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ABC, GA and AG Co-Even Domination Indices of Graph

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Abstract

In this research work, we use the definition for the degree of vertex $v \in V(G)$, called Co-even domination degree of $d(v)$ are introduced and denoted by $d_{coe}(v)$, along with this new degree some Co-even domination indices based on Co-even domination degrees are Introduced. The exact value of Co-even domination ABC, GA and AG of some well-known classes of graphs is established. Lower and upper bounds on the Co-even domination ABC, GA and AG indices of the graph.

Keywords: Co-even dominating sets; Co-even domination degree; Topological Indices; Co-Even Domination ABC, GA AND AG.

2020 Mathematics Subject Classification: 05C69, 05C90, 05C35.

1. Introduction

In this paper, we assume that $G = (V, E)$ is a simple connected graph,Graph theory is considered a new language to deal with all sciences. In mathematics deals with most of its fields such as topological graph [12, 27], fuzzy graph [25] and, and others. In the last decade, graph theory has found considerable use in mathematical chemistry. In this area, we can apply tools of graph theory to model the chemical phenomenon mathematically. This theory contributes a prominent in chemical science. Topological indicators are numeric parameters of a graph that characterize a topology and are usually constant in the graph. Topological indicators are used for example in the development of quantitative structure-activity relationships (QSARs) where the biological activity or other properties of molecules are associated with chemical structure. We can list some major of topological indices such as degree-based-topological indices see [18,19], and distance based-topological indices see [13–17]. For more discussion, see [2–11, 23, 24]. Zagreb Topological indices relies on distance degree, and defined as follows [20, 21]:

$$M_1(G) = \sum_{v \in V(G)} d(v)^2$$

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$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

The Forgotten index $F(G) = \sum_{v \in V(G)} d_1^3(v/G) = \sum_{uv \in E(G)} [d_1^2(v/G) + d_1^2(v/G)]$ [21]. For more information on Zagreb and beyond topological indices, readers are referred to the survey [22]. The maximum and minimum degrees of vertices of a graph G are denoted by $\Delta(G)$ and $\delta(G)$ respectively. A helm graph is obtained by the wheel graph of n vertices by adjoining a pendent edge at each vertex of the cycle. Ladder graph L_n is a planar undirected graph with $2n$ vertices and $3n-2$ edges. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge: $L_{n,1} = P_n \times P_2$. A set $D \subseteq V$ is said to be a dominating set of G, if for any vertex $v \in V - D$ there exists a vertex $u \in D$ such that u and v are adjacent. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set in G. Let G be a graph and D is a dominating set, the set D is called Co-even dominating set if, $deg(v)$ is even number for all $v \in V - D$ [26]. A. M. Hanan Ahmed [18] are introduced the P set degree of the vertex $v \in V(G)$ define as $d_p(v) = |\{S \subseteq V(G) : S \ has \ propertices \ P \ and \ v \in S\}|$ for a graph G, we use the notation $T_{coe}(G)$ to denote the total number of Co-even dominating sets of G. In this paper, we define a new degree of each vertices $v \in V(G)$, called Co-even domination degree of $d(v)$ is introduced and denoted by $d_{coe}(v)$, along with this new degree some Co-even domination indices based on Co-even domination degree are Introduced. The exact value of Co-even domination Zagreb indices of some well-known classes of graphs are established. Lower and upper bounds on the Co-even domination Zagreb indices of the graph.

2. Co-even Domination Indices of Graph

In this section, we present the definitions of Co-Even Domination Forgotten, Modified Forgotten, hyper Zagreb Indices and, we establish the formulaes of the exact values of Co-Even Domination Forgotten, Modified Forgotten, hyper Zagreb Indices of Graph for some well-known graph classes.

Definition 2.1. *[1] Let D be a Co-even dominating set, then the Co-even domination degree of vertex $v \in V(G)$ is the number of Co-even dominating sets which containing v, and denoted by $d_{coe}(v)$ i.e. $d_{coe}(v)$=$|\{D \subseteq V(G)$: D is Co-even dominating set and $v \in D\}|$. The minimum and maximum co-even domination degree of G denoted by $\delta_{coe}(G)$ and $\Delta_{coe}(G)$ respectively, where $\delta_{coe}(G) = \min\{d_{coe}(v) : v \in V(G)\}$, $\Delta_{coe}(G) = \max\{d_{coe}(v) : v \in V(G)\}$*

Definition 2.2. *Let G be a graph and D be a Co-even domination set of graph G. Then the Co-Even Domination Forgotten, Modified Forgotten, hyper Zagreb Indices of Graph are define as:*

$$D_{coe}ABC(\Gamma) = \sum_{xy \in E(\Gamma)} \sqrt{\frac{d_{coe}(x) + d_{coe}(y) - 2}{d_{coe}(x)d_{coe}}}. \tag{1}$$

$$D_{coe}GA(\Gamma) = \sum_{xy \in E(\Gamma)} \frac{2\sqrt{d_{coe}(x)d_{coe}}}{d_{coe}(x) + d_{coe}(y)}. \tag{2}$$

$$D_{coe}AG(\Gamma) = \sum_{xy \in E(\Gamma)} \frac{d_{coe}(x) + d_{coe}(y)}{2\sqrt{d_{coe}(x)d_{coe}}}. \quad (3)$$

Proposition 2.3. *Let S_n stare graph. Then*

1. *If n is even*

$$D_{coe}ABC(S_n) = \sum_{xy \in E(\Gamma)} \sqrt{\frac{1+1-2}{1}} = 0.$$
$$D_{coe}GA(S_n) = \sum_{xy \in E(\Gamma)} \frac{2\sqrt{1}}{1+1} = (n-1).$$
$$D_{coe}AG(S_n) = \sum_{xy \in E(\Gamma)} \frac{1+1}{2\sqrt{1}} = (n-1).$$

2. *If n is odd*

$$D_{coe}ABC(S_n) = \frac{1}{\sqrt{2}}(n-1).$$
$$D_{coe}GA(S_n) = \frac{2\sqrt{2}}{3}(n-1).$$
$$D_{coe}AG(S_n) = \frac{3}{2\sqrt{2}}(n-1).$$

Proof. Let Γ be a graph that is isomorphic to a star graph. Suppose the set of n vertices of Γ is $\{x_1, x_2, ..., x_{n-1}, w\}$, where w is the center vertex.

Case 1: If n is even, then we have only one Co-even dominating set $\{x_1, x_2, ..., x_{n-1}, w\}$. Hence $d_{coe}(x) = 1$, $T_{coe}(x) = 1$, so we have only for all $x \in V(\Gamma)$, then by the Definition 2.2

$$D_{coe}ABC(S_n) = \sum_{xy \in E(\Gamma)} \sqrt{\frac{1+1-2}{1}} = 0.$$
$$D_{coe}GA(S_n) = \sum_{xy \in E(\Gamma)} \frac{2\sqrt{1}}{1+1} = (n-1).$$
$$D_{coe}AG(S_n) = \sum_{xy \in E(\Gamma)} \frac{1+1}{2\sqrt{1}} = (n-1).$$

Case 2: If n is odd, we have $D_{coe1} = \{x_1, x_2, ..., x_{n-1}, w\}$, $D_{coe2} = \{x_1, x_2, ..., x_{n-1}\}$, $T_{coe}(S_n) = 2$. Hence

$$d_{coe}(x) = \begin{cases} 1, & \text{if } x = w; \\ 2, & \text{if } otherwise \text{ .} \end{cases}$$

then by definition 2.2, we get

$$D_{coe}ABC(S_n) = \frac{1}{\sqrt{2}}(n-1).$$

$$D_{coe}GA(S_n) = \frac{2\sqrt{2}}{3}(n-1).$$
$$D_{coe}AG(S_n) = \frac{3}{2\sqrt{2}}(n-1).$$

□

Proposition 2.4. *Let W_n wheel graph. Then*

1. *If n is even*

$$D_{coe}ABC(W_n) = (n-1).$$
$$D_{coe}GA(W_n) = \frac{1}{\sqrt{2}}(n-1).$$
$$D_{coe}AG(W_n) = \frac{3}{2\sqrt{2}}(n-1.$$

2. *If n is odd*

$$D_{coe}ABC(W_n) = \frac{1}{\sqrt{2}}(n-1).$$
$$D_{coe}GA(W_n) = 3(n-1).$$
$$D_{coe}AG(W_n) = \frac{1}{2}(n-1) + \frac{1}{2}(n-1).$$

Proof. We can proof this by similarly Proposition 2.3. □

Proposition 2.5. *Let G be a complete graph K_n,with $n \geq 3$. Then*

1. *If n is even*

$$D_{coe}ABC(\Gamma) = (n-1).$$
$$D_{coe}GA(\Gamma) = \frac{(n-1)^2}{2}.$$
$$D_{coe}AG(\Gamma) = (n-1)^2.$$

2. *If n is odd, then*

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^n-2}{2^n}}(n-1).$$
$$D_{coe}GA(\Gamma) = 2^{1-n}\sqrt{2^{2(n-1)}}\frac{(n-1)^2}{2}$$
$$D_{coe}AG(\Gamma) = n(n-1)2^{n(n-1)}(n-1)^2.$$

Proof. Let K_n be a complete graph, with $n \geq 3$, then
Case 1: If n is even, then there only Co-even dominating set of Γ is $V(K_n)$ i.e, $D_{coe} = \{x_1, x_2, ..., x_n\}$.

Hance $d_{coe}(x) = 1$ for all $v \in V(K_n)$, and

$$D_{coe}ABC(\Gamma) = (n-1).$$
$$D_{coe}GA(\Gamma) = \frac{(n-1)^2}{2}.$$
$$D_{coe}AG(\Gamma) = (n-1)^2.$$

Case 2: In this case if n is odd. The first degree of all vertices are even, hance there are $\binom{n}{1}$ Co-even dominating sets containing only one vertex has only first even degree, there are $\binom{n}{2}$ Co-even dominating sets containing two vertices have even first degree,and there are $\binom{n}{3}$ Co-even dominating sets containing three vertices have even first degree and there are $\binom{n}{n}$ Co-even dominating sets contain all vertices, not that all above Co-even dominating sets contains all vertices of odd first degree. The total number of Co-even dominating sets are $\binom{n}{1} + \binom{n}{k} + ... + \binom{n}{n} = 2^n - 1$ sets. Using the definition of Co-even domination degree we have $d_{coe}(x) = 2^{n-1}$. Applying definition 2.2, we get

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^n - 2}{2^n}}(n-1).$$
$$D_{coe}GA(\Gamma) = 2^{1-n}\sqrt{2^{2(n-1)}}\frac{(n-1)^2}{2}$$
$$D_{coe}AG(\Gamma) = n(n-1)2^{n(n-1)}(n-1)^2.$$

□

Proposition 2.6. *Let $K_{m,n} \cong \Gamma$ be complete bipartite graph with m, n vertices.*

1. *If m is odd and n is even. Then*

$$D_{coe}ABC(\Gamma) = mn\left(\sqrt{\frac{2^m + 2^{(m-1)} - 2}{2^{2m-1}}}\right).$$
$$D_{coe}GA(\Gamma) = 2(2^m + 2^{m-1})^{-1}\sqrt{2^{2m-1}}(mn).$$
$$D_{coe}AG(\Gamma) = mn\left(\frac{2^m + 2^{m-1}}{2\sqrt{2^{2m-1}}}\right).$$

2. *If m is even and n is odd. Then*

$$D_{coe}ABC(\Gamma) = mn\left(\sqrt{\frac{2^m + 2^{(m-1)} - 2}{2^{2m-1}}}\right).$$
$$D_{coe}GA(\Gamma) = 2(2^m + 2^{m-1})^{-1}\sqrt{2^{2m-1}}(mn).$$
$$D_{coe}AG(\Gamma) = mn\left(\frac{2^m + 2^{m-1}}{2\sqrt{2^{2m-1}}}\right).$$

Proof. Let A the set of vertices in the first partite, and B the set of vertices in the second partite, clear $|A| = m$, $|B| = n$.

Case 1: If m is odd and n is even. Then first degree$(d(v))$ is even for all $v \in A$ and $d(v)$ is odd for all $v \in B$. Now, there are $\binom{m}{1}$ Co-even dominating sets contain only one vertex of A, $\binom{m}{2}$ Co-even dominating sets contain two vertices of A and $\binom{m}{m}$ Co-even dominating sets contain all the vertices of A. Not that all vertices of B are present in all Co-even dominating sets. Hence the total number of Co-even dominating sets are the set B and $\binom{m}{1} + \binom{m}{2} + ... + \binom{m}{m} = 2^m$. Using the definition of Co-even domination degree we get the total number is 2^m

$$d_{coe}(v) = \begin{cases} 2^m, \text{ if } v \in B; \\ 2^{m-1}, \text{ if } v \in A \ . \end{cases}$$

Applying Definition 2.2, we get

$$D_{coe}ABC(\Gamma) = mn\left(\sqrt{\frac{2^m + 2^{(m-1)} - 2}{2^{2m-1}}}\right).$$

$$D_{coe}GA(\Gamma) = 2(2^m + 2^{m-1})^{-1}\sqrt{2^{2m-1}}(mn).$$

$$D_{coe}AG(\Gamma) = mn\left(\frac{2^m + 2^{m-1}}{2\sqrt{2^{2m-1}}}\right).$$

Case 2: If m even and n odd. Applying the same method in Case 1. Then we get,

$$d_{coe}(v) = \begin{cases} 2^{n-1}, \text{ if } v \in B; \\ 2^n, \text{ if } v \in A \ . \end{cases}$$

Applying the Definition 2.2, we get

$$D_{coe}ABC(\Gamma) = mn\left(\sqrt{\frac{2^m + 2^{(m-1)} - 2}{2^{2m-1}}}\right).$$

$$D_{coe}GA(\Gamma) = 2(2^m + 2^{m-1})^{-1}\sqrt{2^{2m-1}}(mn).$$

$$D_{coe}AG(\Gamma) = mn\left(\frac{2^m + 2^{m-1}}{2\sqrt{2^{2m-1}}}\right).$$

□

Definition 2.7. *The graph Γ is called $K-$ Co-even domination regular graph if and only if $d_{coe}(v) = k$, for all $v \in V(G)$. For example,*

1. *S_n with n is even is one Co-even domination regular graph.*
2. *$K_n, n \geq 3$ with n even is one-Co-even domination regular graph.*
3. *$K_n, n \geq 3$ with n odd is 2^{n-1} Co-even domination regular graph.*

Observation 2.8. *[1] Let G be graph and $\gamma_{coe}(\Gamma)$ is Co-even domination number of $G = (n, m)$, with $D_1, D_2, ..., D_t$ as Co-even domination sets. Then*

$$t\gamma_{coe}(\Gamma) \leq \sum_{v \in V(\Gamma)} d_{coe}(v) \leq t|V(\Gamma)|$$

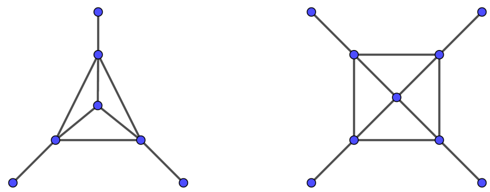

Figure 1: Helm graph H_4, and H_5.

Theorem 2.9. *Let $H_n \cong \Gamma$ be Helm graph with $n \geq 4$. Then*

1. *If n is even*

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^{n-2} + 2^{n-1} - 2}{2^{2n-3}}}(n-1) + \sqrt{\frac{2^{n-2} + 2^{n-2} - 2}{2^{4n-4}}}(n-1)$$
$$D_{coe}GA(\Gamma) = 2\frac{2^{2n-3}}{2^{n-1} + 2^{n-2}}(n-1) + 2\frac{2^{2n-4}}{2^{n-1} + 2^{2n-4}}(n-1)$$
$$D_{coe}AG(\Gamma) = \frac{2^{n-2} + 2^{n-1}}{2\sqrt{2^{2n-3}}}(n-1) + \frac{2^{n-2} + 2^{n-1}}{2\sqrt{2^{2n-4}}}(n-1)$$

2. *If n is odd*

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^{n} + 2^{n-1} - 2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1} + 2^{n-1} - 2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1} + 2^{n-1} - 3}{2^{2n-2}} + 2^{n-1}}(n-1)$$
$$D_{coe}GA(\Gamma) = 2\frac{2^{n-1}2^{n}}{2^{2n-1}}(n-1) + 2\frac{2^{2n-2}}{2^{n-1} + 2^{n-1}}(n-1) + 2\frac{\sqrt{2^{2n-2} + 2^{n-1}}}{2^{2n-2}}(n-1)$$
$$D_{coe}AG(\Gamma) = \frac{2^{n} + 2^{n-1}}{2\sqrt{2^{2n-1}}} + \frac{2^{n-1} + 2^{n-1}}{2\sqrt{2^{2n-2}}} + \frac{2^{n-1} + 2^{n-1} - 1}{2\sqrt{2^{2n-2}} - 2^{n-1}}$$

Proof. Let H_n be a Helm graph with $n \geq 4$.

Case 1: If n is even. In this case there are $n-1$ vertices of even first degree $d(v)$. Hance the number of Co-even dominating sets produced from all the different substitutions for $n-1$ vertices are $\binom{n-1}{1} + \binom{n-1}{2} + ... + \binom{n-1}{n-1} = 2^{n-1} - 1$. Note, that all vertices of odd first degree are present in all $2^{n-1} - 1$ Co-even dominating sets. Also, there is another Co-even dominating sets contains only the vertices of $d(v)$ is odd. Then $T_{coe}(G) = 2^{n-1} - 1 + 1 = 2^{n-1}$. Hance, by using the definition of Co-even domination degree, we get

$$d_{coe}(v) = \begin{cases} 2^{n-2}, \text{ if } d(v) \textit{ is even;} \\ 2^{n-1}, \text{ if } d(v) \textit{ is odd .} \end{cases}$$

Therefore by Definition 2.2, we get

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^{n-2}+2^{n-1}-2}{2^{2n-3}}}(n-1) + \sqrt{\frac{2^{n-2}+2^{n-2}-2}{2^{4n-4}}}(n-1)$$

Let E_1 be the sets of all edges that connect vertices whose first degrees are odd with vertices whose first degrees are even. E_2 be the sets of all edges that connect vertices whose first degrees are even with vertices whose first degrees are even. Therefore

$$\begin{aligned} D_{coe}ABC(\Gamma) &= \sqrt{\frac{2^{n-2}+2^{n-1}-2}{2^{2n-3}}}(n-1) + \sqrt{\frac{2^{n-2}+2^{n-2}-2}{2^{4n-4}}}(n-1) \\ D_{coe}GA(\Gamma) &= 2\frac{2^{2n-3}}{2^{n-1}+2^{n-2}}(n-1) + 2\frac{2^{2n-4}}{2^{n-1}+2^{2n-4}}(n-1) \\ D_{coe}AG(\Gamma) &= \frac{2^{n-2}+2^{n-1}}{2\sqrt{2^{2n-3}}}(n-1) + \frac{2^{n-2}+2^{n-1}}{2\sqrt{2^{2n-4}}}(n-1) \end{aligned}$$

Case 2: If n is odd in this case there are n vertices of even first degree. Hance the number of Co-even dominating sets is equal the number of different substitutions for n is $\binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n} = 2^n - 1$ Co-even dominating sets. Note that all pendent vertices are present in all $2^n - 1$ Co-even dominating sets. Also, there is another Co-even dominating sets that contains all pendent vertices with the center vertex. Hence $T_{coe}(H_n) = 2^n$. Applying the definition of Co-even dominating degree, we get

$$d_{coe}(v) = \begin{cases} 2^n, \text{ if } d(v) \textit{ is odd;} \\ 2^{n-1}, \text{ if } d(v) \textit{ is even expet the center vertices;} \\ 2^{n-1}+1, \text{if } v \textit{ is even and } v \textit{ the center vertices.} \end{cases}$$

Let A be the set of all pendent vertex,and B the set of all vertices on the cycle. Applying the Definition 2.2, we get

$$D_{coe}ABC(\Gamma) = \sqrt{\frac{2^n+2^{n-1}-2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1}+2^{n-1}-2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1}+2^{n-1}-3}{2^{2n-2}}+2^{n-1}}(n-1)$$

Let $E_1 = \{uv : d_{coe}(u) = 2^n,\ d_{coe}(v) = 2^{n-1}\}$, $E_2 = \{u,v : d_{coe}(u) = 2^{n-1},\ d_{coe}(v) = 2^{n-1}\}$, $E_3 = \{u,v : d_{coe}(u) = 2^{n-1},\ d_{coe}(v) = 2^{n-1}+1\}$, where $|E_1| = n-1$, $|E_2| = n-1$, $|E_3| = n-1$. Applying the Definition 2.2, we get

$$\begin{aligned} D_{coe}ABC(\Gamma) &= \sqrt{\frac{2^n+2^{n-1}-2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1}+2^{n-1}-2}{2^{2n-1}}}(n-1) + \sqrt{\frac{2^{n-1}+2^{n-1}-3}{2^{2n-2}}+2^{n-1}}(n-1) \\ D_{coe}GA(\Gamma) &= 2\frac{2^{n-1}2^n}{2^{2n-1}}(n-1) + 2\frac{2^{2n-2}}{2^{n-1}+2^{n-1}}(n-1) + 2\frac{\sqrt{2^{2n-2}+2^{n-1}}}{2^{2n-2}}(n-1) \\ D_{coe}AG(\Gamma) &= \frac{2^n+2^{n-1}}{2\sqrt{2^{2n-1}}} + \frac{2^{n-1}+2^{n-1}}{2\sqrt{2^{2n-2}}} + \frac{2^{n-1}+2^{n-1}-1}{2\sqrt{2^{2n-2}-2^{n-1}}} \end{aligned}$$

□

References

[1] A. Alsinai, V. Mathad, Ahmed A. Omran and H. Ahmed, *Co-Even Domination Zagreb Indices Of Graph*, comencated, (2021).

[2] Ammar Alsinai, Anwar Alwardi, Mohammed Reza Fraahani and D. Soner Nandappa, *On the ψ_k-polynomial of graph*, Eurasian Chemical Communcations, 3(4)(2021), 219-226.

[3] A. Alsinai, B. Basavanagoud, M. Sayyed and M. R. Farahani, *Sombor index of some nanostructures*, Journal of Prime Research in Mathematics, 17(2)(2021), 123-133.

[4] H. Ahmed, A. Alsinai, A. Khan and H. A. Othman, *The Eccentric Zagreb Indices for the Subdivision of Some Graphs and Their Applications*, Appl. Math., 16(3)(2022), 467-472.

[5] A. Alsinai, A. Saleh, H. Ahmed, L. N. Mishra and N. D. Soner, *On fourth leap Zagreb index of graphs*, Discrete Mathematics, Algorithms and Applications, (2022).

[6] A. Alsinai, A. Alwardi and N. D. Soner, *On Reciprocals Leap indices of graphs*, International Journal of Analysis and Applications, 19(1)(2021), 1-19.

[7] A. Alsinai, A. Alwardi and N. D. Soner, *Topological Properties of Graphene Using ψ_k- polynomial*, Proceedings of the Jangjeon Mathematical Society, 24(2021), 375-388.

[8] A. Alsinai, H. Ahmed, A. Alwardi and N. D. Soner, *HDR Degree Bassed Indices and Mhr-Polynomial for the Treatment of COVID-19*, Biointerface Research in Applied Chemistry, 12(6)(2021), 7214-7225.

[9] A. Alsinai, H. M. U. Rehman, Y. Manzoor, M. Cancan, Z. Tas and M. R. Farahani, *Sharp upper bounds on forgotten and SK indices of cactus graph*, Journal of Discrete Mathematical Sciences and Cryptography, (2022), 1-22.

[10] S. Hussain, A. Alsinai, D. Afzal, A. Maqbool, F. Afzal and M. Cancan, *Investigation of Closed Formula and Topological Properties of Remdesivir (C27H35N6O8P)*, Chem. Methodol., 5(6)(2021), 485-497.

[11] A. Hasan, M. H. A. Qasmi, A. Alsinai, M. Alaeiyan, M. R. Farahani and M.Cancan, *Distance and Degree Based Topological Polynomial and Indices of X-Level Wheel Graph*, Journal of Prime Research in Mathematics, 17(2)(2021), 39-50.

[12] K. S. AlDzhabri, A. A. Omran and M. N. Al-Harere, *DG-domination topology in Digraph*, Journal of Prime Research in Mathematics, 17(2)(2021), 93-100.

[13] M. Azari and A. Iranmanesh, *Harary Index of Some Nano-structures*, MATCH Commun. MAath. Comput. Chem., 71(2014), 373-382.

[14] T. Al-Fozan, P. Manuel, I. Rajasingh and R. Sundara Rajan, *Computing Szeged Index of Certain Nanosheets Using Partition Technique*, MATCH Commun. Math. Comput. Chem., 72(2014), 339-353.

[15] F. Afzal, A. Alsinai, S. Hussain, D. Afzal, F. Chaudhry and M. Cancan, *On topological aspects of silicate network using M-polynomial*, Journal of Discrete Mathematical Sciences and Cryptography, 24(7)(2021), 2109-2119.

[16] A. Alsinai, A. Alwardi and N. D. Soner, *On reciprocals leap function of graph*, Journal of Discrete Mathematical Sciences and Cryptography, 24(2)(2021), 307-324.

[17] A. Alsinai, A. Alwardi, H. Ahmed and N. D. Soner, *Leap Zagreb Indices For The Central Graph Of Graph*, Journal of Prime Research in Mathematics, 12(2)(2021), 100-110.

[18] A. M. Hanan Ahmed, A. Alwardi and M. R. Salestina, *On domination topological indices of graphs*, InternationalJournal of Analysis and Applications, 19(1)(2020), 47-64.

[19] A. Alsinai, A. Alwardi and N. D. Soner, *On Reciprocal leap function of graph*, Journal of Discrete Mathematical Sciences and Cryptography, 24(2)(2021), 307-324.

[20] I. Gutman and N. Trinajstic, *Graph theory and molecular orbitals Total f-electron energy of alternant hydrocarbons*, Chem. Phys. Lett., 17(1972), 535-538.

[21] I. Gutman, B. Ruscic, N. Trinajstic and C. F. Wilcox, *Graph theory and molecular orbitals XII Acyclic polyenes*, J. Chem. Phys., 62(1975), 3399-3405.

[22] I. Gutman, E. Milovanovic and I. Milovanovic, *Beyond the Zagreb indices*, AKCE Int. J. Graph Combin., (2018).

[23] S. Javaraju, H. Ahmed, A. Alsinai and N. D. Soner, *Domination topological properties of carbidopa-levodopa used for treatment Parkinson's disease by using* φ_p*-polynomial*, 3(9)(2021), 614-621.

[24] S. Javaraju, A. Alsinai, A. Alwardi, H. Ahmed and N. D. Soner, *Reciprocal leap indices of some wheel related graphs*, Journal of Prime Research in Mathematics, 17(2)(2021), 101-110.

[25] A. A. Jabor and A. A. Omran, *Topological domination in graph theory*, In AIP Conference Proceedings, 2334(1).

[26] M. M. Shalaan and A. A. Omran, *Co-even Domination In Graph*, International Journal of Control and Automation, 13(3)(2020), 330-334.

[27] H. J. Yousif and A. A. Omran, *Inverse 2- Anti Fuzzy Domination in Anti fuzzy graphs*, IOP Publishing Journal of Physics: Conference Series, 1818(2021), 012072.