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$\widehat{\beta}g$ Closed Set in Intuitionistic Fuzzy Topological Spaces

Research Article

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Abstract: The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy $\widehat{\beta}$ generalized closed set and intuitionistic fuzzy $\widehat{\beta}$ generalized open set in intuitionistic fuzzy topological space. We investigate some of their properties. Also we study the application of intuitionistic fuzzy $\widehat{\beta}$ generalized closed set namely intuitionistic fuzzy $\widehat{\beta}T_{1/2}$ space and intuitionistic fuzzy $\widehat{\beta}_gT_q$ space.

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Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\widehat{\beta}$ generalized closed set, Intuitionistic fuzzy $\widehat{\beta}$ generalized open set, Intuitionistic fuzzy $\widehat{\beta}T_{1/2}$ space and Intuitionistic fuzzy $\widehat{\beta}_gT_q$ space.

1. Introduction

The concept of fuzzy sets was introduced by zadeh [10] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduce one of the concepts namely $\widehat{\beta}$ generalized closed set and we have studied some of the basic properties regarding it. We also introduced the applications of intuitionistic fuzzy $\widehat{\beta}$ generalized closed set namely intuitionistic fuzzy $\widehat{\beta}T_{1/2}$ space, intuitionistic fuzzy $\widehat{\beta}_gT_q$ space and obtained some characterizations and several preservation theorems of such spaces.

2. Preliminaries

Definition 2.1 ([1])**.** *Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an objecthaving the form $A = \{\langle x, \mu_A(x), v_A(x)/x \in X\rangle\}$ where the functions $\mu_A(x) : X \to [0,1]$ and $v_A(x) : X \to [0,1]$ denote thedegree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $v_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + v_A(x) \le 1$ for each $x \in X$.*

Definition 2.2 ([1])**.** *Let A and B of IFS's of the forms $A = \{\langle x, \mu_A(x), v_A(x)/x \in X\rangle\}$ and $B = \{\langle x, \mu_B(x), v_B(x)/x \in X\rangle\}$ then*

(a). $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $v_A(x) \ge v_B(x)$ for all $x \in X$.

(b). $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

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(c). $A^c = \{\langle x, v_A(x), \mu_A(x) / x \in X\rangle\}$.

(d). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), v_A(x) \vee v_B(x) / x \in X\rangle\}$.

(e). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), v_A(x) \wedge v_B(x) / x \in X\rangle\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \mu_B\rangle$ *instead of* $A = \{\langle x, \mu_A(x), v_A(x) / x \in X\rangle\}$*. Also for the sake of simplicity we shall use the notation* $A = \langle x, (\mu_A, \mu_B), (v_A, v_B)\rangle$ *instead of* $A = \left\{\left\langle x, \left(\frac{A}{\mu_A}, \frac{B}{\mu_B}\right), \left(\frac{A}{v_A}, \frac{B}{v_B}\right)\right\rangle\right\}$*. The intuitionistic fuzzy sets* $0\sim = \{\langle x, 0, 1\rangle / x \in X\}$ *and* $1\sim = \{\langle x, 1, 0\rangle / x \in X\}$ *are respectively the empty set and the whole set of X.*

Definition 2.3 ([3])**.** *An intuitionistic fuzzy topology (IFT inshort) on a non empty X is a family* τ *of IFS in X satisfying the following axioms:*

(a). $0\sim, 1\sim \in \tau$,

(b). $G_1 \cap G_2 \in \tau$, *for any* $G_1, G_2 \in \tau$,

(c). $\cup G_i \in \tau$ *for any arbitrary family* $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement* A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS for short) in X.*

Definition 2.4 ([3])**.** *Let* (X, τ) *be an IFTS and* $A = \langle x, \mu_A, v_A\rangle$ *bean IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by* $int(A) = \cup\{G/\ G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$, $cl(A) = \cap\{K/\ K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$.

Result 2.5 ([3])**.** *Let A and B be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space* (X, τ)*. Then*

(a). *A is an intuitionistic fuzzy closed set in X* $\Leftrightarrow cl(A) = A$,

(b). *A is an intuitionistic fuzzy open set in X* $\Leftrightarrow int(A) = A$,

(c). $cl(A^c) = (int(cl(A))^c$,

(d). $int(A^c) = (cl(A))^c$,

(e). $A \subseteq B \Rightarrow int(A) \subseteq int(B)$,

(f). $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,

(g). $cl(A \cup B) = cl(A) \cup cl(B)$

(h). $int(A \cap B) = int(A) \cap int(B)$.

Definition 2.6 ([9])**.** *Let* (X, τ) *be an IFTS and* $A = \langle x, \mu_A, v_A\rangle$ *be an IF S in X. Then the semi closure of A (scl(A) in short) and semi interior of A (sint(A) in short) are defined as*

(a). $sint(A) = \cup\{G/G \text{ is an IFSOS in } X \text{ and } G \subseteq A\}$,

(b). $scl(A) = \cap\{K/K \text{ is an IFSCS in } X \text{ and } A \subseteq K\}$.

Result 2.7 ([7])**.** *Let Abe an IFS in* (X, τ)*, then* *(a).* $scl(A) = A \cup int(cl(A))$,

(b). $sint(A) = A \cap cl(int(A))$.

Definition 2.8. *Let (X, τ) be an IFTS and $A = \langle x, \mu_A, v_A \rangle$ be an IFS in X. Then the alpha closure of A ($\alpha cl(A)$ in short) and alpha interior of A ($\alpha int(A)$ in short) are defined as*

$$\alpha int(A) = \cup\{G/G \text{ is an IF}\alpha OS \text{ in } X \text{ and } G \subseteq A\}$$

$$\alpha cl(A) = \cap\{K/K \text{ is an IF}\alpha CS \text{ in } X \text{ and } A \subseteq K\}$$

Result 2.9. *Let A be an IFS in (X, τ), then*

(a). $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(b). $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.10. *An IFS $A = \{\langle x, \mu_A(x), v_A(x)/x \in X \rangle\}$ in an IFTS (X, τ) is called an*

(a). intuitionistic fuzzy semi closed set [4] (IFSCS) if $int(cl(A)) \subseteq A$.

(b). intuitionistic fuzzy α closed set[4] (IFαCS) if $cl(int(cl(A))) \subseteq A$.

(c). intuitionistic fuzzy preclosed set [4] (IFPCS) if $cl(int(A)) \subseteq A$.

(d). intuitionistic fuzzy regular closed set[4] (IFRCS) if $cl(int(A)) = A$.

(e). intuitionistic fuzzy generalised closed set[8] (IFGCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS.

(f). intuitionistic fuzzy generalised semi closed set (IFGSCS) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(g). intuitionistic fuzzy α generalised closed set[4] (IFαGCS) if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFOS.

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IαFGOS) if the complement of A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IαFGCS respectively.

3. Intuitionistic Fuzzy $\widehat{\beta}$ Generalized Closed Set

In this section we introduce intuitionistic fuzzy $\widehat{\beta}$ generalized closed set and study some of its properties.

Definition 3.1. *An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy $\widehat{\beta}$ generalized closed set (IF$\widehat{\beta}$GCS) if $cl(int(cl(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X. The family of all IF$\widehat{\beta}$GCSs of an IFTS (X, τ) is denoted by IF$\widehat{\beta}$GCS(X).*

Example 3.2. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.3), (0.5, 0.6) \rangle$. Then the IFS $A = \langle x, (0.1, 0.2), (0.4, 0.5) \rangle$ is an IF$\widehat{\beta}$GCS in X.*

Theorem 3.3. *Every IFCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFCS in (X,τ). Let U be an intuitionistic fuzzy open set such that $A \subseteq U$. Since A is intuitionistic fuzzy closed $cl(A) = A$ and $cl(A) \subseteq U$. But $cl(int(A)) \subseteq cl(A) \subseteq U$. Therefore, $cl(int(cl(A))) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.4. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. Then the IFS $A = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFCS in X since $cl(A) = T^c \neq A$.*

Theorem 3.5. *Every IFαCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFαCS in X and let $A \subseteq U$ and U is an IFOS in (X,τ). By hypothesis, $cl(int(cl(A))) \subseteq A$ and $A \subseteq U$. Therefore, $cl(int(cl(A))) \subseteq A \subseteq U$. Since A is intuitionistic fuzzy closed $cl(A) = A$. Therefore, $cl(int((A)) \subseteq cl(int(cl(A))) \subseteq A \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.6. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$. Then the IFS $A = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFαCS in X since $A \subseteq T$ but $cl(int(cl(A))) = 1 \nsubseteq A$.*

Theorem 3.7. *Every IFGCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFGCS in X and let $A \subseteq U$ and U is an IFOS in (X,τ). Since, $cl(A) \subseteq U, cl(int(A)) \subseteq cl(A)$. That is, $cl(int(A)) \subseteq cl(A) \subseteq U$. Since A is intuitionistic fuzzy closed $cl(A) = A$. Therefore, $cl(int(cl(A))) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.8. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$. Then the IFS $A = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFGCS in X since, $cl(A) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle \nsubseteq U$.*

Theorem 3.9. *Every IFRCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFRCS in X and let $A \subseteq U$ and U is an IFOS in (X,τ). Since A is IFRCS, $cl(int(A)) = A \subseteq U$. Also since A is intuitionistic fuzzy closed $cl(A) = A$. This implies $cl(int(cl(A))) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.10. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$. The IFS $A = \langle x, (0.2, 0.3), (0.4, 0.5)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFRCS in X since, $cl(int(A)) = 0 \neq A$.*

Theorem 3.11. *Every IFPCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFPCS in X and let $A \subseteq U$ and U is an IFOS in (X,τ). By definition, $cl(int(A)) \subseteq A$ and $A \subseteq U$. Since A is intuitionistic fuzzy closed $cl(A) = A$. Therefore, $cl(int(cl(A))) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.12. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$. Then the IFS $A = \langle x, (0.9, 0.3), (0.1, 0.6)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFPCS in X. Since, $cl(int(A)) = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle \nsubseteq$ A.*

Theorem 3.13. *Every IFαGCS is an IF$\widehat{\beta}$GCS but not conversely.*

Proof. Let A be an IFαGCS in X and let $A \subseteq U$ and U is an IFOS in (X,τ). By definition, $A \cup cl(int(cl(A))) \subseteq U$. This implies $cl(int(A)) \subseteq cl(int(cl(A))) \subseteq U$. Since A is intuitionistic fuzzy closed $cl(A) = A$. Therefore, $cl(int(cl(A))) \subseteq U$. Hence A is an IF$\widehat{\beta}$GCS in X. □

Example 3.14. *Let $X = \{a,b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$. Then the IFS $A = \langle x, (0.3, 0.5), (0.6, 0.5)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFαGCS in X since $\alpha cl(A) = 1 \nsubseteq T$.*

Proposition 3.15. *IFSCS and IF$\widehat{\beta}$GCS are independent to each other which can be seen from the following example.*

Example 3.16. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$. Then the IFS $A = T$ is an IFSCS in X but not an IF$\widehat{\beta}$GCS in X. Since, $A \subseteq T$ but $cl(int(A)) = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle \nsubseteq T$.*

Example 3.17. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.9, 0.7), (0.1, 0.2)\rangle$. Then the IFS $A = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFSGCS in X. Since, $int(cl(A)) = 1 \nsubseteq A$.*

Proposition 3.18. *IFGSCS and IF$\widehat{\beta}$GCS are independent to each other.*

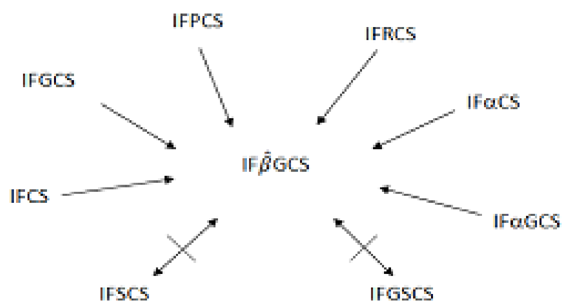
Example 3.19. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.3, 0.5), (0.7, 0.5)\rangle$. Then the IFS $A = T$ is an IF$\widehat{\beta}$GCS in X but not an IFGPCS in X. Since, $A \subseteq T$ but $cl(int(A)) = \langle x, (0.7, 0.5), (0.3, 0.5)\rangle \nsubseteq A$.*

Example 3.20. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X where $T = \langle x, (0.8, 0.9), (0.2, 0.1)\rangle$. Then the IFS $A = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$ is an IF$\widehat{\beta}$GCS in X but not an IFGSCS in X. Since, $scl(A) = 1 \nsubseteq T$.*

Remark 3.21. *The union of any two IF$\widehat{\beta}$GCSs need not be an IF$\widehat{\beta}$GCS in general as seen from the following example.*

Example 3.22. *Let $X = \{a, b\}$ be an IFTs and let $T = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, T, 1\sim\}$ is an IFT on X and the IFSs $A = \langle x, (0.1, 0.8), (0.9, 0.2)\rangle$, $B = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$ are IF$\widehat{\beta}$GCSs but $A \cup B$ is not an IFGSCS in X.*

Relation between intuitionistic fuzzy $\widehat{\beta}$ generalized closed set and other existing intuitionistic fuzzy closed sets are represented in the following diagram:



In this diagram by "$A \to B$" we mean A implies B but not conversely and "$A \nleftrightarrow B$" means A and B are independent of each other. None of them is reversible.

4. Intuitionistic Fuzzy $\widehat{\beta}$ Generalized Open Set

In this section we introduce intuitionistic fuzzy $\widehat{\beta}$ generalized open set and have studied some of its properties.

Definition 4.1. *An IFS A is said to be an intuitionistic fuzzy $\widehat{\beta}$ generalized open set (IF$\widehat{\beta}$GOS in short) in (X, τ) if the complement A^c is an IF$\widehat{\beta}$GCS in X. The family of all IF$\widehat{\beta}$GOS of an IFTS (X, τ) is denoted by IF$\widehat{\beta}$GOS(X).*

Example 4.2. *Let $X = \{a, b\}$ and let $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X, where $T = \langle x, (0.7, 0.5), (0.2, 0.5)\rangle$. Then the IFS $A = \langle x, (0.8, 0.7), (0.2, 0.2)\rangle$ is an IF$\widehat{\beta}$GOS in X.*

Theorem 4.3. *For any IFTS (X, τ), we have the following*

(1). Every IFOS is an IF$\widehat{\beta}$GOS.

(2). Every IFSOS is an IF$\widehat{\beta}$GOS.

(3). Every IFαOS is an IF$\widehat{\beta}$GOS.

(4). Every IFGOS is an IF$\widehat{\beta}$GOS.

But the converses are not true in general.

The converse of the above statement need not be true in general which can be seen from the following examples.

Example 4.4. *Let $X = \{a, b\}$ and $T = \langle x, (0.7, 0.5), (0.3, 0.4)\rangle$. Then $\tau = \{0\sim, T, 1\sim\}$ is an IFT on X. The IFS $A = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ is an IF$\widehat{\beta}$GOS in (X, τ) but not an IFOS in X.*

Example 4.5. *Let $X = \{a, b\}$ and $\tau = \{0\sim, T, 1\sim\}$ where $T = \langle x, (0.1, 0.2), (0.9, 0.7)\rangle$. Then the IFS $A = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ is an IF$\widehat{\beta}$GOS in (X, τ) but not an IFSOS in X.*

Example 4.6. *Let $X = \{a, b\}$ and $\tau = \{0\sim, T, 1\sim\}$ be an IFT on X, where $T = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$. Then the IFS $A = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$ is an IF$\widehat{\beta}$GOS but not an IFαOS in X.*

Example 4.7. *Let $X = \{a, b\}$ and $T = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$. Then $\tau = \{0\sim, T, 1\sim\}$ is an IFT on X. The IFS $A = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ is an IF$\widehat{\beta}$GOS but not an IFPOS in X.*

Remark 4.8. *The intersection of any two IF$\widehat{\beta}$GOSs need not bean IF$\widehat{\beta}$GOS in general.*

Example 4.9. *Let $X = \{a, b\}$ be an IFTS and let $T = \langle x, (0.6, 0.8), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, T, 1\sim\}$ is an IFT on X. The IFSs $A = \langle x, (0.9, 0.2), (0.1, 0.8)\rangle$ and $B = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$ are IF$\widehat{\beta}$GOSs but $A \cap B$ is not an IF$\widehat{\beta}$GOS in X.*

Theorem 4.10. *An IFS A of an IFTS (X, τ) is an IF$\widehat{\beta}$GOS if and only if $F \subseteq int(cl(A))$ whenever F is an IFCS and $F \subseteq A$.*

Proof. **Necessity:** Suppose A is an IF$\widehat{\beta}$GOS in X. Let Fbe an IFCS and $F \subseteq A$. Then F^c is an IFOS in X such that A^cF^c. Since A^c is an IF$\widehat{\beta}$GCS, $cl(int(A^c))F^c$. Hence, $(int(cl(A)))^cF^c$. This implies $F \subseteq int(cl(A))$.

Sufficiency: Let A be an IFS of X and $F \subseteq int(cl(A))$ whenever F is an IFCS and $F \subseteq A$. Then A^cF^c and F^c is an IFOS. By hypothesis, $(int(cl(A)))^cF^c$. Hence, $cl(int(A^c))F^c$. Hence, A is an IF$\widehat{\beta}$GOS of X. □

5. Applications of Intuitionistic Fuzzy $\widehat{\beta}$ Generalized Closed Set

In this section, we introduce intuitionistic fuzzy $\widehat{\beta}T_{1/2}$ space and $\widehat{\beta}_gT_q$ space, which utilize intuitionistic fuzzy $\widehat{\beta}$ generalized closed set and its characterizations are proved.

Definition 5.1. *An IFTS (X, τ) is called an intuitionistic fuzzy $\widehat{\beta}T_{1/2}$ (IF$\widehat{\beta}T_{1/2}$ in short) space if every IF$\widehat{\beta}$GCS in X is an IFCS in X.*

Definition 5.2. *An IFTS (X, τ) is called an intuitionistic fuzzy $\widehat{\beta}_gT_q$ (IF$\widehat{\beta}_gT_q$ in short) space if every IF$\widehat{\beta}$GCS in X is an IFPCS in X.*

Theorem 5.3. *Every IF$\widehat{\beta}T_{1/2}$ space is an IF$\widehat{\beta}_gT_q$ space. But the converse is not true in general.*

Proof. Let X be an IF$\widehat{\beta}T_{1/2}$ space and let A be an IF$\widehat{\beta}$GCS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFPCS, A is an IFPCS in X. Hence X is an IF$\widehat{\beta}_gT_q$ space. □

The converses need not be true which can be seen from the following examples.

Example 5.4. *Let $X = \{a, b\}$ and $\tau = \{0\sim, T, 1\sim\}$ where $T = \langle x, (0.9, 0.9), (0.1, 0.1)\rangle$. Then (X, τ) is an $IF_{wg}T_q$ space. But it is not an IF$\widehat{\beta}T_{1/2}$ space. Since, the IFS $A = \langle x, (0.2, 0.3), (0.8, 0.7)\rangle$ is IF$\widehat{\beta}$GCS but not an IFCS in X.*

Theorem 5.5. *Let (X, τ) be an IFTS and X is an IF$\widehat{\beta}T_{1/2}$ space then*

(1). Any union of IF$\widehat{\beta}$GCS is an IF$\widehat{\beta}$GCS

(2). Any intersection of IF$\widehat{\beta}$GOS is an IF$\widehat{\beta}$GOS.

Proof.

(1). Let $\{A_i\}_{i\in J}$ be a collection of IF$\widehat{\beta}$GCS in an $\widehat{\beta}T_{1/2}$ space (X, τ). Therefore, every IF$\widehat{\beta}$GCS is an IFCS. But the union of IFCS is an IFCS. Hence, the union of IF$\widehat{\beta}$GCS is an IF$\widehat{\beta}$GCS in X.

(2). It can be proved by taking complement in (1). □

Theorem 5.6. *An IFTS X is an IF$\widehat{\beta}_gT_q$ space if and only if $IF\widehat{\beta}GOS(X) = IFPOS(X)$.*

Proof. **Necessity:** Let A be an IF$\widehat{\beta}$GOS in X. Then A^c is an IF$\widehat{\beta}$GCS in X. By hypothesis, A^c is an IFPCS in X. Therefore, A is an IFPOS in X. Hence IF$\widehat{\beta}GOS(X) = IFPOS(X)$.
Sufficiency: Let A be an IF$\widehat{\beta}$GCS in X. Then A^c is an IF$\widehat{\beta}$GOS in X. By hypothesis A^c is an IFPOS in X. Therefore A is an IFPCS in X. Hence X is an IF$\widehat{\beta}_gT_q$ space. □

Theorem 5.7. *An IFTS X is an IF$\widehat{\beta}T_{1/2}$ space if and only if $IF\widehat{\beta}GOS(X) = IFOS(X)$.*

Proof. **Necessity:** Let A be an IF$\widehat{\beta}$GOS in X. Then A^c is an IF$\widehat{\beta}$GCS in X. By hypothesis A^c is an IFCS in X. Therefore A is an IFOS in X. Hence, IF$\widehat{\beta}GOS(X) = IFOS(X)$.
Sufficiency: Let A be an IF$\widehat{\beta}$GCS in X. Then A^c is an IF$\widehat{\beta}$GOS in X. By hypothesis A^c is an IFPOS in X. Therefore A is an IFPCS in X. Hence X is an IF$\widehat{\beta}T_{1/2}$ space. □

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