



SD Prime Cordial Labeling of Some Special Graphs

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Abstract: In this paper we investigate the SD - prime cordial labeling for $G \cup (P_n \odot K_1)$, $G \cup K_{1,n,n}$, $G \cup PS_n$ and $G \cup P_n$.

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1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [3]. For standard terminology and notations related to number theory we refer to Burton [1] and graph labeling, we refer to Gallian [2]. The notion of prime labeling for graphs originated with Roger Entringer and was introduced in a paper by Tout et al. [9] in the early 1980's and since then it is an active field of research for many scholars. Sundaram et al. introduced the notion of prime cordial labeling in [8]. Lau et al was introduced SD - prime labeling of graph in [4]. In [5], Lau et al. introduced SD - prime cordial labeling and they discussed SD - prime cordial labeling for some standard graphs. In [6], Lourdasamy et al. investigated some new construction of SD-prime cordial graph. In this paper, we presented SD - prime cordial labeling of some disconnected graph $G \cup (P_n \odot K_1)$, $G \cup K_{1,n,n}$, $G \cup PS_n$ and $G \cup P_n$.

2. Preliminaries

Definition 2.1. A complete bipartite graph $K_{1,n}$ is called a star and it has $n + 1$ vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Definition 2.2. The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to all the vertices in the i^{th} copy of G_2 .

Definition 2.3. Comb is a graph obtained by joining a single pendant edge to each vertex of a path. In other words $P_n \odot K_1$ is a comb graph.

Definition 2.4. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3 .

Definition 2.5. The n -pan graph is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

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Definition 2.6. Paw graph is the 3-pan graph.

Definition 2.7. The paw snake PS_n is obtained from the path P_n by replacing each edge of the path by a paw graph.

Definition 2.8. Let $G = (V, E)$ be a graph with n vertices. A function $f : V(G) \rightarrow \{1, 2, 3, \dots, n\}$ is said to be a prime labeling, if it is bijective and for every pair of adjacent vertices u and v , $\gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 2.9. Given a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$, we associate two integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in $E(G)$. The labeling f induces an edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ such that for any edge uv in $E(G)$, $f^*(uv) = 1$ if $\gcd(S, D) = 1$, and $f^*(uv) = 0$. Otherwise, we say f is SD-prime labeling if $f^*(uv) = 1$ for all $uv \in E(G)$. Moreover, G is SD - prime if it admits SD - prime labeling. Let $e_{f^*}(i)$ be the number of edges labeled with $i \in \{0, 1\}$. We say f is SD - prime cordial labeling if $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$. Moreover G is SD - prime cordial if it admits SD - prime cordial labeling.

3. SD Prime Cordial Labeling

Theorem 3.1. G is a SD - prime cordial graph with p vertices and q edges, then $G \cup (P_n \odot K_1)$ is a SD-prime cordial graph, where $n \geq 2$.

Proof. G is a SD - prime cordial graph with v_1, v_2, \dots, v_p vertices and e_1, e_2, \dots, e_q edges and f is SD - prime cordial labeling of G . Let $P_n \odot K_1$ be a comb graph. Let u_1, u_2, \dots, u_{2n} be the vertices and $s_1, s_2, \dots, s_{2n-1}$ be the edges of $P_n \odot K_1$. Let $G_1 = G \cup (P_n \odot K_1)$. Then $|V(G_1)| = p + 2n$ and $|E(G_1)| = q + 2n - 1$. Define $g : V(G_1) \rightarrow \{1, 2, \dots, p + 2n\}$ as follows:
 $g(v_i) = f(v_i)$ if $1 \leq i \leq p$.

Case 1 : q is even.

$$g(u_i) = \begin{cases} p + 2i - 1 & \text{if } 1 \leq i \leq n, \\ p + 2i - 2n & \text{if } n + 1 \leq i \leq 2n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq n - 1, \\ 1 & \text{if } n \leq i \leq 2n - 1. \end{cases}$$

In view of the above defined labeling pattern, we have $e_{g^*}(0) + 1 = e_{g^*}(1) = n$ in $P_n \odot K_1$. Here q is even, then $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G . Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) + 1 = e_{g^*}(1) = n$ in $P_n \odot K_1$, Therefore $e_{g^*}(0) = \frac{q}{2} + n - 1$ and $e_{g^*}(1) = \frac{q}{2} + n$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + n - 1) - (\frac{q}{2} + n)| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 2 : q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$.

$$\begin{aligned} g(u_1) &= p + 4, \\ g(u_{n+1}) &= p + 2, \\ g(u_{n+2}) &= p + 1, \\ g(u_i) &= \begin{cases} p + 2i - 1 & \text{if } 2 \leq i \leq n, \\ p + 2i - 2n & \text{if } n + 3 \leq i \leq 2n. \end{cases} \end{aligned}$$

Then induced labels are

$$\begin{aligned} g^*(s_1) &= 1, \\ g^*(s_i) &= 0 & \text{if } 2 \leq i \leq n - 1, \\ g^*(s_n) &= 0, \\ g^*(s_{n+1}) &= 0, \\ g^*(s_i) &= 1 & \text{if } n + 2 \leq i \leq 2n - 1. \end{aligned}$$

In view of the above defined labeling pattern, we have $e_{g^*}(0) = e_{g^*}(1) + 1 = n$ in $P_n \odot K_1$. Here q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$ in G . Thus, $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) + 1 = n$ in $P_n \odot K_1$, Therefore $e_{g^*}(0) = \frac{q-1}{2} + n$ and $e_{g^*}(1) = \frac{q+1}{2} + n - 1$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q-1}{2} + n) - (\frac{q+1}{2} + n - 1)| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 3 : q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$

$$g(u_i) = \begin{cases} p + 2i - 1 & \text{if } 1 \leq i \leq n, \\ p + 2i - 2n & \text{if } n + 1 \leq i \leq 2n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq n - 1, \\ 1 & \text{if } n \leq i \leq 2n - 1. \end{cases}$$

In view of the above defined labeling pattern, we have $e_{g^*}(0) + 1 = e_{g^*}(1) = n$ in $P_n \odot K_1$. Here q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G . Thus, $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G and $e_{g^*}(0) + 1 = e_{g^*}(1) = n$ in $P_n \odot K_1$, Therefore $e_{g^*}(0) = \frac{q+1}{2} + n - 1$ and $e_{g^*}(1) = \frac{q-1}{2} + n$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + n - 1) - (\frac{q-1}{2} + n)| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Hence $G(P_n \odot K_1)$ is a SD - prime cordial graph, where $n \geq 2$. □

Example 3.2. The SD prime cordial labeling is in Figure 1 for $P_5 \cup (P_4 \odot K_1)$

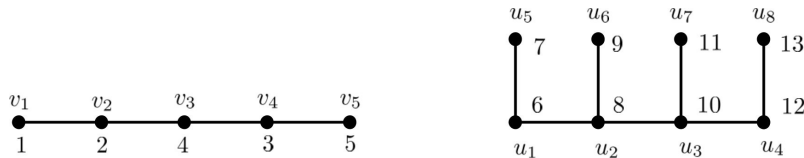


Figure 1.

Theorem 3.3. G is a SD - prime cordial graph with p vertices and q edges, then $G \cup K_{1,n,n}$ is a SD - prime cordial graph, where $n \geq 2$.

Proof. G is a SD - prime cordial graph with v_1, v_2, \dots, v_p vertices and e_1, e_2, \dots, e_q edges and f is SD - prime cordial labeling of G . Let $u, u_1, u_2, \dots, u_{2n}$ be the vertices and s_1, s_2, \dots, s_{2n} be the edges of $K_{1,n,n}$. Let $G_1 = G \cup K_{1,n,n}$. Then $|V(G_1)| = p + 2n + 1$ and $|E(G_1)| = q + 2n$. Define $g : V(G_1) \rightarrow \{1, 2, \dots, p + 2n + 1\}$ as follows :

$$\begin{aligned} g(v_i) &= f(v_i) && \text{if } 1 \leq i \leq p, \\ g(u) &= p + 1, \\ g(u_i) &= \begin{cases} p + 2i + 1 & \text{if } 1 \leq i \leq n, \\ p + 2i - 2n & \text{if } n + 1 \leq i \leq 2n. \end{cases} \end{aligned}$$

Then induced edge labels are

$$g^*(e_i) = \begin{cases} 0 & \text{if } 1 \leq i \leq n, \\ 1 & \text{if } n + 1 \leq i \leq 2n. \end{cases}$$

In view of the above defined labeling pattern, we have $e_{g^*}(0) = e_{g^*}(1) = n$ in $K_{1,n,n}$.

Case 1 : q is even.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = n$ in $K_{1,n,n}$. Therefore $e_{g^*}(0) = \frac{q}{2} + n$ and $e_{g^*}(1) = \frac{q}{2} + n$ in G_1 .

Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + n) - (\frac{q}{2} + n)| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph for $n \geq 2$.

Case 2 : q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = n$ in $K_{1,n,n}$. Therefore $e_{g^*}(0) = \frac{q-1}{2} + n$ and $e_{g^*}(1) = \frac{q+1}{2} + n$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q-1}{2} + n) - (\frac{q+1}{2} + n)| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph for $n \geq 2$.

Case 3 : q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = n$ in $P_n \odot K_1$. Therefore $e_{g^*}(0) = \frac{q+1}{2} + n$ and $e_{g^*}(1) = \frac{q-1}{2} + n$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + n) - (\frac{q-1}{2} + n)| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph for $n \geq 2$.

Hence $G \cup K_{1,n,n}$ is a SD-prime cordial graph, where $n \geq 2$. □

Example 3.4. The SD prime cordial labeling is in Figure 2 for $C_6 \cup K_{1,4,4}$

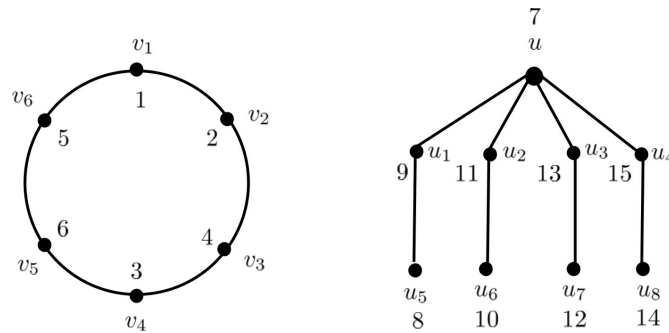


Figure 2.

Theorem 3.5. G is a SD - prime cordial graph with p vertices and q edges, then $G \cup PS_n$ is a SD - prime cordial graph, where $n \geq 2$.

Proof. G is a SD - prime cordial graph with v_1, v_2, \dots, v_p vertices and e_1, e_2, \dots, e_q edges and f is SD - prime cordial labeling of G . Let $u_1, u_2, \dots, u_n, w_1, w_2, \dots, w_{n-1}, x_1, x_2, \dots, x_{n-1}$ be the vertices and $s_1, s_2, \dots, s_{4n-4}$ be the edges of PS_n , where $s_i = w_i x_i, s_{n+2i-2} = u_i w_i, s_{n+2i-1} = w_i u_{i+1}$ and $s_{3n-3+i} = u_i u_{i+1}$, for $1 \leq i \leq n - 1$. Let $G_1 = G \cup PS_n$. Then $|V(G_1)| = p + 3n - 2$ and $|E(G_1)| = q + 4n - 4$. Define $g : V(G_1) \rightarrow \{1, 2, \dots, p + 3n - 3\}$ as follows: $g(v_i) = f(v_i)$ if $1 \leq i \leq p$.

Case 1 : $p \equiv 0 \pmod{3}$.

$$g(u_i) = \begin{cases} p + 3i - 1 & \text{if } 1 \leq i \leq n - 1, \\ p + 3i - 2 & \text{if } i = n. \end{cases}$$

$$g(w_i) = p + 3i \quad \text{if } 1 \leq i \leq n - 1,$$

$$g(x_i) = p + 3i - 2 \quad \text{if } 1 \leq i \leq n - 1.$$

Then induced edge labels are

$$g^*(s_i) = 0 \text{ if } 1 \leq i \leq n - 1,$$

$$g^*(s_{n+2i-2}) = 1 \text{ if } 1 \leq i \leq n - 1,$$

$$g^*(s_{n+2i-1}) = 0 \text{ if } 1 \leq i \leq n - 2,$$

$$g^*(s_{n+2i-1}) = 1 \text{ if } i = n - 1,$$

$$g^*(s_{3n-3+i}) = 1 \text{ if } 1 \leq i \leq n - 2,$$

$$g^*(s_{3n-3+i}) = 0 \text{ if } i = n - 1.$$

Case 2 : $p \equiv 1 \pmod{3}$.

$$\begin{aligned}
 g(u_1) &= p + 4, \\
 g(u_i) &= p + 3i - 4 \quad \text{if } 2 \leq i \leq n, \\
 g(w_i) &= p + 3i \quad \text{if } 1 \leq i \leq n - 1, \\
 g(x_1) &= p + 1, g(x_1) = p + 1, \\
 g(x_i) &= p + 3i + 1 \quad \text{if } 2 \leq i \leq n - 1.
 \end{aligned}$$

Then induced edge labels are

$$\begin{aligned}
 g^*(s_1) &= 0, \\
 g^*(s_i) &= 1 \quad \text{if } 2 \leq i \leq n - 1, \\
 g^*(s_i) &= 1, \text{ if } i = n, \\
 g^*(s_{n+2i-2}) &= 0 \quad \text{if } 2 \leq i \leq n - 1, \\
 g^*(s_{n+2i-1}) &= 1 \quad \text{if } 1 \leq i \leq n - 1, \\
 g^*(s_{3n-3+i}) &= 0 \quad \text{if } 1 \leq i \leq n - 1.
 \end{aligned}$$

Case 3 : $p \equiv 2 \pmod{3}$.

$$\begin{aligned}
 g(u_i) &= p + 3i - 2 \quad \text{if } 1 \leq i \leq n, \\
 g(w_i) &= p + 3i - 1 \quad \text{if } 1 \leq i \leq n - 1, \\
 g(x_i) &= p + 3i \quad \text{if } 1 \leq i \leq n - 1.
 \end{aligned}$$

Then induced edge labels are

$$\begin{aligned}
 g^*(s_i) &= 1 \quad \text{if } 1 \leq i \leq n - 1, \\
 g^*(s_{n+2i-2}) &= 1 \quad \text{if } 1 \leq i \leq n - 1, \\
 g^*(s_{n+2i-1}) &= 0 \quad \text{if } 1 \leq i \leq n - 1, \\
 g^*(s_{3n-3+i}) &= 0 \quad \text{if } i \leq i \leq n - 1.
 \end{aligned}$$

In view of the above defined labeling pattern, we have $e_{g^*}(0) = e_{g^*}(1) = 2n - 2$ in PS_n .

For q is even. Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = 2n - 2$ in PS_n . Therefore $e_{g^*}(0) = \frac{q}{2} + 2n - 2$ and $e_{g^*}(1) = \frac{q}{2} + 2n - 2$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + 2n - 2) - (\frac{q}{2} + 2n - 2)| = 0$ in G_1 . Thus G_1 is a SD - prime cordial graph for $n \geq 2$.

For q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$. Thus, $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = 2n - 2$ in PS_n . Therefore $e_{g^*}(0) = \frac{q-1}{2} + 2n - 2$ and $e_{g^*}(1) = \frac{q+1}{2} + 2n - 2$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q-1}{2} + 2n - 2) - (\frac{q+1}{2} + 2n - 2)| = 1$ in G_1 . Thus G_1 is a SD - prime cordial graph for $n \geq 2$.

For q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$. Thus, $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = 2n - 2$ in PS_n . Therefore $e_{g^*}(0) = \frac{q+1}{2} + 2n - 2$ and $e_{g^*}(1) = \frac{q-1}{2} + 2n - 2$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + 2n - 2) - (\frac{q-1}{2} + 2n - 2)| = 1$ in G_1 . Thus G_1 is a SD - prime cordial graph for $n \geq 2$.

Hence $G \cup PS_n$ is a SD - prime cordial graph, where $n \geq 2$. □

Example 3.6. The SD prime cordial labeling is in Figure 3 for $C_3 \odot K_1 \cup PS_3$

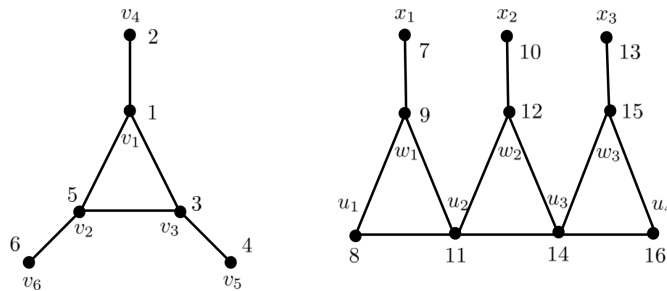


Figure 3.

Theorem 3.7. G is a SD - prime cordial graph with p vertices and q edges, then $G \cup P_n$ is a SD - prime cordial graph, where $n \geq 2$.

Proof. G is a SD - prime cordial graph with v_1, v_2, \dots, v_p vertices and e_1, e_2, \dots, e_q edges and f is SD - prime cordial labeling of G . Let u_1, u_2, \dots, u_n be the vertices and s_1, s_2, \dots, s_{n-1} be the edges of P_n . Let $G_1 = G \cup P_n$. Then $|V(G_1)| = p + n$ and $|E(G_1)| = q + n - 1$. Define $g : V(G_1) \rightarrow \{1, 2, \dots, p + n\}$ as follows: $g(v_i) = f(v_i)$ if $1 \leq i \leq p$.

Case 1 : $n \equiv 1, 3 \pmod{4}$.

$$g(u_i) = \begin{cases} p + i & \text{if } i \equiv 0, 1 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + 1 + i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + i - 1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } i \text{ is odd,} \\ 1 & \text{if } i \text{ is even.} \end{cases}$$

In view of the above defined labeling pattern, we have, $e_{g^*}(0) = e_{g^*}(1) = \frac{n-1}{2}$ and $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$.

Sub Case 1.1 : q is even.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = \frac{n-1}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q}{2} + \frac{n-1}{2}$ and $e_{g^*}(1) = \frac{q}{2} + \frac{n-1}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + \frac{n-1}{2}) - (\frac{q}{2} + \frac{n-1}{2})| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Sub Case 1.2 : q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = \frac{n-1}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q-1}{2} + \frac{n-1}{2}$ and $e_{g^*}(1) = \frac{q+1}{2} + \frac{n-1}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q-1}{2} + \frac{n-1}{2}) - (\frac{q+1}{2} + \frac{n-1}{2})| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Sub Case 1.3 : q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = \frac{n-1}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q+1}{2} + \frac{n-1}{2}$ and $e_{g^*}(1) = \frac{q-1}{2} + \frac{n-1}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + \frac{n-1}{2}) - (\frac{q-1}{2} + \frac{n-1}{2})| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 2 : $n \equiv 0 \pmod{4}$ and $e_{g^*}(0) \neq e_{g^*}(1) + 1$ in G .

$$g(u_i) = \begin{cases} p + i & \text{if } i \equiv 0, 1 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + 1 + i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + i - 1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } i \text{ is odd,} \\ 1 & \text{if } i \text{ is even.} \end{cases}$$

In view of the above defined labeling pattern, we have, $e_{g^*}(0) = e_{g^*}(1) = \frac{n}{2}$ and $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$.

Sub Case 2.1 : q is even.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q}{2} + \frac{n}{2}$ and $e_{g^*}(1) = \frac{q}{2} + \frac{n}{2} - 1$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + \frac{n}{2}) - (\frac{q}{2} + \frac{n}{2} - 1)| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Sub Case 2.2 : q is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q-1}{2} + \frac{n}{2}$ and $e_{g^*}(1) = \frac{q+1}{2} + \frac{n}{2} - 1$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q-1}{2} + \frac{n}{2}) - (\frac{q+1}{2} + \frac{n}{2} - 1)| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 3 : $n \equiv 0 \pmod{4}$ and $e_{g^*}(0) = e_{g^*}(1) + 1$ in G .

$$g(u_i) = \begin{cases} p + i & \text{if } i \equiv 1, 2 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + 1 + i & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n, \\ p + i - 1 & \text{if } i \equiv 0 \pmod{4} \text{ and } 1 \leq i \leq n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 1 & \text{if } i \text{ is odd,} \\ 0 & \text{if } i \text{ is even.} \end{cases}$$

In view of the above defined labeling pattern, we have, $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{n}{2}$ and $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$.

Thus, $e_{g^*}(0) = e_{g^*}(1) + 1 \implies q$ is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$. Then $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q+1}{2} + \frac{n}{2} - 1$ and $e_{g^*}(1) = \frac{q-1}{2} + \frac{n}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + \frac{n}{2} - 1) - (\frac{q-1}{2} + \frac{n}{2})| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 4 : $n \equiv 2 \pmod{4}$ and $e_{g^*}(0) + 1 \neq e_{g^*}(1)$ in G .

$$g(u_i) = \begin{cases} p+i & \text{if } i \equiv 0, 1 \pmod{4} \text{ and } 1 \leq i \leq n-1, \\ p+1+i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n-1, \\ p+i-1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n-1, \\ p+n & \text{if } i = n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } i \text{ is odd and } 1 \leq i \leq n-2, \\ 1 & \text{if } i \text{ is even and } 1 \leq i \leq n-2, \\ 1 & \text{if } i = n-1. \end{cases}$$

In view of the above defined labeling pattern, we have, $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{n}{2}$ and $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$.

Sub Case 4.1 : q is even

Thus $e_{g^*}(0) = e_{g^*}(1) = \frac{q}{2}$ in G and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q}{2} + \frac{n}{2} - 1$ and $e_{g^*}(1) = \frac{q}{2} + \frac{n}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q}{2} + \frac{n}{2} - 1) - (\frac{q}{2} + \frac{n}{2})| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 4.2 : q is odd and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$.

Thus, $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{q+1}{2}$ in G and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q+1}{2} + \frac{n}{2} - 1$ and $e_{g^*}(1) = \frac{q-1}{2} + \frac{n}{2}$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| = |(\frac{q+1}{2} + \frac{n}{2} - 1) - (\frac{q-1}{2} + \frac{n}{2})| = 1$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Case 5 : $n \equiv 2 \pmod{4}$ and $e_{g^*}(0) + 1 = e_{g^*}(1)$ in G .

$$g(u_i) = \begin{cases} p+i & \text{if } i \equiv 0, 1 \pmod{4} \text{ and } 1 \leq i \leq n-2, \\ p+1+i & \text{if } i \equiv 2 \pmod{4} \text{ and } 1 \leq i \leq n-2, \\ p+i-1 & \text{if } i \equiv 3 \pmod{4} \text{ and } 1 \leq i \leq n-2, \\ p+n & i = n-1, \\ p+n-1 & i = n. \end{cases}$$

Then induced edge labels are

$$g^*(s_i) = \begin{cases} 0 & \text{if } i \text{ is odd and } 1 \leq i \leq n-3, \\ 1 & \text{if } i \text{ is even and } 1 \leq i \leq n-3, \\ 0 & \text{if } i = n-2, \\ 1 & \text{if } i = n-1. \end{cases}$$

In view of the above defined labeling pattern, we have, $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{n}{2}$ and $|e_{g^*}(0) - e_{g^*}(1)| \leq 1$.

Thus, $e_{g^*}(0) + 1 = e_{g^*}(1) \implies q$ is odd and $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$. Then $e_{g^*}(0) + 1 = e_{g^*}(1) = \frac{q+1}{2}$ in G and $e_{g^*}(0) = e_{g^*}(1) + 1 = \frac{n}{2}$ in P_n . Therefore $e_{g^*}(0) = \frac{q-1}{2} + \frac{n}{2}$ and $e_{g^*}(1) = \frac{q+1}{2} + \frac{n}{2} - 1$ in G_1 . Now $|e_{g^*}(0) - e_{g^*}(1)| =$

$|(\frac{q-1}{2} + \frac{n}{2}) - (\frac{q+1}{2} + \frac{n}{2} - 1)| = 0$ in G_1 . Thus in this case G_1 is a SD - prime cordial graph.

Hence $G \cup P_n$ is a SD - prime cordial graph, where $n \geq 2$. □

Example 3.8. The SD prime cordial labeling is in Figure 4 for $K_{1,8,8} \cup P_8$

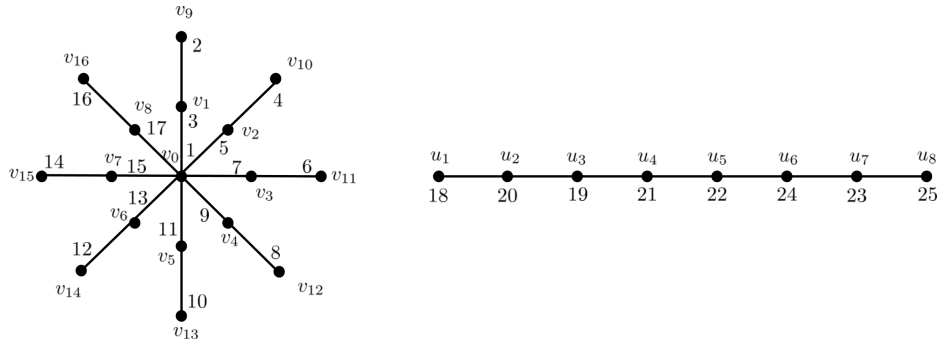


Figure 4.

4. Conclusion

Labeling of graph is the topic of current interest for many researchers as it has diversified applications. It is also very interesting to investigate graph or families of graph which admits particular type of labeling. We derived four new results by investigating SD prime cordial labeling of graphs. Similar investigations are possible for other graph families and parallel results can be investigated corresponding to other graph labeling techniques.

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