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Intuitionistic Fuzzy Meet Semi Bi-filter

Research Article

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Abstract: This paper contained the concept of intuitionistic fuzzy meet semi bi-ideal is defined. Discuss some theorems on intuitionistic fuzzy meet semi bi-ideal.

Keywords: Bi-ideal, Intuitionistic fuzzy meet semi bi-ideal, intuitionistic fuzzy set.

1. Introduction

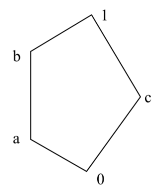
In [5, 10] N.Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in semigroups. In [4] Kim and Jun introduced the concept of intuitionistic fuzzy ideals of semigroups. Shabir [7], introduced the concept of prime bi-ideals, strongly prime bi-ideal and semiprime bi-ideals of a semigroup and studied those semigroups for which each bi-ideal is semiprime and strongly irreducible. In [9], authors define the fuzzy bi-ideals in ordered semigroups and give some theorem which characterizes the bi-ideals in terms of fuzzy bi-ideals and characterize the left and right simple, the completely regular, and the strongly regular ordered semigroups by means of fuzzy bi-ideals.

2. Main Results

Definition 2.1. *An intuitionistic fuzzy subsemilattice $A = \langle \mu_A, \nu_A \rangle$ of L is called an intuitionistic fuzzy meet bi-filter of L if*

1. $\mu(x \wedge y \wedge z) \leq \max\{\mu_A(x), \mu_A(y)\}$
2. $\nu(x \wedge y \wedge z) \geq \min\{\nu_A(x), \nu_A(y)\}$

Example 2.2. *Let $A = \{0, a, b, c, 1\}$. Let $\mu : A \to [0,1]$ and $\nu : A \to [0,1]$ is an intuitionistic fuzzy set in L by $\mu_A(0) = 0.3$, $\mu_A(a) = 0.5$, $\mu_A(b) = 0.4$, $\mu_A(c) = 0.3$, $\mu_A(1) = 0.3$, $\nu_A(0) = 0.3$, $\nu_A(a) = 0.3$, $\nu_A(b) = 0.4$, $\nu_A(c) = 0.5$, $\nu_A(1) = 0.6$.*



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Hence A is an intuitionistic fuzzy meet semi bi-filter of L.

Proposition 2.3. *If A is a meet semi bi-filter of L, then $\bar{A} = \langle \chi_A, \chi_{\bar{A}} \rangle$ is an intuitionistic fuzzy meet semi bi-filter of L.*

Proof. Since A is a subsemilattice of L, we obtain $\bar{A} = \langle \chi_A, \chi_{\bar{A}} \rangle$ is an intuitionistic fuzzy meet subsemilattice. Let $x, y, z \in L$. From the hypothesis $x \wedge y \wedge z \in A$ if $x, z \in A$. In this case $\chi_A(x \wedge y \wedge z) = 1 = \max\{\chi_A(x), \chi_A(z)\}$ and $\chi_{\bar{A}}(x \wedge y \wedge z) = 1 - \chi_A(x \wedge y \wedge z) = 1 - 1 = 0$. $\chi_{\bar{A}}(x \wedge y \wedge z) \geq \min\{\chi_{\bar{A}}(x), \chi_{\bar{A}}(z)\}$. If $x \notin A$ or $z \notin A$, then $\chi_A(x) = 0$ or $\chi_A(z) = 0$. Thus

$$\chi_A(x \wedge y \wedge z) \leq \max\{\chi_A(x), \chi_A(z)\}$$

$$\chi_{\bar{A}}(x \wedge y \wedge z) = 1 - \chi_A(x \wedge y \wedge z)$$

$$\min\{\chi_{\bar{A}}(x), \chi_{\bar{A}}(z)\} = \min\{1 - \chi_A(x), 1 - \chi_A(z)\} = 0$$

$$\chi_{\bar{A}}(x \wedge y \wedge z) \geq \min\{\chi_{\bar{A}}(x), \chi_{\bar{A}}(z)\}$$

Hence $\bar{A} = \langle \chi_A, \chi_{\bar{A}} \rangle$ is an intuitionistic fuzzy meet semi bi-filter of L. □

Theorem 2.4. *If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy meet semi bi-filter of L, then $\Box A$ and $\Diamond A$ are intuitionistic fuzzy meet semi bi-filter of L.*

Proof.

(1). Let A be an intuitionistic fuzzy meet semi bi-filter of L. Then $\Box A = \langle \mu_A, 1 - \mu_A \rangle$ and let $B = \Box A$. Then $\mu_B = \mu_A$ and $\nu_B = 1 - \mu_A$. Let $x, y, z \in L$. Then

$$\begin{aligned}
\mu_B(x \wedge y \wedge z) &= \mu_A(x \wedge y \wedge z) \\
&\leq \max\{\mu_A(x), \mu_A(z)\} \\
\mu_B(x \wedge y \wedge z) &\leq \max\{\mu_B(x), \mu_B(z)\} \\
\nu_B(x \wedge y \wedge z) &= 1 - \mu_A(x \wedge y \wedge z) \\
&\geq 1 - \max\{\mu_A(x), \mu_A(z)\} \\
&= \min\{1 - \mu_A(x), 1 - \mu_A(z)\} \\
&= \min\{\nu_B(x), \nu_B(z)\} \\
\nu_B(x \wedge y \wedge z) &\geq \min\{\nu_B(x), \nu_B(z)\}
\end{aligned}$$

Hence $\Box A$ is an intuitionistic fuzzy meet semi bi-filter of L.

(2). Let A be an intuitionistic fuzzy meet semi bi-filter of L. Then $\Diamond A = \langle 1 - \nu_A, \nu_A \rangle$ and let $B = \Diamond A$. Then $\mu_B = 1 - \nu_A$ and $\nu_B = \nu_A$. Let $x, y, z \in L$. Then

$$\begin{aligned}
\nu_B(x \wedge y \wedge z) &= \nu_A(x \wedge y \wedge z) \\
&\geq \min\{\nu_A(x), \nu_A(z)\} \\
\nu_B(x \wedge y \wedge z) &\geq \min\{\nu_B(x), \nu_B(z)\} \\
\mu_B(x \wedge y \wedge z) &= 1 - \nu_A(x \wedge y \wedge z) \\
&\leq 1 - \min\{\nu_A(x), \nu_A(z)\}
\end{aligned}$$

$$= \max\{1 - \nu_A(x), 1 - \nu_A(z)\}$$

$$= \max\{\mu_B(x), \mu_B(z)\}$$

$$\mu_B(x \wedge y \wedge z) \leq \max\{\mu_B(x), \mu_B(z)\}$$

Hence $\Diamond A$ is an intuitionistic fuzzy meet semi bi-filter of L. □

Theorem 2.5. *If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy meet semi bi-filter of L, then $B = \langle \mu_A, 0 \rangle$ and $C = \langle 0, 1 - \mu_A \rangle$ are intuitionistic fuzzy meet semi bi-filters of L.*

Proof. Suppose A is an intuitionistic fuzzy meet semi bi-filter of L. Then $\mu_A(x \wedge y \wedge z) \leq \max\{\mu_A(x), \mu_A(z)\}$ and $\nu_A(x \wedge y \wedge z) \leq \max\{\nu_A(x), \nu_A(z)\}$

(1). Let $B = \langle \mu_A, 0 \rangle$. Then $\mu_B = \mu_A$, $\nu_B = 0$. Let $x, y, z \in L$. Then

$$\mu_B(x \wedge y \wedge z) = \mu_A(x \wedge y \wedge z)$$

$$\leq \max\{\mu_A(x), \mu_A(z)\}$$

$$= \max\{\mu_B(x), \mu_B(z)\}$$

$$\mu_B(x \wedge y \wedge z) \leq \max\{\mu_B(x), \mu_B(z)\}$$

$$\nu_B(x \wedge y \wedge z) = 0 \geq \min\{\nu_B(x), \nu_B(z)\}$$

Hence B is an intuitionistic fuzzy meet semi bi-filter of L.

(2). Let $C = \langle 0, 1 - \mu_A \rangle$. Then $\mu_C = 0$, $\nu_C = 1 - \mu_A$. Let $x, y, z \in L$. Then

$$\nu_C(x \wedge y \wedge z) = 1 - \mu_A(x \wedge y \wedge z)$$

$$\geq 1 - \max\{\mu_A(x), \mu_A(z)\}$$

$$= \min\{1 - \mu_A(x), 1 - \mu_A(z)\}$$

$$\nu_C(x \wedge y \wedge z) \geq \min\{\nu_C(x), \nu_C(z)\}$$

$$\mu_C(x \wedge y \wedge z) \leq \max\{\mu_C(x), \mu_C(z)\}$$

Hence C is an intuitionistic fuzzy meet semi bi-filter of L. □

Theorem 2.6. *An intuitionistic fuzzy set $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy meet semi bi-filter of L if and only if the fuzzy sets μ_A and $\nu_{\bar{A}}$ are fuzzy meet semi bi-filters of L.*

Proof. Let $A = \langle \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy meet semi bi-filter of L. Then clearly μ_A is a fuzzy meet semi bi-filter of L. Let $x, y, z \in L$. Then

$$\nu_{\bar{A}}(x \wedge y) = 1 - \nu_A(x \wedge y) \leq 1 - \min\{\nu_A(x), \nu_A(y)\}$$

$$= \max\{1 - \nu_A(x), 1 - \nu_A(y)\}$$

$$= \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\}$$

$$\nu_{\bar{A}}(x \wedge y) \leq \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\}$$

$$\nu_{\bar{A}}(x \wedge y \wedge z) = 1 - \nu_A(x \wedge y \wedge z) \leq 1 - \min\{\nu_A(x), \nu_A(z)\}$$

$$= \max\{1 - \nu_A(x), 1 - \nu_A(z)\}$$

$$= \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(z)\}$$

Hence $\nu_{\bar{A}}$ is a fuzzy meet semi bi-filter of L.

Conversely, suppose that μ_A and $\nu_{\bar{A}}$ are fuzzy meet semi bi-filters of L. Let $x, y, z \in L$. Then

$$\begin{aligned} 1 - \nu_A(x \wedge y) = \nu_{\bar{A}}(x \wedge y) &\leq \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(y)\} \\ &= \max\{1 - \nu_A(x), 1 - \nu_A(y)\} \\ &= 1 - \min\{\nu_A(x), \nu_A(y)\} \\ \nu_A(x \wedge y) &\geq \min\{\nu_A(x), \nu_A(y)\} \\ 1 - \nu_A(x \wedge y \wedge z) = \nu_{\bar{A}}(x \wedge y \wedge z) &\leq \max\{\nu_{\bar{A}}(x), \nu_{\bar{A}}(z)\} \\ &= \max\{1 - \nu_A(x), 1 - \nu_A(z)\} \\ &= 1 - \min\{\nu_A(x), \nu_A(z)\} \end{aligned}$$

Therefore $\nu_A(x \wedge y \wedge z) \geq \min\{\nu_A(x), \nu_A(z)\}$. Hence $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy meet semi bi-filter. □

Theorem 2.7. *Let $f : S \to T$ be a homomorphism of semilattices. If $B = \langle \mu_B, \nu_B \rangle$ is an intuitionistic fuzzy meet semi bi-filter of T, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of B under f is an intuitionistic fuzzy meet semi bi-filter of L.*

Proof. Assume that $B = \langle \mu_B, \nu_B \rangle$ is an intuitionistic fuzzy meet bi-filter of T and let $x, y \in L$. Then

$$\begin{aligned} f^{-1}(\mu_B(x \wedge y)) &= \mu_B(f(x \wedge y)) \\ &= \mu_B(f(x) \wedge f(y)) \\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y)) \quad \text{as B is aintuitionistic fuzzy meet sub-semilattice} \\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\} \\ f^{-1}(\mu_B(x \wedge y)) &\geq \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\} \\ f^{-1}(\nu_B(x \wedge y)) &= \nu_B(f(x \wedge y)) \\ &= \nu_B(f(x) \wedge f(y)) \\ &\leq \max\{\nu_B(f(x)), \nu_B(f(y)) \\ &= \max\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(y))\} \end{aligned}$$

Therefore $f^{-1}(\nu_B(x \wedge y)) \leq \max\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(y))\}$. Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an intuitionistic fuzzy meet sub-semilattice of L. Let $x, y, z \in L$. Then

$$\begin{aligned} f^{-1}(\mu_B(x \wedge y \wedge z)) &= \mu_B(f(x \wedge y \wedge z)) \\ &= \mu_B(f(x) \wedge f(y) \wedge f(z)) \\ &\leq \max\{\mu_B(f(x)), \mu_B(f(z)) \\ &= \max\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(z))\} \\ f^{-1}(\mu_B(x \wedge y \wedge z)) &\leq \max\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(z))\} \\ f^{-1}(\nu_B(x \wedge y \wedge z)) &= \nu_B(f(x \wedge y \wedge z)) \\ &= \nu_B(f(x) \wedge f(y) \wedge f(z)) \\ &\geq \min\{\nu_B(f(x)), \nu_B(f(z)) \\ &= \min\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(z))\} \end{aligned}$$

Therefore $f^{-1}(\nu_B(x \wedge y \wedge z)) \geq \min\{f^{-1}(\nu_B(x)), f^{-1}(\nu_B(z))\}$. Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an intuitionistic fuzzy meet semi bi-filter of L. □

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