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$IF\alpha^{**}g$ Closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract: In this paper, we introduce and study new classes of sets called $IF\alpha^{**}g$ closed set in intuitionistic fuzzy topological spaces and $IF\alpha^{**}g$ open set in intuitionistic fuzzy topological spaces. We focus upon the some of their basic properties.

Keywords: Intuitionistic fuzzy topology, $IF\alpha^{**}g$ closed set in intuitionistic fuzzy topological spaces, $IF\alpha^{**}g$ open set in intuitionistic fuzzy topological spaces.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [2] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce $IF\alpha^{**}g$ closed set in intuitionistic fuzzy topological spaces and IF**g open set in intuitionistic fuzzy topological spaces.

2. Preliminaries

Definition 2.1 ([1])**.** *Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. We denote the set of all intuitionistic fuzzy sets in X, by IFS (X).*

Definition 2.2 ([1])**.** *Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then*

(a). $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

(b). $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(c). $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$.

(d). $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$.

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(e). $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ *instead of* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. *Also for the sake of simplicity, we shall use the notation* $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B)\rangle$ *instead of* $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ *are respectively the empty set and the whole set of* X.

Definition 2.3 ([2])**.** *An intuitionistic fuzzy topology (IFT in short) on* X *is a family* t *of IFSs in* X *satisfying the following axioms.*

(a). $0_\sim, 1_\sim \in \tau$.

(b). $G_1 \cap G_2 \in \tau$, *for any* $G_1, G_2 \in \tau$.

(c). $\bigcup G_i \in \tau$ *for any family* $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in* X. *The complement* A^c *of an IFOS* A *in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in* X.

Definition 2.4. *An IFS* A *of an IFTS* (X, τ) *is an*

(a). Intuitionistic fuzzy interior of A [2] if $int(A) = \bigcup \{G/G$ *is an IFOS in* X *and* $G \subseteq A\}$.

(b). Intuitionistic fuzzy closure of A [2] if $cl(A) = \bigcap \{K/K$ *is an IFCS in* X *and* $A \subseteq K\}$.

(c). Intuitionistic fuzzy semi closed set [4] (IFSCS in short) if $int(cl(A)) \subseteq A$.

(d). Intuitionistic fuzzy pre open set [4] (IFPOS in short) if $A \subseteq int(cl(A))$.

(e). Intuitionistic fuzzy pre closed set [4] (IFPCS in short) if $cl(int(A)) \subseteq A$.

(f). Intuitionistic fuzzy pre open set [4] (IFPOS in short) if $A \subseteq int(cl(A))$.

(g). Intuitionistic fuzzy a-open set [4] (IF α OS in short) if $A \subseteq int(cl(int(A)))$.

(h). Intuitionistic fuzzy a-closed set [4] (IF α CS in short) if $cl(int(cl(A)) \subseteq A$.

(i). Intuitionistic fuzzy γ-open set [5] (IF γ OS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$.

(j). Intuitionistic fuzzy γ-closed set [5] (IF γ CS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

(k). Intuitionistic fuzzy semi pre open set [4] (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

(l). Intuitionistic fuzzy semi pre closed set [4] (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$.

(m). Intuitionistic fuzzy regular open set [9] (IFROS in short) if $A = int(cl(A))$.

(n). Intuitionistic fuzzy regular closed set [9] (IFRCS in short) if $A = cl(int(A))$.

(o). Intuitionistic fuzzy generalized closed set [9] (IFGCS in short) if $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is an IFOS in* X.

(p). Semi closure of A [7] (scl(A) in short) is defined as $scl(A) = \bigcap \{K/K$ *is an IFSCS in* X *and* $A \subseteq K\}$.

(q). Semi interior of A [7] (sint(A) in short) is defined as $sint(A) = \bigcup \{K/K$ *is an IFSOS in* X *and* $K \subseteq A\}$.

(r). Intuitionistic fuzzy generalized semi closed set [10] (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq UR$ and U is an IFOS in X.

(s). Regular open [7] if $A = int(cl(A))$.

(t). π open [7] if A is the union of regular open sets.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

3. $IF\alpha^{**}g$ Closed Sets in Intuitionistic Fuzzy Topological Spaces

In this section we introduce intuitionistic fuzzy a**g closed sets and study some of their properties.

Definition 3.1 ([6]). *An IFS A is said to be an intuitionistic fuzzy $\alpha^{**}g$ closed set ($IF\alpha^{**}g$ CS in short) in (X, τ) if $\alpha cl(A) \subseteq int(cl(U))$ whenever, $A \subseteq U$ and U is an IFOS in X. The family of all $IF\alpha^{**}g$ CSs of an IFTS (X, τ) is denoted by $IF\alpha^{**}g$ CS(X).*

Example 3.2. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.6), (0.1, 0.2)\rangle$. Here $\mu_G(a) = 0.7$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.1$ and $\nu_G(b) = 0.2$. Let us consider the IFS $A = \langle x, (0.1, 0.1), (0.7, 0.6)\rangle$ is an $IF\alpha^{**}g$ CSin (X, τ).*

Theorem 3.3. *Every IFCS in (X, τ) is an $IF\alpha^{**}g$ CS (X, τ) but not conversely.*

Proof. Assume that A is an IFCS in (X, τ). Let us consider an IFS $A \subseteq U$ and U be an IFOS in (X, τ). Since $\alpha cl(A) \subseteq cl(A)$ and A is an IFCS in X, $\alpha cl(A) \subseteq cl(A) = A \subseteq int(cl(U))$. That is $\alpha cl(A) \subseteq int(cl(U))$. Therefore, A is an $IF\alpha^{**}g$ CS in X. □

Example 3.4. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ where $G = \langle x, (0.3, 0.1), (0.5, 0.6)\rangle$. Consider the IFS $A = \langle x, (0.4, 0.3), (0.4, 0.5)\rangle$. Clearly $A \subseteq 1_\sim$ and $\alpha cl(A) = \langle x, (0.5, 0.6), (0.3, 0.1)\rangle \subseteq 1_\sim$. Hence, A is an $IF\alpha^{**}g$ CS. But A is not an IFCS in X.*

Theorem 3.5. *Every IFRCS in (X, τ) is an $IF\alpha^{**}g$ CS in (X, τ) but not conversely.*

Proof. Let A be an IFRCS in (X, τ). By definition, $A = cl(int(A))$. This implies cl(A)=A. That is A is an IFCS in X. Since every IFRCS is an IFCS, A is an IFCS in X. Hence by Theorem 3.3, A is an $IF\alpha^{**}g$ CS in X. □

Example 3.6. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.1, 0.2), (0.5, 0.6)\rangle$. Then the IFS $A = \langle x, (0.1, 0.1), (0.7, 0.6)\rangle$ is an $IF\alpha^{**}g$ CS in X but not an IFRCS in X.*

Theorem 3.7. *Every $IF\alpha CS$ in (X, τ) is an $IF\alpha^{**}g$ CS in (X, τ) but not conversely.*

Proof. Let us consider an IFS $A \subseteq U$ and U be an IFOS in (X, τ). Also let A is an $IF\alpha CS$ in X. By hypothesis, $cl(int(cl(A))) \subseteq A$, therefore $int(cl(A)) \subseteq A$. This implies $\alpha cl(A) \subseteq A = int(cl(A)$. Since, A is an IFRCS in X. Hence by Theorem 3.5, A is an $IF\alpha^{**}g$ CS in X. □

Example 3.8. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$, where $G_1 = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$, $G_2 = \langle x, (0.8, 0.8), (0.2, 0.2)\rangle$. Then the IFS $A = \langle x, (0.4, 0.4), (0.5, 0.6)\rangle$ is an $IF\alpha^{**}g$ CS in X. But A is not an $IF\alpha CS$ in X because $cl(int(cl(A))) = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle \nsubseteq A$.*

Theorem 3.9. *Every IFGCS in (X, τ) is an $IF\alpha^{**}g$ CS in (X, τ) but its converse may not be true in general.*

Proof. Assume that A be an IFGCS in (X,τ). Let $A \subseteq U$ and U be an IFOS in X. By hypothesis, $cl(A) \subseteq U$. Clearly $\alpha cl(A) \subseteq cl(A) = int(cl(U))$. Whenever, $A \subseteq U$ and U is an IFOS in X. Hence A is an $IF\alpha^{**}g$ CS in X. □

Example 3.10. *Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x,(0.1,0.7),(0.2,0)\rangle$. Then the IFS $A = \langle x,(0.1,0),(0.3,0.8)\rangle$ is an $IF\alpha^{**}g$ CS but A is not an IFGCS in X, as $cl(A) \nsubseteq G$ even though $A \subseteq G$ and G is an IFOS in X.*

Theorem 3.11. *Every $IF\alpha^{**}g$ CS in (X,τ) is an IFGSCS in (X,τ) but its converse may not be true in general.*

Proof. Assume that A is an $IF\alpha^{**}g$ CS in (X,τ). Let an IFS $A \subseteq U$ and U be an IFOS in (X,τ). By hypothesis, $\alpha cl(A) \subseteq int(cl(U))$. By Theorem 3.5, every IFRCS in (X,τ) is an $IF\alpha^{**}g$ CS in (X,τ). This implies, $A \cup int(cl(A)) \subseteq U$. Therefore, $scl(A) = A \cup int(cl(A))) \subseteq U$. Hence, A is an IFGSCS in X. □

Example 3.12. *Let $X = \{a,b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x,(0.2,0.3),(0.4,0.5)\rangle$. Then the IFS $A = \langle x,(0.1,0),(0.5,0.6)\rangle$ is an IFGSCS in X but A is not an $IF\alpha^{**}g$ CS in X, since $\alpha cl(A) \nsubseteq int(cl(U))$ even though $A \subseteq G$ and G is an IFOS in (X,τ).*

Remark 3.13. *An IFGSPCS in (X,τ) is need not be an $IF\alpha^{**}g$ CS in (X,τ).*

Example 3.14. *Let $X = \{a,b\}$ and $G = \langle x,(0,0.8),(0.4,0.1)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0,0.2),(0.6,0.6)\rangle$ is an IFGSPCS but A is not an $IF\alpha^{**}g$ CS in X since $\alpha cl(A) \nsubseteq int(cl(U))$ even though $A \subseteq G$ and G is an IFOS in (X,τ).*

Remark 3.15. *An $IF\alpha^{**}g$ CS closedness is independent of an IFP closedness.*

Example 3.16. *Let $X = \{a,b\}$ and $G = \langle x,(0,0.8),(0.4,0.1)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0,0.2),(0.6,0.6)\rangle$ is an IFPCS but not an $IF\alpha^{**}g$ CS in X, as $\alpha cl(A) \nsubseteq int(cl(U))$ even though $A \subseteq G$ and G is an IFOS in X.*

Example 3.17. *Let $X = \{a,b\}$ and $G = \langle x,(0.2,0.2),(0.5,0.7)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.3,0.2),(0.5,0.5)\rangle$ is an $IF\alpha^{**}g$ CS but not an IFPCS in X, as $cl(int(A)) \nsubseteq A$.*

Remark 3.18. *An $IF\alpha^{**}g$ CS closedness is independent of an IFSP closedness.*

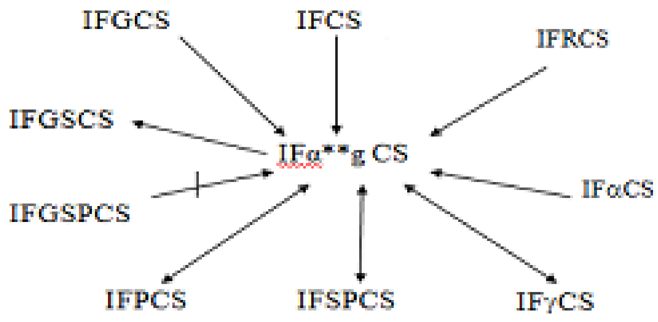
Example 3.19. *Let $X = \{a,b\}$ and $G = \langle x,(0.1,0.9),(0.6,0.1)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Consider an IFS $A = \langle x,(0,0.4),(0.7,0.6)\rangle$ in X. Since $\alpha cl(A) \nsubseteq int(cl(U))$ even though $A \subseteq G$ and G is an IFOS in X, A is not an $IF\alpha^{**}g$ CS in X. But A is an IFSPCS in (X,τ).*

Example 3.20. *Let $X = \{a,b\}$ and $G = \langle x,(0.6,0.7),(0.3,0.2)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.7,0.8),(0.1,0.2)\rangle$ is an $IF\alpha^{**}g$ CS but not an IFSPCS in X since $int(cl(int(A))) = 1_\sim \nsubseteq A$.*

Remark 3.21. *An $IF\alpha^{**}g$ CS closedness is independent of IFγ closedness.*

Example 3.22. *Let $X = \{a,b\}$ and $G = \langle x,(0.2,0.3),(0.5,0.6)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.2,0.3),(0.5,0.6)\rangle$ is an IFγ CS but not an $IF\alpha^{**}g$ CS in X, as $\alpha cl(A) \nsubseteq int(cl(U))$ even though $A \subseteq G$ and G is an IFOS in X.*

Example 3.23. *Let $X = \{a,b\}$ and $G = \langle x,(0.4,0),(0.4,0.8)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.6,0.7),(0.1,0)\rangle$ is an $IF\alpha^{**}g$ CS but not an IFγ CS in X, as $cl(int(A)) \cap int(cl(A)) \nsubseteq A$.*



Remark 3.24. *The union of two $IF\alpha^{**}g$ C sets is an $IF\alpha^{**}g$ CS in (X,τ).*

Example 3.25. *Let $X = \{a,b\}$ and $G = \langle x,(0.4,0),(0.1,1)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.6,0),(0.2,1)\rangle$, $B = \langle x,(0.2,1),(0.5,0)\rangle$ are $IF\alpha^{**}g$ CS. Now $A\cup B = \langle x,(0.6,1),(0.2,0)\rangle$. Since $\alpha cl(A\cup B) \subseteq int(cl(U))$, $A\cup B$ is an $IF\alpha^{**}g$ CS in X.*

Theorem 3.26. *Let (X,τ) be an IFTS. If A is an IFS of X then $\alpha cl(A^c) = (\alpha int(A))^c$.*

Proof. By Theorem, $\alpha cl(A) = A\cup cl(int(cl(A)))$. Replacing A by A^c, we get $\alpha cl(A^c) = A^c \cup cl(int(cl(A^c)))$. $\alpha cl(A^c) = A^c \cup cl(int(int(A))^c)$. That is $\alpha cl(A^c) = A^c \cup cl(cl(int(A)))^c$. This implies $\alpha cl(A^c) = A^c \cup (int(cl(int(A))))^c$. That is $\alpha cl(A^c) = (A\cap int(cl(int(A))))^c = (\alpha int(A))^c$. □

Remark 3.27. *The intersection of any two $IF\alpha^{**}g$ CS is not an $IF\alpha^{**}g$ CS in general as seen from the following example.*

Example 3.28. *Let $X = \{a,b\}$ and $G = \langle x,(0.5,0),(0.1,1)\rangle$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X. Then the IFS $A = \langle x,(0.2,1),(0.7,0)\rangle$, $B = \langle x,(0.6,0),(0.3,1)\rangle$ are $IF\alpha^{**}g$ CS. Now $A\cap B = \langle x,(0.2,0),(0.7,1)\rangle$. Since $\alpha cl(A\cap B) \nsubseteq int(cl(U))$ even though $A\cap B \subseteq G$ and G is an IFOS in X, $A\cap B$ is not an $IF\alpha^{**}g$ CS in X.*

Theorem 3.29. *Let (X,τ) be an IFTS. Then for every $A \in IF\alpha^{**}g$ CS (X) and for every $B \in IFS(X)$, $A \subseteq B \subseteq \alpha cl(A)$ implies $B \in IF\alpha^{**}g$ CS.*

Proof. Let an IFS $B \subseteq U$ and U be an IFOS in X. Since $A \subseteq B$, $A \subseteq U$ and A is an $IF\alpha^{**}g$ CS, $\alpha cl(A) \subseteq int(cl(U))$. By hypothesis, $B \subseteq \alpha cl(A)$, $\alpha cl(B) \subseteq \alpha cl(A) \subseteq int(cl(U))$. Therefore, $\alpha cl(B) \subseteq int(cl(U))$. Hence B is an $IF\alpha^{**}g$ CS of X. □

Theorem 3.30. *If A is an IFOS in (X,τ) and an $IF\alpha^{**}g$ CS in (X,τ), then A is an $IF\alpha CS$ in X.*

Proof. Let A be an IFOS in X. Since $A \subseteq A$, by hypothesis, $\alpha cl(A) \subseteq A = int(cl(A))$ by Theorem 3.5. But $A \subseteq \alpha cl(A)$. Therefore, $\alpha cl(A) = A$. Hence A is an $IF\alpha CS$ of X. □

Theorem 3.31. *The union of $IF\alpha^{**}g$ CS A and B is an $IF\alpha^{**}g$ CS in (X,τ), if A and B are IFCS in (X,τ).*

Proof. Since A and B are IFCS in X, $cl(A) = A$ and $cl(B) = B$. Assume that A and B are $IF\alpha^{**}g$ CS in (X,τ). Let $A\cup B \subseteq U$ and U be IFOS in X. Then $cl(int(cl(A\cup B))) = cl(int(A\cup B)) \subseteq cl(A\cup B) = A\cup B \subseteq U$. That is $\alpha cl(A\cup B) \subseteq int(cl(U))$. Therefore, the union of A and B is an $IF\alpha^{**}g$ CS in (X,τ). □

Theorem 3.32. *Let (X,τ) be an IFTS and A be an IFS in X. Then A is an $IF\alpha^{**}g$ CS if and only if $A\overline{q}F$ implies $\alpha cl(A)\overline{q}F$ for every IFCS F of X.*

Proof. Necessity: Let F be an IFCS in X and let $A\overline{q}F$. Then $A \subseteq F^c$, where F^c is an IFOS in X. Therefore $\alpha cl(A) \subseteq F^c$, by hypothesis. Hence $\alpha cl(A)\overline{q}F$.

Sufficiency: Let F be an IFCS in X and let A be an IFS in X. Then by hypothesis, $A\overline{q}F$ implies $\alpha cl(A)\overline{q}F$. Then $\alpha cl(A) \subseteq F^c$ whenever $A \subseteq F^c$ and F^c is an IFOS in X. Hence A is an $IF\alpha^{**}g$ CS in X. □

Theorem 3.33. *Let (X, τ) be an IFTS. Then $IF\alpha O(X) = IF\alpha C(X)$ if and only if every IFS in (X, τ) is an $IF\alpha^{**}g$ CS in X.*

Proof. Necessity: Suppose that $IF\alpha O(X) = IF\alpha C(X)$. Let $A \subseteq U$ and U be an IFOS in X. This implies $\alpha cl(A) \subseteq \alpha cl(U)$. Since U is an IFOS, U is an $IF\alpha OS$ in X. Since by hypothesis U is an $IF\alpha CS$ in X, $\alpha cl(U) = U$. This implies $\alpha cl(A) \subseteq int(cl(U))$. Therefore, A is an $IF\alpha^{**}g$ CS of X.

Sufficiency: Suppose that every IFS in (X, τ) is an $IF\alpha^{**}g$ CS in X. Let $U \in IFO(X)$, then $U \in IF\alpha O(X)$. Since $U \subseteq U$ and U is IFOS in X. By hypothesis, $\alpha cl(U) \subseteq U$. But clearly $U \subseteq \alpha cl(U)$. Hence $U = \alpha cl(U)$. That is $U \in IF\alpha C(X)$. Hence $IF\alpha O(X) \subseteq IF\alpha C(X)$. Let $A \in IF\alpha C(X)$ then A^c is an $IF\alpha OS$ in X. But $IF\alpha O(X) \subseteq IF\alpha C(X)$. Therefore, $A^c \in IF\alpha C(X)$. Hence, $A \in IF\alpha O(X)$. This implies, $IF\alpha C(X) \subseteq IF\alpha O(X)$. Thus $IF\alpha O(X) = IF\alpha C(X)$. □

Theorem 3.34. *If A is an IFOS and an $IF\alpha^{**}g$ CS in (X, τ), then*

(1). A is an IFROS in X

(2). A is an IFRCS in X.

Proof.

(1). Let A be an IFOS and an $IF\alpha^{**}g$ CS in X. Then $\alpha cl(A) \subseteq A$. This implies, $cl(int(cl(A))) \subseteq A$. That is $int(cl(A)) \subseteq A$. Since A is an IFOS, A is an IFPOS in X. Hence $A \subseteq int(cl(A))$. Therefore $A = int(cl(A))$. Hence, A is an IFROS in X.

(2). Let A be an IFOS and an $IF\alpha GCS$ in X. Then $cl(int(cl(A))) \subseteq A$. That is $cl(int(A)) \subseteq A$. Since A is an IFOS, A is an IFSOS in X. Hence, $A \subseteq cl(int(A))$. Therefore $A = cl(int(A))$. Hence A is an IFRCS in X. □

4. $IF\alpha^{**}g$ Open Sets in Intuitionistic Fuzzy Topological Spaces

In this section, we have introduced intuitionistic fuzzy $\alpha^{**}g$ open sets and studied some of their properties.

Definition 4.1. *An IFS A is said to be an intuitionistic fuzzy $\alpha^{**}g$ open set ($IF\alpha^{**}g$ OS in short) in (X, τ), if the complement A^c is an $IF\alpha^{**}g$ CS in X. The family of all $IF\alpha^{**}g$ OS of an IFTS (X, τ) is denoted by $IF\alpha^{**}gO(X)$.*

Theorem 4.2. *For any IFTS (X, τ), every IFOS is an $IF\alpha^{**}g$ OS but not conversely.*

Proof. Let A be an IFOS in X. Then A^c is an IFCS in X. By Theorem 3.3, A^c is an $IF\alpha^{**}g$ CS in X. Hence, A is an $IF\alpha^{**}g$ OS in X. □

Example 4.3. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.1), (0.5, 0.6)\rangle$. Then the IFS $A = \langle x, (0.4, 0.5), (0.4, 0.3)\rangle$ is an $IF\alpha^{**}g$ OS in X, but A is not an IFOS in X.*

Theorem 4.4. *For any IFTS (X, τ), every IFROS is an $IF\alpha^{**}g$ OS but not conversely.*

Proof. Let A be an IFROS in X. Then A^c is an IFRCS in X. By Theorem 3.5, A^c is an $IF\alpha^{**}g$ CS in X. Hence, A is an $IF\alpha^{**}g$ OS in X. □

Example 4.5. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.1, 0.2), (0.5, 0.6)\rangle$. Then the IFS $A = \langle x, (0.7, 0.6), (0.1, 0.1)\rangle$ is an $IF\alpha^{**}g$ OS but not an IFROS in X.*

Theorem 4.6. *In any IFTS (X, τ), every $IF\alpha OS$ is an $IF\alpha^{**}g$ OS but not conversely.*

Proof. Let A be an $IF\alpha OS$ in X. Then A^c is an $IF\alpha CS$ in X. By Theorem 3.7, A^c is an $IF\alpha^{**}g$ CS in X. Hence, A is an $IF\alpha^{**}gOS$ in X. □

Example 4.7. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ be an IFT on X, where $G_1 = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ and $G_2 = \langle x, (0.8, 0.8), (0.2, 0.2)\rangle$. Then the IFS $A = \langle x, (0.5, 0.6), (0.4, 0.4)\rangle$ is an $IF\alpha^{**}g$ OS in X but A is not an $IF\alpha OS$ in X.*

Theorem 4.8. *For any IFTS (X, τ), every IFGOS is an $IF\alpha^{**}g$ OS but its converse may not be true in general.*

Proof. Let A be an IFGOS in X. Then A^c is an IFGCS in X. By Theorem 3.9, A^c is an $IF\alpha^{**}g$ CS in X. Hence, A is an $IF\alpha^{**}g$ OS in X. □

Example 4.9. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.1, 0.7), (0.2, 0)\rangle$. Then the IFS $A = \langle x, (0.3, 0.8), (0.1, 0)\rangle$ is an $IF\alpha^{**}g$ OS in X but A is not an IFGOS in X, as $cl(A^c) \nsubseteq G$ even though $A^c \subseteq G$ and G is an IFOS in (X, τ).*

Remark 4.10. *An $IF\alpha^{**}g$ OS in (X, τ) is not an IFGSOS in (X, τ).*

Example 4.11. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.4, 0.5)\rangle$. Then the IFS $A = \langle x, (0.5, 0.6), (0.1, 0)\rangle$ is an $IF\alpha^{**}g$ OS in X but A is not an IFGSOS in X, as $scl(A^c) \nsubseteq G$ even though $A^c \subseteq G$ and G is an IFOS in X.*

Theorem 4.12. *For any IFTS (X, τ), every $IF\alpha^{**}g$ OS is an IFGSPOS but its converse may not be true in general.*

Proof. Let A be an $IF\alpha^{**}g$ OS in X. Then A^c is an $IF\alpha^{**}g$ CS in X. By Theorem 3.13, A^c is an IFGSPCS in X. Hence, A is an IFGSPOS in X. □

Example 4.13. *Let $X = \{a, b\}$ and let $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X, where $G = \langle x, (0, 0.8), (0.4, 0.1)\rangle$. Then the IFS $A = \langle x, (0.6, 0.6), (0, 0.2)\rangle$ is an IFGSPOS in X but A is not an $IF\alpha^{**}g$ OS in X, as $\alpha cl(A^c) \nsubseteq G$ even though $A^c \subseteq G$ and G is an IFOS in X.*

Theorem 4.14. *Let (X, τ) be an IFTS. If A is an IFS of X then the following properties are equivalent:*

(i) $A \in IF\alpha^{**}gO(X)$.

(ii) $V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFCS in X

(iii) There exists IFOS G_1 such that $G_1 \subseteq V \subseteq int(cl(G))$, where $G = int(A)$, $V \subseteq A$ and V is an IFCS in X.

Proof. *(i)* ⇒ *(ii)* Let $A \in IF\alpha^{**}gO(X)$. Then A^c is an $IF\alpha^{**}g$ CS in X. Therefore $\alpha cl(A^c) \subseteq U$ whenever $A^c \subseteq U$ and U is an IFOS in X. That is $cl(int(cl(A^c))) \subseteq U$. Taking complement on both sides, we get $(cl(int(cl(A^c))))^c = int(int(cl(A^c)))^c = int(cl(cl(A^c))^c) = int(cl(int(A^c)^c)) = int(cl(int(A))) \supseteq U^c$. This implies $U^c \subseteq int(cl(int(A)))$ whenever $U^c \subseteq A$ and U^c is an IFCS in X. Replacing U^c by V, $V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFCS in X.
(ii) ⇒ *(iii)* Let $V \subseteq int(cl(int(A)))$ whenever $V \subseteq A$ and V is an IFCS in X. Hence $int(V) \subseteq V \subseteq int(cl(int(A)))$. Then there exist IFOS G_1 in X such that $G_1 \subseteq V \subseteq int(cl(G))$ where $G = int(A)$ and $G_1 = int(V)$.

$(iii) \Rightarrow (i)$ Suppose that there exists IFOS G_1 such that $G_1 \subseteq V \subseteq int(cl(G))$ where $G = int(A)$, $V \subseteq A$ and V is an IFCS in X. It is clear that $(int(cl(G)))^c \subseteq V^c$. That is $(int(cl(int(A))))^c \subseteq V^c$. This implies that $cl(cl(int(A)))^c = cl(int(cl(A^c))) \subseteq V^c$, $A^c \subseteq V^c$ and V^c is IFOS in X. Hence, $\alpha cl(A^c) \subseteq V^c$. That is A^c is an $IF\alpha^{**}g$ CS in X. Which implies $A \in IF\alpha^{**}gO(X)$. □

Theorem 4.15. *Let (X, τ) be an IFTS. Then for every $A \in IF\alpha^{**}gO(X)$ and for every $B \in IFS(X)$, $\alpha int(A) \subseteq B \subseteq A$ implies $B \in IF\alpha^{**}gO(X)$.*

Proof. By hypothesis, $\alpha int(A) \subseteq B \subseteq A$. Taking complement on both sides, we get $A^c \subseteq B^c \subseteq (\alpha int(A))^c$. Let $B^c \subseteq U$ and U be an IFOS in X. Since $A^c \subseteq B^c$, $A^c \subseteq U$. Since A^c is an $IFpG\alpha CS$, $\alpha cl(A^c) \subseteq U$. Also $B^c \subseteq (\alpha int(A))^c = \alpha cl(A^c)$. Therefore $\alpha cl(B^c) \subseteq \alpha cl(A^c) \subseteq U$. Hence, B^c is an $IF\alpha GCS$ in X. This implies that B is an $IF\alpha^{**}gOS$ of X. That is $B \in IF\alpha^{**}gO(X)$. □

Remark 4.16. *The union of any two $IF\alpha^{**}g$ OS in (X, τ) is an $IF\alpha^{**}g$ OS in (X, τ).*

Example 4.17. *Let $X = \{a, b\}$ and let $G = \langle x, (0.4, 0), (0.1, 1)\rangle$. Then $\tau = \{0_\sim, G, 1_\sim\}$ is an IFT on X and the IFS $A = \langle x, (0.2, 1), (0.6, 0)\rangle$, $B = \langle x, (0.5, 0), (0.2, 1)\rangle$ are $IF\alpha^{**}g$ OS in X but $A \cup B = \langle x, (0.5, 1), (0.2, 0)\rangle$ is an $IF\alpha^{**}gOS$ in X.*

Theorem 4.18. *An IFS A of an IFTS (X, τ) is an $IF\alpha^{**}g$ OS if and only if $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is an IFCS in X.*

Proof. Necessity: Suppose A is an $IF\alpha^{**}g$ OS in X. Let F be an IFCS in X and $F \subseteq A$. Then F^c is an IFOS in X such that $A^c \subseteq F^c$. Since A^c is an $IF\alpha^{**}g$ CS, we have $\alpha cl(A^c) \subseteq F^c$. Hence $(\alpha int(A))^c \subseteq F^c$. Therefore $F \subseteq \alpha int(A)$.
Sufficiency: Let A be an IFS in X and let $F \subseteq \alpha int(A)$ whenever F is an IFCS in X and $F \subseteq A$. Then $A^c \subseteq F^c$ and F^c is an IFOS. By hypothesis, $(\alpha int(A))^c \subseteq F^c$, which implies $\alpha cl(A^c) \subseteq F^c$. Therefore A^c is an $IF\alpha^{**}g$ OS of X. Hence A is an $IF\alpha^{**}g$ OS of X. □

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