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Some Properties of Contra *gpr*-continuous Maps

Research Article

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Abstract: In this paper, we introduce contra gpr-continuous maps, study some of their properties and discuss its relationships with some topological maps and separation axioms.

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1. Introduction and Preliminaries

In 1997, Gnanambal [3] introduced generalized preregular closed sets (briefly, gpr-closed sets) in topological spaces. In 1996, Dontchev [2] introduced a new class of functions called contra-continuous functions. He defined a function $f: X \to Y$ to be contra-continuous if the preimage of every open subset of Y is closed in X. Quite recently, Jafari and Noiri [5–7] introduced and investigated the notions of contra-g-continuity, contra-precontinuity and contra-α-continuity as a continuation of research done by Dontchev [2] on the interesting notion of contra-continuity. In this direction, in this paper, we introduce the notion of contra gpr-continuity via the notion of gpr-closed sets and study some of their basic properties.

Throughout this paper, (X, τ), (Y, σ) and (Z, ρ) (briefly X, Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. Let us recall the following definitions which are often used.

Definition 1.1. *A subset A of a space X is called*

(1). a preopen set [11] if $A \subseteq int(cl(A))$;

(2). a semi-open set [8] if $A \subseteq cl(int(A))$;

(3). an α-open set [13] if $A \subseteq int(cl(int(A)))$;

(4). regular open [19] if $A = int(cl(A))$.

The complements of the above mentioned open sets are called their respective closed sets. The preclosure (α-closure) of a subset A of X is, denoted by pcl(A) (αcl(A)), defined as the intersection of all preclosed (α-closed) sets containing A.

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Definition 1.2. *A subset A of a space X is called*

(1). g-closed [9] if cl(A)⊆U whenever A⊆U and U is open in X;

(2). rg-closed [16] if cl(A)⊆U whenever A⊆U and U is regular open in X;

(3). α*g-closed [10] if* α*cl(A)⊆U whenever A⊆U and U is open in X;*

(4). gp-closed [14] if pcl(A)⊆U whenever A⊆U and U is open in X;

(5). gpr-closed [3] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.3. *A map* $f : X \to Y$ *is called*

(1). contra-continuous [2] if $f^{-1}(V)$ *is closed in X for every open set V of Y;*

(2). contra g-continuous [5] if $f^{-1}(V)$ *is g-closed in X for every open set V of Y;*

(3). contra precontinuous [6] if $f^{-1}(V)$ *is preclosed in X for every open set V of Y;*

(4). contra rg-continuous [17] if $f^{-1}(V)$ *is rg-closed in X for every open set V of Y;*

(5). contra α*g-continuous [7] if* $f^{-1}(V)$ *is* α*g-closed in X for every open set V of Y;*

(6). gpr-irresolute [3] if $f^{-1}(V)$ *is gpr-closed in X for every gpr-closed set V of Y;*

(7). gpr-continuous [3] if $f^{-1}(V)$ *is gpr-closed in X for every closed set V of Y;*

(8). contra gp-continuous [18] if $f^{-1}(V)$ *is gp-closed in X for every open set V of Y;*

(9). semi-continuous [8] if $f^{-1}(V)$ *is semi-open in X for every open set V of Y;*

(10). gpr-continuous [4] if* $f^{-1}(V)$ *is gpr-closed in X for every preclosed set V of Y;*

(11). completely continuous [1] if $f^{-1}(V)$ *is regular open in X for every open set V of Y;*

(12). perfectly continuous [15] if $f^{-1}(V)$ *is clopen in X for each open set V of Y.*

Definition 1.4. *A space X is called*

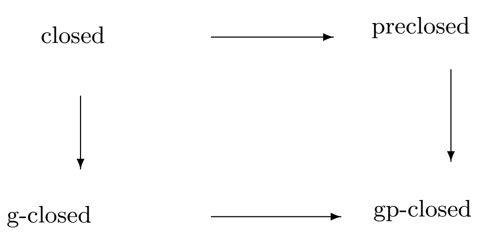
(1). locally indiscrete [12] if every open subset of X is closed in X;

(2). preregular $T_{1/2}$ *space [3] if every gpr-closed subset of X is preclosed in X;*

(3). locally g-indiscrete [5] if every g-open subset of X is closed in X;

(4). gp-space [18] if every gp-closed subset of X is closed in X.

Remark 1.5 ([18])**.** *We have the following diagram of implications.*



None of the above implications is reversible.

Remark 1.6 ([3]). *The following statements are true in a topological space* (X, τ).

(1). Every rg-closed set is gpr-closed but not conversely.

(2). Every gp-closed set is gpr-closed but not conversely.

(3). Every preclosed set is gpr-closed but not conversely.

(4). Every αg*-closed set is gpr-closed but not conversely.*

Remark 1.7 ([3]). *Every regular open gpr-closed set is preclosed and hence clopen.*

Remark 1.8 ([4]). *Every semi-open gpr-closed set is rg-closed.*

2. Properties of Contra gpr-continuous Maps

We introduce the following definition.

Definition 2.1. *A map* $f : X \to Y$ *is called contra gpr-continuous if* $f^{-1}(V)$ *is gpr-closed in* X *for every open set* V *of* Y.

Theorem 2.2.

(1). Every contra rg-continuous map is contra gpr-continuous.

(2). Every contra gp-continuous map is contra gpr-continuous.

The following Examples support that the converses of the above Theorems are not true.

Example 2.3. *Let* $X = Y = \{a, b, c, d, e\}$. *Let* $\tau = \{\phi, X, \{a, b\}, \{c, d\}, \{a, b, c, d\}\}$ *and* $\sigma = \{\phi, Y, \{a\}\}$. *Let* $f : X \to Y$ *be the identity map. Then f is contra gpr-continuous map but it is not contra rg-continuous.*

Example 2.4. *Let* $X = Y = \{a, b, c\}$. *Let* $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{\phi, Y, \{c\}, \{a, b\}\}$. *Let* $f : X \to Y$ *be the identity map. Then f is contra gpr-continuous map but it is not contra gp-continuous.*

Theorem 2.5. *Every contra* αg*-continuous map is contra gpr-continuous.*

The following Example supports that the converse of the above Theorem is not true.

Example 2.6. *Let* $X = Y =\{a, b, c\}$. *Let* $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{\phi, Y, \{c\}, \{a, b\}\}$. *Let* $f : X \to Y$ *be the identity map. Then f is contra gpr-continuous map but it is not contra* αg*-continuous.*

Theorem 2.7. *If* $f : X \to Y$ *is contra gpr-continuous map and X is preregular* $T_{1/2}$ *space, then f is contra precontinuous map.*

Theorem 2.8. *Let* $f : X \to Y$ *be a map. Then the following statements are equivalent.*

(1). f is contra gpr-continuous.

(2). The inverse image of each open set in Y is gpr-closed in X.

(3). The inverse image of each closed set in Y is gpr-open in X.

Theorem 2.9. *If* $f : X \to Y$ *is contra gpr-continuous map where Y is locally g-indiscrete and* $g : Y \to Z$ *is contra g-continuous map, then* $g\ o\ f : X \to Z$ *is contra gpr-continuous map.*

Proof. Let F be a closed set of Z. Since g is contra g-continuous, $g^{-1}(F)$ is g-open in Y. Since Y is locally g-indiscrete, $g^{-1}(F)$ is closed in Y. Since f is contra gpr-continuous, $f^{-1}(g^{-1}(F)) = (g \text{ o } f)^{-1}(F)$ is gpr-open in X. Therefore g o f is contra gpr-continuous map. □

Definition 2.10. *A map $f : X \to Y$ is said to be pre gpr-closed if for every gpr-closed set V of X, f(V) is gpr-closed set in Y.*

Theorem 2.11. *Let $f : X \to Y$ be surjective gpr-irresolute and pre gpr-closed and $g : Y \to Z$ be any map. Then g o $f : X \to Z$ is contra gpr-continuous if and only if g is contra gpr-continuous.*

Proof. Let g o f : X→ Z be contra gpr-continuous map. Let F be an open subset of Z. Then $(g \text{ o } f)^{-1}$ (F) = $f^{-1}(g^{-1}(F))$ is a gpr-closed subset of X. Since f is pre gpr-closed, $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is gpr-closed in Y. Thus g is contra gpr-continuous map.

Conversely, let g : Y→Z be contra gpr-continuous. Let G be an open subset of Z. Since g is contra gpr-continuous, $g^{-1}(G)$ is gpr-closed in Y. Since f is gpr-irresolute, $f^{-1}(g^{-1}(G)) = (g \text{ o } f)^{-1}(G)$ is gpr-closed in X. Hence g o f is contra gpr-continuous map. □

Theorem 2.12. *The composition of two contra gpr-continuous maps need not be contra gpr-continuous map.*

The following Example supports the above Theorem.

Example 2.13. *Let $X = Y = Z = \{a, b, c\}$. Let $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\phi, Y, \{a\}, \{b, c\}\}$ and $\rho = \{\phi, Z, \{b\}, \{b, c\}\}$. Let $f : X \to Y$ be defined as f(a)=b; f(b)=c and f(c)=a. Let $g : Y \to Z$ be the identity map. Then f and g are contra gpr-continuous maps. But their composition g o f is not contra gpr-continuous.*

3. Miscellaneous Results

Theorem 3.1. *Let $f : X \to Y$ be gpr-continuous map where Y is gp-space. Let $g : Y \to Z$ be contra gp-continuous map. Then g o $f : X \to Z$ is contra gpr-continuous map.*

Proof. Let F be an open set in Z. Since g is contra gp-continuous, $g^{-1}(F)$ is gp-closed in Y. But Y is gp-space, $g^{-1}(F)$ is closed in Y. Since f is gpr-continuous, $f^{-1}(g^{-1}(F))$ is gpr-closed in X. Therefore g o f is contra gpr-continuous map. □

Theorem 3.2. *Let $f : X \to Y$ be gpr-irresolute map and $g : Y \to Z$ be contra gpr-continuous map. Then g o $f : X \to Z$ is contra gpr-continuous map.*

Proof. Let F be an open set in Z. Since g is contra gpr-continuous, $g^{-1}(F)$ is gpr-closed in Y. Since f is gpr-irresolute, $f^{-1}(g^{-1}(F)) = (g \text{ o } f)^{-1}$ (F) is gpr-closed in X. Therefore g o f is contra gpr-continuous map. □

Theorem 3.3. *If a map $f : X \to Y$ is semi-continuous and contra gpr-continuous map, then f is contra rg-continuous map.*

Proof. Let F be an open subset of Y. Since f is semi-continuous and contra gpr-continuous, $f^{-1}(F)$ is semi-open and gpr-closed in X. It implies $f^{-1}(F)$ is rg-closed in X. Therefore f is contra rg-continuous map. □

Theorem 3.4. *If a map $f : X \to Y$ is completely continuous and contra gpr-continuous map, then f is contra precontinuous and hence perfectly continuous map.*

Theorem 3.5. *Let $\{X_\lambda : \lambda \in \Omega\}$ be any family of topological spaces. If $f : X \to \Pi X_\lambda$ is a contra gpr-continuous map, then Pr_λ o $f : X \to X_\lambda$ is contra gpr-continuous for each $\lambda \in \Omega$, where Pr_λ is the projection of ΠX_λ onto X_λ.*

Proof. We shall consider a fixed $\lambda \in \Omega$. Suppose U_λ is an arbitrary open set in X_λ. Then ${Pr_\lambda}^{-1}(U_\lambda)$ is open in ΠX_λ. Since f is contra gpr-continuous, we have by definition $f^{-1}({Pr_\lambda}^{-1}(U_\lambda)) = (Pr_\lambda \text{ o } f)^{-1}(U_\lambda)$ is gpr-closed in X. Therefore Pr_λ o f is contra gpr-continuous. □

Theorem 3.6. *Let $f : X \to Y$ be contra-continuous map. Then f is contra gpr-continuous map.*

The converse of the above Theorem is not true as seen from the following Example.

Example 3.7. *Let $X = Y = \{a, b, c\}$. Let $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\phi, Y, \{a, b\}\}$. Let $f : X \to Y$ be the identity map. Then f is contra gpr-continuous map but it is not contra-continuous.*

Definition 3.8. *A space X is said to be locally gpr-indiscrete if every gpr-open subset of X is closed in X.*

Theorem 3.9. *If a map $f : X \to Y$ is contra gpr-continuous where X is locally gpr-indiscrete, then f is continuous map.*

Theorem 3.10. *If a map $f : X \to Y$ is gpr*-continuous where Y is preregular $T_{1/2}$ space and $g : Y \to Z$ is contra gpr-continuous, then g o f : $Y \to Z$ is contra gpr-continuous map.*

Proof. Let F be an open set in Z. Since g is contra gpr-continuous, $g^{-1}(F)$ is gpr-closed in Y. But Y is preregular $T_{1/2}$ space, $g^{-1}(F)$ is preclosed in Y. Since f is gpr*-continuous, $f^{-1}(g^{-1}(F)) = (g \text{ o } f)^{-1}(F)$ is gpr-closed in X. Therefore g o f is contra gpr-continuous. □

4. gpr-connected and gpr-compact Spaces

Definition 4.1. *A space X is called gpr-connected provided that X is not the union of two disjoint non-empty gpr-open sets.*

Theorem 4.2. *If $f : X \to Y$ is contra gpr-continuous surjection and X is gpr-connected, then Y is connected.*

Proof. Suppose that Y is not connected. There exist non-empty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore V_1 and V_2 are clopen in Y. Since f is contra gpr-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are gpr-open in X. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non-empty disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not gpr-connected. This contradicts that Y is not connected assumed. Hence Y is connected. □

Definition 4.3. *A space X is said to be*

(1). gpr-compact (strongly S-closed [2]) if every gpr-open (closed) cover of X has a finite subcover;

(2). countably gpr-compact (strongly countably S-closed) if every countable cover of X by gpr-open (closed) sets has a finite subcover;

(3). gpr-Lindelöf (strongly S-Lindelöf) if every gpr-open (closed) cover of X has a countable subcover.

Theorem 4.4. *The contra gpr-continuous image of gpr-compact (gpr-Lindelöf, countably gpr-compact) spaces are strongly S-closed (strongly S-Lindelöf, strongly countably S-closed).*

Proof. Suppose that $f : X \to Y$ is a contra gpr-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any closed cover of Y. Since f is contra gpr-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an gpr-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup \{V_\alpha : \alpha \in I_0\}$ and Y is strongly S-closed.
The other proofs can be obtained similarly. □

Definition 4.5. *A space X is said to be*

(1). gpr-closed-compact if every gpr-closed cover of X has a finite subcover;

(2). countably gpr-closed-compact if every countable cover of X by gpr-closed sets has a finite subcover;

(3). gpr-closed-Lindelöf if every gpr-closed cover of X has a countable subcover.

Theorem 4.6. *The contra gpr-continuous image of gpr-closed-compact (gpr-closed-Lindelöf, countably gpr-closed-compact) spaces are compact (Lindelöf, countably compact).*

Proof. Suppose that $f : X \to Y$ is a contra gpr-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any open cover of Y. Since f is contra gpr-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a gpr-closed cover of X. Since X is gpr-closed-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup \{V_\alpha : \alpha \in I_0\}$ and Y is compact.
The other proofs can be obtained similarly. □

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