact space is semi*regular compact space.*
2. *Every semi-pre-compact space is semi*regular compact space.*
3. *Every semi α-compact space is semi*regular compact space.*
4. *Every semi*-compact space is semi*regular compact space.*

5. *Every semi*pre-compact space is semi*regular compact space.*

6. *Every semi*α-compact space is semi*regular compact space.*

Theorem 4.6. *Every semi*regular closed subset of a semi*regular compact space X is semi*regular compact relative to X.*

Definition 4.7. *Let A be a semi*regular closed subset of a semi*regular compact space X. Let B be a semi*regular open cover of A. Then $B \cup \{XnA\}$ is a semi*regular open cover of X. Since X is semi*regular compact, this cover contains a finite subcover of X and hence contains a finite subcollection of B that covers A. This shows that A is semi*regular compact relative to X.*

Theorem 4.8. *A space X is semi*regular compact if and only if for every family of semi*regular closed sets in X which has empty intersection has a finite subfamily with empty intersection.*

Proof. Suppose X is semi*regular compact and $\{F\alpha : \alpha \in \Delta\}$ is a family of semi*regular closed sets in X such that $\cap\{F\alpha : \alpha \in \Delta\} = \emptyset$. Then $\cup\{XnF\alpha : \alpha \in \Delta\}$ is a semi*regular open cover for X. Since X is semi*regular compact, this cover has a finite subcover $\{XnF\alpha_1, XnF\alpha_2, \ldots, XnF\alpha_n\}$. That is, $X = \cup\{XnF\alpha_i : i = 1, 2, \ldots, n\}$. On taking the complements on both sides we get $\bigcap_{i=1}^{n} F_{\alpha_i} = \emptyset$.
Conversely, suppose that every family of semi*regular closed sets in X which has empty intersection has a finite subfamily with empty intersection. Let $\{U\alpha : \alpha \in \Delta\}$ be a semi*regular open cover for X. Then $\cup\{U\alpha : \alpha \in \Delta\} = X$. Taking the complements, we get $\cap\{XnU\alpha : \alpha \in \Delta\} = \emptyset$. Since $XnU\alpha$ is semi*regular closed for each $\alpha \in \Delta$, by the assumption, there is a finite sub family, $\{XnU\alpha_1, XnU\alpha_2, \ldots, XnU\alpha_n\}$ with empty intersection. That is $\bigcap_{i=1}^{n}(X \backslash U_{\alpha_i}) = \emptyset$. Taking the complements on both sides, we get $\bigcup_{i=1}^{n}(X \backslash U_{\alpha_i}) = X$ Hence X is semi*regular compact. □

Theorem 4.9. *Let $f : X \to Y$ be a semi*r-irresolute bijection. If X is semi*r-compact, then so is Y.*

Proof. Let $f : X \to Y$ be a semi*r-irresolute bijection and X be semi*r-compact. Let $\{V_\alpha\}$ be a semi*regular open cover for Y. Then $\{f^{-1}(V_\alpha)\}$ is a cover of X by semi*regular open sets. Since X is semi*r-compact, $\{f^{-1}(V_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(V\alpha_1), f^{-1}(V\alpha_2), \ldots, f^{-1}(V_n)\}$. Then $\{V\alpha_1, V\alpha_2, \ldots, V\alpha_n\}$ is a finite subcover for Y. Thus Y is semi*r-compact. □

Theorem 4.10. *Let $f : X \to Y$ be a semi*r-continuous bijection and X be semi*r-compact. Then Y is compact.*

Proof. Let $f : X \to Y$ be a semi*r-continuous bijection and X be semi*α-compact. Let $\{V\alpha\}$ be an open cover for Y. Then $\{f^{-1}(V\alpha)\}$ is a cover of X by semi*regular open sets. Since X is semi*r-compact, $\{f^{-1}(V\alpha)\}$ contains a finite sub cover, namely $\{f^{-1}(V\alpha_1), f^{-1}(V\alpha_2), \ldots, f^{-1}(V\alpha_n)\}$. Then $\{V\alpha_1, V\alpha_2, \ldots, V\alpha_n\}$ is a cover for Y. Thus Y is compact. □

Theorem 4.11. *Let $f : X \to Y$ be a pre-semi*regular open injection. If Y is semi*r-compact, then so is X .*

Proof. Let $\{V\alpha\}$ be a semi*regular open cover for X. Then $\{f(V\alpha)\}$ is a cover of Y by semi*regular open sets. Since Y is semi*r-compact, $\{f(V\alpha)\}$ contains a finite subcover, namely $\{f(V\alpha_1), f(V\alpha_2), ..., f(V\alpha_n)\}$. Since f is semi*regular open injection, $\{V\alpha_1, V\alpha_2, ..., V\alpha_n\}$ is a finite subcover for X. Therefore X is semi*r-compact. □

Theorem 4.12. *If $f : X \to Y$ is a semi*regular open injection and Y is semi*r-compact, then X is compact.*

Proof. Let $\{V\alpha\}$ be an open cover for X. Then $\{f(V\alpha)\}$ is a cover of Y by semi*regular open sets. Since Y is semi*r-compact, $\{f(V\alpha)\}$ contains a finite subcover, namely $\{f(V\alpha_1), f(V\alpha_2), ..., f(V\alpha_n)\}$. Since f is semi*regular open injection, $\{V\alpha_1, V\alpha_2, ..., V\alpha_n\}$ is a finite sub cover for X. Thus X is compact. □

Theorem 4.13. *Let $f : X \to Y$ be a contra-semi*r-continuous function and Y be T_1. If X is semi*r-compact, then the range of f is finite. Further if Y is infinite, f cannot be onto.*

Proof. Since Y is T_1, for each $y \in Y$, $\{y\}$ is closed in Y. Since f is contra-semi*r-continuous, by Theorem 2.15 (3), $f^{-1}(\{y\})$ is semi*regular open in X. Therefore $\{f^{-1}(\{y\}) : y \in Y\}$ is a semi*regular open cover for X. Since X is semi*r-compact, there are $y_1, y_2, ..., y_n$ in Y such that $\{f^{-1}(\{y_i\}) : i = 1, 2, ...n\}$ is a cover of X by semi*regular open sets. Therefore $\cup\{f^{-1}(\{y_i\}) : i = 1, 2, ...n\} = X$. That is, $f^{-1}(\{y_1, y_2, ..., y_n\}) = X$. This implies $f(X) = \{y_1, y_2, ..., y_n\}$. Thus the range of f is finite. If Y is infinite, $f(X) \neq Y$. Hence f cannot be onto. □

Theorem 4.14. *Let $f : X \to Y$ be a contra-semi*r-irresolute function and Y be semi*r-T_1. If X is semi*r-compact,then the range of f is finite. Further if Y is infinite, f cannot be onto.*

Proof. Since Y is semi*r-T_1, for each $y \in Y$, $\{y\}$ is semi*regular closed in Y. Since f is contra-semi*r-continuous, by Theorem 2.15 (4), $f^{-1}(\{y\})$ is semi*regular open in X. Therefore $\{f^{-1}(\{y\}) : y \in Y\}$ is a semi*regular open cover for X. Since X is semi*r-compact, there are $y_1, y_2, ..., y_n$ in Y such that $\{f^{-1}(\{y_i\}) : i = 1, 2, ...n\}$ is a cover of X by semi*regular open sets . Therefore $\cup\{f^{-1}(\{yi\}) : i = 1, 2, ...n\} = X$. That is, $f^{-1}(\{y_1, y_2, ..., y_n\}) = X$. This implies $f(X) = \{y_1, y_2, ..., y_n\}$. Thus the range of f is finite. If Y is infinite, $f(X) \neq Y$. Hence f cannot be onto. □

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