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Growth Analysis of Entire Functions Based on Vector Valued Dirichlet Series

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Abstract

Biswas [11] introduced the idea of (p,q)-th relative ritt order and (p,q)-th relative ritt type. In this paper, we establish some results of the growth analysis of entire function represented by vector valued Dirichlat series $f(s) = \sum\limits_{n=1}^{\infty} a_n e^{s\lambda_n}$ on the basis of (p,q)-th relative ritt L-order and (p,q)-th relative ritt L-lower order of an entire function represented by vector valued Dirichlat series.

Keywords: Growth; vector valued dirichlet series; entire function.

2020 Mathematics Subject Classification: 30D30, 30D15, 30D50.

1. Introduction

Let $f(s)$ be an entire function of the complex variable $s = \sigma + it$ (σ and t are real variables) defined by everywhere absolutely convergent vector valued Dirichlet series in brief known as VVDS,

$$f(s) = \sum_{n=1}^{\infty} a_n e^{s\lambda_n}, \tag{1}$$

where a_n's belong to a Banach space $(E, \|.\|)$ and λ_n's are non-negative real numbers such that $0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n < \lambda_{n+1}$ $(n \geq 1)$, $\lambda_n \to +\infty$ as $n \to +\infty$ and satisfy the condition $\limsup\limits_{n\to+\infty} \frac{logn}{\lambda_n} = D < +\infty$ and $\limsup\limits_{n\to+\infty} \frac{log\|a_n\|}{\lambda_n} = -\infty$. If σ_c and σ_a denote respectively abscissa of convergence and absolute convergence of (1), then in this case clearly $\sigma_a = \sigma_c = +\infty$. The function $M_f(\sigma)$ known as maximum modulus function corresponding to an entire function $f(s)$ defined by (1), is written as follows,

$$M_f(\sigma) = \sup_{-\infty<t<+\infty} \|f(\sigma+it)\|.$$

For $x \in [0,\infty)$ and $k \in \mathbb{N}$, we define $\log^{[k]} x = \log(\log^{[k-1]} x)$ and $\exp^{[k]} = \exp(\exp^{[k-1]})x$, where $k \in \mathbb{N}$ is the set of positive integers are also denote $\log^{[0]} x = x$, $\exp^{[0]} = x$. Juneja and Kapoor [4] introduced the definition of (p,q)th-Ritt order and (p,q)th-Ritt lower order of an entire function $f(s)$ represented by VVDS are defined as follows:

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Definition 1.1. *[4] Let f be an entire function represented by VVDS, (p,q)th-Ritt order and (p,q)th-Ritt lower order of an entire function $f(s)$ represented by VVDS are respectively defined as,*

$$\rho^{(p,q)}(f) = \limsup_{\sigma\to\infty} \frac{\log^{[p]} M_f(\sigma)}{\log^{[q]} \sigma} = \limsup_{\sigma\to\infty} \frac{\log^{[p]} \sigma}{\log^{[q]} M_f^{-1}(\sigma)},$$

and

$$\lambda^{(p,q)}(f) = \liminf_{\sigma\to\infty} \frac{\log^{[p]} M_f(\sigma)}{\log^{[q]} \sigma} = \liminf_{\sigma\to\infty} \frac{\log^{[p]} \sigma}{\log^{[q]} M_f^{-1}(\sigma)},$$

where p and q are integers $p \geq q+1 \geq 1$.

Definition 1.2. *[10] An entire function f is said to be index pair (p,q) if $b < \rho_f^{(p,q)} < \infty$ and $\rho_f^{(p-1,q-1)}$ is not a non-zero finite number , where $b = 1$, if $p = q$ and $b = 0$ otherwise. Moreover, if $0 < \rho_f^{(p,q)} < \infty$, then*

$$\begin{cases} \rho_f^{(p-n,q)} = \infty, & \text{for } n < p; \\ \rho_f^{(p,q-n)} = 0, & \text{for } n < q; \\ \rho_f^{(p+n,q+n)} = 1, & \text{for } n = 1,2,\dots \end{cases}$$

Similarly for $0 < \lambda_f^{(p,q)} < \infty$,

$$\begin{cases} \lambda_f^{(p-n,q)} = \infty, & \text{for } n < p; \\ \lambda_f^{(p,q-n)} = 0, & \text{for } n < q; \\ \lambda_f^{(p+n,q+n)} = 1, & \text{for } n = 1,2,\dots \end{cases}$$

An entire function f (represented by VVDS) for which (p,q)th-Ritt order and (p,q)th-Ritt lower order are equal, is said to be of regular (p,q)th-Ritt growth, otherwise f is said to be of irregular (p,q)th-Ritt growth. R. Thamizharasi and D. Somasundaram [7] introduced the notions of ritt L-order and ritt L-lower order of an entire function, where $L \equiv L(\sigma)$ in a positive continuous function increasing slowly i.e. $L(a\sigma) \sim L(\sigma)$ as $\sigma \to \infty$ for every positive constants $'a'$. S. Sarkar [9] introduce the definition of (p,q)th Ritt L-order for an entire function represented by VVDS in the following manner:

Definition 1.3. *[7] Let f be an entire function represented by VVDS. The (p,q)th-Ritt L-order of f and denoted by $\rho_f^{(p,q)}(L)$ is defined as,*

$$\rho^{L(p,q)}(f) = \limsup_{\sigma\to\infty} \frac{log^{[p]} M_f(\sigma)}{log^{[q]}\, (\sigma L(\sigma))},$$

similarly, (p,q)th Ritt L-lower order of f $\lambda_f^{L(p,q)}$ in the following way:

$$\lambda^{L(p,q)}(f) = \liminf_{\sigma\to\infty} \frac{log^{[p]} M_f(\sigma)}{log^{[q]}\, (\sigma L(\sigma))}.$$

Datta and Biswas [8], introduced the concepts of (p,q)th relative Ritt L-order and (p,q)th relative Ritt L-lower order of an entire function f (represented as VVDS) with respect to an other entire function g which is also represented by VVDS, in the following way:

Definition 1.4. *[8] Let f and g be any two entire functions represented by VVDS with index-pair (m,q) and (m,p), respectively, where p, q, m are integers such that $m \geq q+1 \geq 1$ and $m \geq p+1 \geq 1$. Then the (p,q)th-relative Ritt L-order and (p,q)th-relative Ritt L- lower order of f with respect to g are defined as,*

$$\rho_g^{L(p,q)}(f) = \limsup_{\sigma\to\infty} \frac{log^{[p]} M_g^{-1} M_f(\sigma)}{log^{[q]}\left(\sigma L(\sigma)\right)},$$

and

$$\lambda_g^{L(p,q)}(f) = \liminf_{\sigma\to\infty} \frac{log^{[p]} M_g^{-1} M_f(\sigma)}{log^{[q]}\left(\sigma L(\sigma)\right)}.$$

Definition 1.5. *[10] Let f and g be any two entire functions represented by VVDS with index-pairs (m,q) and (m,p) respectively where p, q, m are integers such that $m \geq q+1 \geq 1$ and $m \geq p+1 \geq 1$. Then the entire function f is said to be relative index-pair (p,q) with respect to another entire function g, if $b < \rho_g^{L[p,q]}(f) < \infty$ and $\rho_g^{L(p-1,q-1)}(f)$ is not a non-zero finite number, where $b=1$, if $p=q$ and $b=0$ otherwise. Moreover, if $0 < \rho_g^{L(p,q)}(f) < \infty$, then*

$$\begin{cases} \rho_g^{L(p-n,q)}(f) = \infty, & \text{for } n < p; \\ \rho_g^{L(p,q-n)}(f) = 0, & \text{for } n < q; \\ \rho_g^{L(p+n,q+n)}(f) = 1, & \text{for } n = 1,2,\ldots \end{cases}$$

Similarly for $0 < \lambda_f^{L(p,q)} < \infty$,

$$\begin{cases} \lambda_g^{L(p-n,q)}(f) = \infty, & \text{for } n < p; \\ \lambda_g^{L(p,q-n)}(f) = 0, & \text{for } n < q; \\ \lambda_g^{L(p+n,q+n)}(f) = 1, & \text{for } n = 1,2,\ldots \end{cases}$$

An entire function f (represented by VVDS) for which (p,q)th-relative Ritt L-order and (p,q)th-relative Ritt L-lower order with respect to another entire function g (represented by VVDS) are same, is said to be function f of regular relative (p,q)th-Ritt L-growth with respect to g. Otherwise, f is said to be irregular relative (p,q)th-Ritt L-growth with respect to g.

During the past decades, the several authors [1–3, 5, 6, 8] made closed investigation the growth properties of entire funtions represented by vector valued Dirichlet series related to (p,q)th Ritt order. In this paper, we want to establish some results related to the growth rates of entire functions represented by vector valued Dirichlet series (1) on the basis of (p,q)th-relative Ritt L-order and (p,q)th-relative Ritt L-lower order.

2. Main Results

Theorem 2.1. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \lambda_h^{L(m,q)}(f) < \rho_h^{L(m,q)}(f) < +\infty$, and $0 < \lambda_k^{L(n,q)}(g) < \rho_k^{L(n,q)}(g) < +\infty$, where q, m, are all positive integers then*

$$\frac{\lambda_h^{L(m,q)}(f)}{\rho_k^{L(n,q)}(g)} \leq \liminf_{\sigma\to\infty} \frac{log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(m,q)}(f)}{\lambda_k^{L(n,q)}(g)}$$
$$\leq \limsup_{\sigma\to\infty} \frac{log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,q)}(f)}{\lambda_k^{L(n,q)}(g)}.$$

Proof. From the definition of $\lambda_h^{L(m,q)}(f)$ and $\rho_k^{L(n,q)}(g)$, we get for arbitrary positive $\epsilon > 0$, for all large values of σ,

$$\log^{[m]} M_h^{-1} M_f(\sigma) \geq (\lambda_h^{L(m,q)}(f) - \epsilon) \log^{[q]} (\sigma L(\sigma)), \tag{2}$$

and

$$\log^{[n]} M_k^{-1} M_g(\sigma) \leq (\rho_k^{L(n,q)}(g) + \epsilon) \log^{[q]} (\sigma L(\sigma)). \tag{3}$$

Now from (2) and (3), it follows that for all sufficiently large values of σ.

$$\frac{log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\lambda_h^{L(m,q)}(f) - \epsilon}{\rho_k^{L(n,q)}(g) + \epsilon}.$$

As $\epsilon > 0$ is arbitrary, we get

$$\liminf_{\sigma\to\infty} \frac{log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\lambda_h^{L(m,q)}(f)}{\rho_k^{L(n,q)}(g)}. \tag{4}$$

Again for a sequence of values of σ tending to infinity, we get

$$\log^{[m]} M_h^{-1} M_f(\sigma) \leq (\lambda_h^{L(n,q)}(f) + \epsilon) \log^{[q]} (\sigma L(\sigma)), \tag{5}$$

and for all sufficiently large values of σ

$$\log^{[n]} M_k^{-1} M_g(\sigma) \geq (\lambda_k^{L(m,q)}(g) - \epsilon) \log^{[q]} (\sigma L(\sigma)). \tag{6}$$

Now from (5) and (6), we obtain for a sequence of values of σ tending to infinity

$$\frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(m,q)}(f) + \epsilon}{\lambda_k^{L(n,q)}(g) - \epsilon}.$$

Since $\epsilon > 0$ is arbitrary, it follows that

$$\liminf_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(m,q)}(f)}{\lambda_k^{L(n,q)}(g)}. \tag{7}$$

Also for a sequence of values of σ tending to infinity, we get

$$\log^{[n]} M_k^{-1} M_g(\sigma) \leq (\lambda_k^{L(n,q)}(g) + \epsilon) \log^{[q]}(\sigma L(\sigma)). \tag{8}$$

Combining (2) and (8), we obtain for a sequence of values of σ tending to infinity

$$\frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\lambda_h^{L(m,q)}(f) - \epsilon}{\lambda_k^{L(n,q)}(g) + \epsilon}.$$

Since $\epsilon > 0$ is arbitrary, we get

$$\limsup_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\lambda_h^{L(m,q)}(f)}{\lambda_k^{L(n,q)}(g)}. \tag{9}$$

Also for a sequence of values of σ tending to infinity, we get

$$\log^{[m]} M_h^{-1} M_f(\sigma) \leq (\rho_h^{L(n,q)}(f) + \epsilon) \log^{[q]}(\sigma L(\sigma)). \tag{10}$$

Now, it follows that (6) and (10), we obtain for a sequence of values of σ tending to infinity

$$\frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,q)}(f) + \epsilon}{\lambda_k^{L(n,q)}(g) - \epsilon}.$$

Since $\epsilon > 0$ is arbitrary, we get

$$\limsup_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,q)}(f)}{\lambda_k^{L(n,q)}(g)}. \tag{11}$$

□

Thus, the theorem follows from (4), (7), (9) and (11). Now, we state the following three Theorems without proof which can be easily carried out from the definitions of (p,q)th relative ritt L-order and (p,q)th relative ritt L-lower order with the help of Theorem 2.1.

Theorem 2.2. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \lambda_h^{L(m,p)}(f) < \rho_h^{L(m,p)}(f) < +\infty$, and $0 < \lambda_k^{L(n,p)}(g) < \rho_k^{L(n,p)}(g) < +\infty$, where p, m, n are all positive integers then,*

$$\frac{\lambda_h^{L(m,p)}(f)}{\rho_k^{L(n,p)}(g)} \leq \liminf_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(m,p)}(f)}{\lambda_k^{L(n,p)}(g)}$$

$$\leq \limsup_{\sigma\to\infty} \frac{log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,p)}(f)}{\lambda_k^{L(n,p)}(g)}.$$

Theorem 2.3. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \lambda_h^{L(p,m)}(f) < \rho_h^{L(p,m)}(f) < +\infty$, and $0 < \lambda_k^{L(q,m)}(g) < \rho_k^{L(q,m)}(g) < +\infty$, where p, q, m, are all positive integers then,*

$$\frac{\lambda_h^{L(p,m)}(f)}{\rho_k^{L(q,m)}(g)} \leq \liminf_{\sigma\to\infty} \frac{\log^{[p]} M_h^{-1} M_f(\sigma)}{log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(p,m)}(f)}{\lambda_k^{L(q,m)}(g)}$$
$$\leq \limsup_{\sigma\to\infty} \frac{log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(p,m)}(f)}{\lambda_k^{L(q,m)}(g)}.$$

Theorem 2.4. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \lambda_h^{L(p,n)}(f) < \rho_h^{L(p,n)}(f) < +\infty$, and $0 < \lambda_k^{L(q,n)}(g) < \rho_k^{L(q,n)}(g) < +\infty$, where p, q, n, are all positive integers then,*

$$\frac{\lambda_h^{L(p,n)}(f)}{\rho_k^{L(q,n)}(g)} \leq \liminf_{\sigma\to\infty} \frac{log^{[p]} M_h^{-1} M_f(\sigma)}{log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\lambda_h^{L(p,n)}(f)}{\lambda_k^{L(q,n)}(g)}$$
$$\leq \limsup_{\sigma\to\infty} \frac{log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(p,n)}(f)}{\lambda_k^{L(q,n)}(g)}.$$

Theorem 2.5. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \rho_h^{L(m,q)}(f) < +\infty$, and $0 < \rho_k^{L(n,q)}(g) < +\infty$, where q, m, n are all positive integers then,*

$$\liminf_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,q)}(f)}{\rho_k^{L(n,q)}(g)} \leq \limsup_{\sigma\to\infty} \frac{log^{[m]} M_h^{-1} M_f(\sigma)}{log^{[n]} M_k^{-1} M_g(\sigma)}.$$

Proof. From the definition $\rho_k^{L(n,q)}(g)$, we get for a sequence of values of σ tending to infinity,

$$\log^{[n]} M_k^{-1} M_g(\sigma) \geq (\rho_k^{L(n,q)}(g) - \epsilon) \log^{[q]}(\sigma L(\sigma)). \tag{12}$$

Now from (10) and (12), we get for a sequence of values of σ tending to infinity,

$$\frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(n,q)}(f) + \epsilon}{\rho_k^{L(n,q)}(g) - \epsilon}.$$

As $\epsilon > 0$ is arbitrary, we get

$$\liminf_{\sigma\to\infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(n,q)}(f)}{\rho_k^{L(n,q)}(g)}. \tag{13}$$

Also for a sequence of values of σ tending ti infinity, we get

$$\log^{[n]} M_k^{-1} M_g(\sigma) \leq (\rho_k^{L(n,q)}(g) + \epsilon) \log^{[q]}(\sigma L(\sigma)). \tag{14}$$

Now from (3) and (14), we get for a sequence of values of σ tending to infinity,

$$\frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\rho_h^{L(n,q)}(f) - \epsilon}{\rho_k^{L(n,q)}(g) + \epsilon}.$$

As $\epsilon > 0$ is arbitrary, we get

$$\limsup_{\sigma \to \infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \geq \frac{\rho_h^{L(n,q)}(f)}{\rho_k^{L(n,q)}(g)}. \tag{15}$$

□

Thus the theorem follows from (13) and (15). Now we state the following three theorems without proof which can easily be carried out from the definitions of (p,q)th relative ritt L-order and (p,q)th relative ritt L-lower order and with the help of Theorem 2.5.

Theorem 2.6. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \rho_h^{L(m,p)}(f) < +\infty$, and $0 < \rho_k^{L(n,p)}(g) < +\infty$, where p, m, n are all positive integers then,*

$$\liminf_{\sigma \to \infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(m,p)}(f)}{\rho_k^{L(n,p)}(g)} \leq \limsup_{\sigma \to \infty} \frac{\log^{[m]} M_h^{-1} M_f(\sigma)}{\log^{[n]} M_k^{-1} M_g(\sigma)}.$$

Theorem 2.7. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \rho_h^{L(p,m)}(f) < +\infty$, and $0 < \rho_k^{L(q,m)}(g) < +\infty$, where p, q, m are all positive integers then,*

$$\liminf_{\sigma \to \infty} \frac{\log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(p,m)}(f)}{\rho_k^{L(q,m)}(g)} \leq \limsup_{\sigma \to \infty} \frac{log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)}.$$

Theorem 2.8. *Let f, g, h and k be any four entire function VVDS defined by (1) such that $0 < \rho_h^{L(p,n)}(f) < +\infty$, and $0 < \rho_k^{L(q,n)}(g) < +\infty$, where p, q, n are all positive integers then,*

$$\liminf_{\sigma \to \infty} \frac{\log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)} \leq \frac{\rho_h^{L(p,n)}(f)}{\rho_k^{L(q,n)}(g)} \leq \limsup_{\sigma \to \infty} \frac{\log^{[p]} M_h^{-1} M_f(\sigma)}{\log^{[q]} M_k^{-1} M_g(\sigma)}.$$

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