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AHP Based on Trapezoidal Linguistic Fuzzy Preference Relation

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Abstract: Consistency is an important issue in any decision making problem. Lack of consistency yields to unreliable solution. This paper proposes Trapezoidal fuzzy linguistic preference relation to improve the consistency of Fuzzy analytic Hierarchy process. In this study using Trapezoidal fuzzy linguistic preference relation and using only n-1 pairwise comparisons, a pairwise comparison matrix is constructed. The matrix satisfies additive reciprocal property and consistency. The proposed method is applied to the example given in [14] and the results are compared. This study provides the same ranking as in [14] without much involvement of decision maker.

Keywords: Fuzzy AHP, consistency, Trapezoidal fuzzy number, linguisticvariable, Decision making.

1. Introduction

AHP [1] introduced by T.L.Satty (1968) is an extensively used approach of Multi criteria decision making. It is very easy to understand and relies on decision makers judgment. Since human thinking is full of uncertainty and ambiguity, this method cannot model imprecise or vagueness. Fuzzy set theory [2] introduced by Zadeh is very useful to deal with uncertainty and vagueness. So AHP is extended to fuzzy environment, FAHP has been studied by many authors and applied in diverse fields ([3–8]). The study of consistency is important to avoid unreliable and unacceptable solution. Consistency check is a part of AHP procedure to identify and remove inconsistency. Since in fuzzyAHP, the comparison ratios are given by fuzzy numbers, the possible of having inconsistency ratios are far greater than classical AHP. It is difficult for a decision maker to provide the fuzzy comparison ratios which include the consistency. Thus it is necessary for a mechanism to figure out inconsistency and make this as a consistent one.

However, to establish a comparison matrix it requires $\frac{n(n-1)}{2}$ comparisons for a level with n criteria. As the number of criteria increases, the number of comparison increases and consequently the decision makers judgment are inconsistent one. Preference relations are most common form of matrix to represent decision makersjudgments. These preference relations can be categorized into multiplicative preference relation, Fuzzy preference relation [9], and linguistic preference relation [10]. In [11] the author proposed Fuzzy linguistic preference relation method based on additive transitivity property to construct a complete consistent matrix using only $n-1$ preferences. However in all the above method the elements of the decision matrix are crisp number. In ([12, 13]) the author extends Fuzzy linguistic preference relation using Triangular

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Fuzzy number. To model real life situations, crisp numbers are not adequate since it involves vagueness. So this paper, extends Fuzzy Lin Pre Ra method using Trapezoidal fuzzy number. To construct a consistent comparison matrix this paper uses only $n-1$ preference value. From this comparison matrices the weight vectors can be derived, based on which the alternatives are ranked. To do this so this paper is arranged as follows. The first section gives the brief introduction of the problem. In second section preliminaries are given. Section 3 the brief introduction of Trapezoidal Fuzzy Number is given. Section 4 introduces Trapezoidal Fuzzy Lin Pre Ra approach. In section 5 this method is applied a solved to a problem in the literature. In last the conclusion is given.

2. Preliminaries

Definition 2.1 ([1, 16]). *A multiplicative preference relation A on a $X = \{x_1, x_2, ..., x_n\}$ set is represented by a matrix $A = (a_{ij})_{n\times n}$ where a_{ij} is the intensity of preference of x_i over x_j and satisfies the reciprocal condition $d_{ij}a_{ji} = 1 \;\forall\; x_i, x_j \in X$.*

Definition 2.2. *Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set, then $R = (r_{ij})_{n\times n}$ is called fuzzy Preference relation on $X \times X$ with the condition $r_{ij} \geq 0$, $r_{ij} + r_{ji} = 1$, $i, j = 1, 2, ..., n$ where r_{ij} denotes the degree that the alternative x_i is prior to the alternative x_j.*

Proposition 2.3 ([11]). *The transformation from multiplicative preference relation $A = (a_{ij})_{n\times n}$ to fuzzy preference relation $\tilde{P} = \left(\tilde{P}\right)_{n\times n}$ is using the formula $p_{ij} = \frac{1}{2}(1 + log_9 \; a_{ij})$.*

Proposition 2.4 ([1]). *For a reciprocal fuzzy preference relation the following statements are equivalent.*

(a). $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \;\forall\; i, j, k$

(b). $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \;\forall\; i < j < k$

Proposition 2.5 ([11]). *For a reciprocal fuzzy preference relation $P = (p_{ij})$ the following statements are equivalent.*

(a). $p_{ij} + p_{jk} + p_{ki} = \frac{3}{2} \;\forall\; i < j < k$

(b). $p_{i(i+1)} + p_{(i+1)(i+2)} + ... + p_{(i+k)i} = \frac{k+1}{2} \;\forall\; i < j$

Using the above preposition, a consistent fuzzy preference relation is constructed from the set of values $\{p_{12}, p_{23}, ..., p_{1-1n}\}$. If the elements of the decision matrix are not in the interval [0, 1] this can be transformed accordingly using the transformation function $f(x) = \frac{x+k}{1+2k}$.

3. Trapezoidal Fuzzy Number

Fuzzy numbers are fuzzy sets on the real line. It is possible to use various fuzzy number depends in the situation. Triangular and trapezoidal fuzzy number are usually adopted fuzzy numbers in the literature. It is easy and convenient to express the vagueness and ambiguity using these numbers. Triangular Fuzzy numbers are special case of Trapezoidal fuzzy number. Therefore, a trapezoidal fuzzy number can deal with general situations. In this study, linguistic variables are used to describe the decision makers opinion that are expressed as trapezoidal Fuzzy number.

Definition 3.1. *A trapezoidal fuzzy number, denoted by $\tilde{A} = (l, m, n, u)$has the following membership function*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l} & l \leq x \leq m \\ 1 & m \leq x \leq n \\ \frac{u-x}{u-n} & n \leq x \leq u \end{cases}$$

where l and u represent the lower and upper bound of $\tilde{A}$. The following are the operational laws of two Trapezoidal Fuzzy number $\tilde{A}_1$ and $\tilde{A}_2$.

$$\begin{aligned}
\tilde{A}_1 + \tilde{A}_2 =& (l_1 + l_2, m_1 + m_2, n_1 + n_2, u_1 + u_2) \\
\tilde{A}_1 - \tilde{A}_2 =& (l_1 - u_2, m_1 - n_2, n_1 - m_2, u_1 - l_2) \\
\tilde{A}_1 * \tilde{A}_2 =& (l_1 l_2, m_1 m_2, n_1 n_2, u_1 u_2) \\
\tilde{A}_1 \div \tilde{A}_2 =& (l_1/u_2, m_1/n_2, n_1/m_2, u_1/l_2) \\
log_n \tilde{A} =& (log_n l, log_n m, log_n n, log_n u) \\
(\tilde{A})^{-1} =& \left(\frac{1}{u}, \frac{1}{n}, \frac{1}{m}, \frac{1}{l}\right)
\end{aligned}$$

4. Trapezoidal Fuzzy Lin Pre Ra method

This study applies Trapezoidal Fuzzy Lin pre Ra method to enhance the consistency of fuzzy AHP method. This method constructs fuzzy preference relation matrices $\tilde{P} = \tilde{P_{ij}} = \left(\tilde{P}_{ij}^{l}, \tilde{P}_{ij}^{m}, \tilde{P}_{ij}^{n}, \tilde{P}_{ij}^{u}\right)$ based on consistent fuzzy preference relation and fuzzy linguistic assessment variables. Table 1 shows the Trapezoidal Fuzzy linguistic assessment variables.

Linguistic variable	Fuzzy number	Reciprocal Fuzzy number
Equally important	(0.5, 0.5, 0.5, 0.5)	
Moderately important	(0.5,0.55,0.65,0.7)	(0.3,0.35,0.45,0.5)
Strongly important	(0.6,0.65,0.75,0.8)	(0.2,0.25,0.35,0.4)
Very strongly important	(0.7,0.75,0.85,0.9)	(0.1,0.15,0.25,0.3)
Absolutely important	(0.8,0.85,0.9,0.95)	(0.05,0.1,0.15,0.2)

Table 1.

Definition 4.1. *A positive reciprocal matrix $\tilde{A} = (\tilde{a}_{ij})$ is reciprocal if and only if $\tilde{a}_{ij} \otimes \tilde{a}_{jk} \approx \tilde{a}_{ij}$.*

Definition 4.2. *A fuzzy positive matrix $\tilde{A} = (\tilde{a}_{ij})$ is reciprocal if and only if $\tilde{a}_{ij} = \tilde{a}_{ij}^{-1}$.*

Proposition 4.3. *Given a set of alternatives $X = \{x_1, x_2, ..., x_n\}$associated with fuzzy multiplicative preference relation $\tilde{A}$ with $\tilde{a}_{ij} \in \left[\frac{1}{9}, 9\right]$ and the corresponding Trapezoidal fuzzy reciprocal linguistic preference relation $\tilde{P} = \tilde{P}_{ij} = (\tilde{P}_{ij}^{l}), \tilde{P}_{ij}^{m}, \tilde{P}_{ij}^{n}, \tilde{P}_{ij}^{u})$ with $\tilde{P}_{ij} \in [0, 1]$ verifies the additives reciprocal then the following statements are equivalent*

(a). $\tilde{P_{ij}^{l}} + \tilde{P_{ji}^{u}} = 1 \;\forall\; i, j \in \{1, 2, ..., n\}$.

(b). $\tilde{P_{ij}^{m}} + \tilde{P_{ji}^{n}} = 1 \;\forall\; i, j \in \{1, 2, ..., n\}$.

(c). $\tilde{P_{ij}^{n}} + \tilde{P_{ji}^{m}} = 1 \;\forall\; i, j \in \{1, 2, ..., n\}$.

(d). $\tilde{P_{ij}^{u}} + \tilde{P_{ji}^{l}} = 1 \;\forall\; i, j \in \{1, 2, ..., n\}$.

Proof. Let $\tilde{A} = (\tilde{a}_{ij})$ being a fuzzy multiplicative preference relation $\tilde{a}_{ij} = \tilde{a}_{ij}^{-1} \Rightarrow a_{ij} \otimes a_{ji} = 1$ taking logarithms on both the sides yields,

$$\begin{aligned}
log_9\, \tilde{a}_{ij} \oplus log_9\, \tilde{a}_{ji} = 0 \,\forall\, i, j \in \{1, 2, ..., n\} \\
\frac{1}{2}\left(1 \oplus \; log_9\, \tilde{a}_{ij}\right) \oplus \frac{1}{2}\left(1 \oplus \; log_9\, \tilde{a}_{ji}\right) = 1 \forall\, i, j \in \{1, 2, ...n\} \\
\tilde{P}_{ij} \oplus \tilde{P}_{ji} = 1 \,\forall\, i, j, \in \{1, 2, ..., n\}
\end{aligned}$$

$$\left(p_{ij}^{l}, p_{ij}^{m}, p_{ij}^{n}, p_{ij}^{u}\right) \oplus \tilde{P}_{ji} = (1,1,1,1)$$

$$\left(p_{ji}^{l}, p_{ji}^{m}, p_{ji}^{n}, p_{ji}^{u}\right) = \left(1 - p_{ij}^{u}, 1 - p_{ij}^{n} 1 - p_{ij}^{m} 1 - p_{ij}^{l}\right)$$

$$p_{ji}^{l} = 1 - p_{ij}^{u}, p_{ji}^{m} = 1 - p_{ij}^{n}, 1 - p_{ji}^{n}, 1 - p_{ij}^{m}, 1 - p_{ji}^{u} = 1 - p_{ij}^{l}$$

Hence

$$\begin{aligned} p_{ji}^{l} + p_{ij}^{u} &= 1 \\ p_{ji}^{m} + p_{ij}^{n} &= 1 \\ p_{ji}^{l} + p_{ij}^{m} &= 1 \\ p_{ji}^{l} + p_{ij}^{l} &= 1 \end{aligned}$$

□

Proposition 4.4. *For a Trapezoidal fuzzy reciprocal linguistic preference relation* $\tilde{P} = (\tilde{p}_{ij}) = \left(p_{ij}^{l}, p_{ij}^{m}, p_{ij}^{n}, p_{ij}^{u}\right)$ *to be constant, verifies the additive consistency then the following statements are equivalent:*

(a). $P_{ij}^{l} + P_{jk}^{l} + P_{ki}^{u} = \frac{3}{2} \, \forall \, i < j < k.$

(b). $P_{ij}^{m} + P_{jk}^{m} + P_{ki}^{n} = \frac{3}{2} \, \forall \, i < j < k.$

(c). $P_{ij}^{n} + P_{jk}^{n} + P_{ki}^{m} = \frac{3}{2} \, \forall \, i < j < k.$

(d). $P_{ij}^{u} + P_{jk}^{u} + P_{ki}^{i} = \frac{3}{2} \, \forall \, i < j < k.$

Proof. Let $\tilde{A} = (\tilde{a}_{ij})$ is constant if $\tilde{a}_{ij} \otimes \tilde{a}_{jk} \;\; \tilde{a}_{ik} \, \forall \, i, j, k$.

$$\begin{aligned} log_9 \, \tilde{a}_{ij} \oplus log_9 \, \tilde{a}_{jk} &= log_9 \, \tilde{a}_{ik} \\ log_9 \, \tilde{a}_{ij} \oplus log_9 \, \tilde{a}_{jk} - log_9 \, \tilde{a}_{ik} &= 0 \\ log_9 \, \tilde{a}_{ij} \oplus log_9 \, \tilde{a}_{jk} \oplus log_9 \, \tilde{a}_{ki} &= 0 \end{aligned}$$

Adding 3 and diving by 2 in the above equation

$$\begin{aligned} \frac{1}{2}\left(1 \oplus log_9 \, \tilde{a}_{ij}\right) \oplus \frac{1}{2}\left(1 \oplus log_9 \, \tilde{a}_{jk}\right) \oplus \frac{1}{2}\left(1 \oplus log_9 \, \tilde{a}_{ki}\right) &= \frac{3}{2} \, \forall \, i, j, k \\ \tilde{P}_{ij} \oplus \tilde{P}_{jk} \oplus \tilde{P}_{ki} &= \frac{3}{2} \\ \left(p_{ij}^{l}, p_{ij}^{m}, p_{ij}^{n}, p_{ij}^{u}\right) \oplus \left(p_{jk}^{l}, p_{jk}^{m}, p_{jk}^{n}, p_{jk}^{u}\right) \oplus \left(p_{ki}^{l}, p_{ki}^{m}, p_{ki}^{n}, p_{ki}^{u}\right) &= \left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right) \\ p_{ki}^{u} &= \frac{3}{2} - p_{ij}^{1} - p_{jk}^{1} \\ p_{ki}^{n} &= \frac{3}{2} - p_{ij}^{m} - p_{jk}^{m} \\ p_{ki}^{m} &= \frac{3}{2} - p_{ij}^{n} - p_{jk}^{n} \\ p_{ki}^{l} &= \frac{3}{2} - p_{ij}^{u} - p_{jk}^{u} \end{aligned}$$

The values of the matrix $\tilde{P}$ may not be in the interval [0, 1]. To transform all the elements of $\tilde{P}$ within the interval [0, 1] the following transformation function is used. $f(x^{l}) = \frac{x^{l}+c}{1+2c}$; $f(x^{m}) = \frac{x^{m}+c}{1+2c}$; $f(x^{n}) = \frac{x^{n}+c}{1+2c}$; $f(x^{u}) = \frac{x^{u}+c}{1+2c}$, Where the values of $\tilde{P}$ is in the interval $[-c, 1+c]$. □

4.1. The procedure of Trapezoidal Fuzzy Lin Pre Ra AHP is given as follows

Step: 1 The problem is decomposed into simple hierarchical structures.

Step: 2 The alternatives or criteria are compared with each other with help of trapezoidal fuzzy linguistic variable given in Table 1 to get the elements $\left\{\tilde{P}_{12}, \tilde{P}_{23}, ..., \tilde{P}_{n-ln}\right\}$, of the matrix $\tilde{P} = \left(\tilde{P}_{ij}\right)_{n\times n}$.

Step: 3 The remaining elements are calculated by Proposition.

Step: 4 Calculate the weight $\tilde{w}_i = \frac{\tilde{A}_i}{\sum\limits_{i=1}^{n} \tilde{a}_i}$ where $\tilde{A}_i = \frac{\sum\limits_{i=1}^{n} \tilde{P}_{ij}}{n}$ for all i.

Step: 5 To rank the alternative the fuzzy weight values are made as crisp one by using the defuzzification formula $\tilde{U}_i = \frac{l+2m+2n+u}{6}$.

5. Illustrative Example

C.Kahramam in his paper [14] applied FAHP based on extend analysis method to select the facilitation location for a Turkish Motors company NEYEK. For that he has taken four criteria and three alternatives are given in Figure [1]. The proposed procedure is applied to the problem [1]. Pairwise comparison of four criteria with respect to goal.

Goal	C_1	C_2	C_3	C_4
C_1	(0.5,0.5,0.5,0.5)	Moderately important		
C_2		(0.5,0.5,0.5,0.5)	Very strongly Not Important	
C_3			(0.5,0.5,0.5,0.5)	Absolutely Important
C_4				(0.5,0.5,0.5,0.5)

Table 2.

Comparison of Alternative with respect to C_1.

	A_1	A_2	A_3
A_1	(0.5,0.5,0.5,0.5)	Moderately Not important	
A_2		(0.5,0.5,0.5,0.5)	Moderately Not important
A_3			(0.5,0.5,0.5,0.5)

Table 3.

Comparison of Alternative with respect to C_2.

	A_1	A_2	A_3
A_1	(0.5,0.5,0.5,0.5)	Very absolutely Not important	
A_2		(0.5,0.5,0.5,0.5)	Very strongly Not important
A_3			(0.5,0.5,0.5,0.5)

Table 4.

Comparison of Alternative with respect to C_3.

	A_1	A_2	A_3
A_1	(0.5,0.5,0.5,0.5)	Very absolutely Not important	
A_2		(0.5,0.5,0.5,0.5)	Moderately Not important
A_3			(0.5,0.5,0.5,0.5)

Table 5.

Comparison of Alternative with respect to C_4.

	A_1	A_2	A_3
A_1	(0.5,0.5,0.5,0.5)	Very absolutely Not important	
A_2		(0.5,0.5,0.5,0.5)	Very strongly Important
A_3			(0.5,0.5,0.5,0.5)

Table 6.

The remaining elements in the above table are calculated by the procedure which is prescribed in this paper. The complete comparison matrix of criteria and alternatives are as follows. Pairwise comparison of four criteria with respect to goal.

Goal	C_1	C_2	C_3	C_4
C_1	(0.5,0.5,0.5,0.5)	(0.5,0.55,0.65,0.7)	(0.1,0.2,0.35,0.55)	(0.4,0.55,0.75,1)
C_2	(0.3,0.35,0.45,0.5)	(0.5, 0.5,0.5, 0.5)	(0.1,0.15,0.2,0.35)	(0.4,0.85,0.9,0.95)
C_3	(0.45,0.65,0.8,0.9)	(0.65,0.8,0.85,0.9)	(0.5, 0.5,0.5, 0.5)	(0.8,0.85,0.9,0.95)
C_4	(0,0.250.45,0.6)	(0.2,0.4,0.5,0.6)	(0.05,0.1,0.15,0.2)	(0.5, 0.5,0.5, 0.5)

Table 7.

Comparison of Alternative with respect to C_1.

	A_1	A_2	A_3
A_1	(0.5, 0.5, 0.5, 0.5)	(0.3,0.35,0.45,0.5)	(0.1,0.2,0.4,0.5)
A_2	(0.5,0.55,0.65,0.7)	(0.5, 0.5, 0.5, 0.5)	(0.3,0.35,0.45,0.5)
A_3	(0.5,0.6,0.8,0.9)	(0.5,0.55,0.65,0.7)	(0.5, 0.5, 0.5, 0.5)

Table 8.

Comparison of Alternative with respect to C_2.

	A_1	A_2	A_3
A_1	(0.5, 0.5, 0.5, 0.5)	(0,0.05,0.1,0.25)	(0.3,0.4,0.5,0.7)
A_2	(0.75,0.9,0.95,1)	(0.5, 0.5, 0.5, 0.5)	(0.8,0.85,0.9,0.95)
A_3	(0.75,1,1.1,1.2)	(0.5,0.6,0.65,0.7)	(0.5, 0.5, 0.5, 0.5)

Table 9.

Comparison of Alternative with respect to C_3.

	A_1	A_2	A_3
A_1	(0.5, 0.5, 0.5, 0.5)	(0,0.05,0.1,0.25)	(0,0.2,0.35,0.55)
A_2	(0.75,0.9,0.95,1)	(0.5, 0.5, 0.5, 0.5)	(0.6,0.65,0.75,0.8)
A_3	(0.45,0.65,0.8,1)	(0.2,0.25,0.35,0.5)	(0.5, 0.5, 0.5, 0.5)

Table 10.

Comparison of Alternative with respect to C_4.

	A_1	A_2	A_3
A_1	(0.5, 0.5, 0.5, 0.5)	(0.0.05,0.1,0.25)	(0,0.2,0.35,0.55)
A_2	(0.75,0.9,0.95,1)	(0.5, 0.5, 0.5, 0.5)	(0.5,0.65,0.75,0.8)
A_3	(0.45,0.65,0.8,1)	(0.2,0.25,0.35,0.5)	(0.5, 0.5, 0.5, 0.5)

Table 11.

Some of the elements in the Table-8 is not in the interval [0, 1].But it can be made within the interval [0,1] by using the formula

$$f(x^l) = \frac{x^l + c}{1+2c};\ f(x^m) = \frac{x^m + c}{1+2c}; f(x^n) = \frac{x^n + c}{1+2c}; f(x^u) = \frac{x^u + c}{1+2c}.$$

The weight vectors are calculated by using the formula $\tilde{w}_i = \frac{\tilde{A}_i}{\sum\limits_{i=1}^{n} \tilde{A}_i}$. The weight values (trapezoidal fuzzy number) are defuzzified and they are made as crisp values.

The crisp weight vectors from table -6 are (0.27, 0.24, 0.37, 0.18).

The crisp weight vectors from table -7 are (0.24, 0.52, 0.26).

The crisp weight vectors from table -8 are (0.28, 0.34, 0.41).

The crisp weight vectors from table -9 are (0.23, 0.37,0.41).

The crisp weight vectors from table -10 are (0.21, 0.48,0.35).

The comparison of overall weight vectors from the method prescribed in this paper and from the paper[14]are as follows.

Overall alternative Weights	A_1	A_2	A_3
In this paper	0.25	0.37	0.43
In Paper [14]	0.13	0.40	0.47

Table 12.

This shows that both the paper has the same weight .Hence ranking also same.

6. Conclusion

In this paper, to derive comparison matrices Trapezoidal Fuzzy Lin Pre Ra method is used. Trapezoidal fuzzy number is very useful to express decision maker's vagueness and ambiguity. The method is applied to a solved problem in the literature. It is found that this method yields the same result as that of the method in [14] without of much involvement of decision maker. This is the main advantage of this method.

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