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Odd Sum Labeling of Tree Related Graphs

Research Article

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Abstract: An injective function $f : V(G) \to \{0,1,2,...,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all $uv \in E(G)$ is a bijective and $f^*(E(G)) = \{1,3,5,...,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of some more graphs.

MSC: 05C78.

Keywords: Odd sum Labeling, Odd Sum Graph, Twig graph, H-graph.

1. Introduction

Through out this paper, by a graph we mean a finite undirected simple graph. Let $G(V, E)$ be a graph with p vertices q edges. For notation and terminology, we follow [7]. For detailed survey of graph labeling we refer to Gallian [4].

In [9], the concept of mean labeling was introduced and further studied in [5, 6]. An injective function $f : V(G) \to \{0,1,2,...,q\}$ is said to be a mean labeling if the induced edge labeling f^* defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if f(u)+f(v) is even} \\ \frac{f(u)+f(v)+1}{2}, & \text{if f(u)+f(v) is odd} \end{cases}$$

is injective and $f^*(E(G)) = \{0,1,2...,q\}$.

A graph G is said to be an odd mean graph if there exists an injective function $f : V(G) \to \{0,1,2,...,2q-1\}$ such that the induced map $f^*(E(G)) = \{1,3,5,...,2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if f(u)+f(v) is even} \\ \frac{f(u)+f(v)+1}{2}, & \text{if f(u)+f(v) is odd} \end{cases}$$

is a bijection[8].

In [1], the concept of odd sum labeling was introduced and studied [2,3]. An injective function $f : V(G) \to \{0,1,2,...,q\}$ is an odd sum labeling if the induced edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all $uv \in E(G)$ is a bijective and $f^*(E(G)) = \{1,3,5,...,2q-1\}$. A graph is said to be an odd sum graph if it admits an odd sum labeling. In this paper we investigate odd sum labeling of tree related graphs.

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2. Main Results

Definition 2.1. *The H Graph of the path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n+1}{2}}$ and $v_{\frac{n}{2}}$ if n is even.*

Theorem 2.2. *The graph $H_n(n \geq 3)$ is a odd sum graph.*

Proof. Let $\{v_i, v_i',\ 1 \leq i \leq n\}$ be the vertices and $\{e_i, e_i', e,\ 1 \leq i \leq n-1\}$ be edges which denoted as in Figure 1. First we label the vertices as follows.

Define $f : V \to \{0, 1, 2, ..., q\}$ by

For $1 \leq i \leq n-1$

$$f(v_i) = i - 1$$

$$f(v_i') = n + i - 1$$

Then the induced edge labels are:

For $1 \leq i \leq n-1$

$$f^*(e_i) = 2i - 1$$

$$f^*(e_i') = 2n + 2i - 1$$

$$f^*(e) = 2n - 1$$

Therefore $f^*(E) = \{1, 3, 5, ..., 2q-1\}$. So f is a odd sum labeling and hence, the graph $H_n(n \geq 3)$ is a odd sum graph. Odd sum graph H_3 is shown in Figure 2.

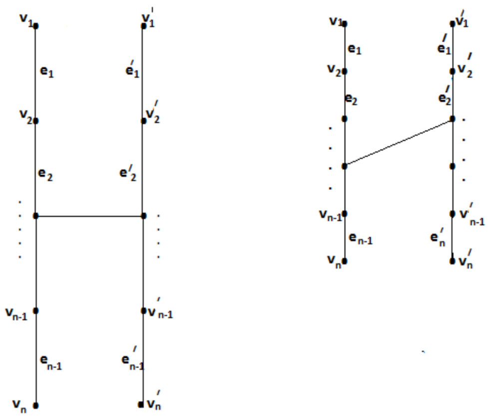

Figure 1. **Ordinary labeling of H_n**

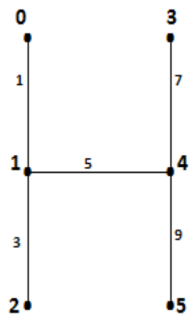

Figure 2. **Odd sum labeling of H_3**

□

Theorem 2.3. *The graph $H_m \Theta nK_1(n \geq 2, m \geq 3)$ is a odd sum graph.*

Proof. Let $\{v_{ij}, v_{ij}', 1 \leq i \leq m, 1 \leq j \leq n, v_i, v_i', 1 \leq i \leq m\}$ be the vertices and $\{e_{ij}, e_{ij}', 1 \leq i \leq m-1, 1 \leq j \leq n,$ $e, e_i, e_i', 1 \leq i \leq m-1\}$ be the edges which are denoted as in Figure 3. First we label the vertices as follows. Define $f : V \to \{0, 1, 2, ..., q\}$ by

For $1 \leq i \leq m$, $1 \leq j \leq n$

$$f(v_{ij}) = \begin{cases} 2(j-1)+(n+1)(i-1) & i \ is \ odd \\ 2j+1+(n+1)(i-2) & i \ is \ even \end{cases}$$

$$f(v_i) = \begin{cases} (n+1)(i-1)+1 & i \ is \ odd \\ (n+1)(i-2)+2n & i \ is \ even \end{cases}$$

$$f(v'_{ij}) = \begin{cases} m(n+1)-n+2j+(n+1)(i-1) & i \ is \ odd \\ m(n+1)+n+2j-1+(n+1)(i-2) & i \ is \ even \end{cases}$$

$$f(v'_i) = \begin{cases} m(n+1)+n-1+(n+1)(i-1) & i \ is \ odd \\ m(n+1)+n+2+(n+1)(i-2) & i \ is \ even \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq m-1$, $1 \leq j \leq n$

$$f^*(e_{ij}) = 2j-1+2(n+1)(i-1)$$

$$f^*(e_i) = 2n+1+2(n+1)(i-1)$$

$$f^*(e) = 2m(n+1)-1$$

$$f^*(e'_{ij}) = 2m(n+1)+2j-1+2(n+1)(i-1)$$

$$f^*(e'_i) = 2m(n+1)+2n+1+2(n+1)(i-1)$$

Therefore $f^*(E) = \{1, 3, 5, ..., 2q-1\}$. So f is a odd sum labeling and hence, the graph $H_m \Theta nK_1 (n \geq 2, m \geq 3)$ is a odd sum graph. Odd sum of the graph $H_6 \Theta 4K_1$ is shown in Figure 4.

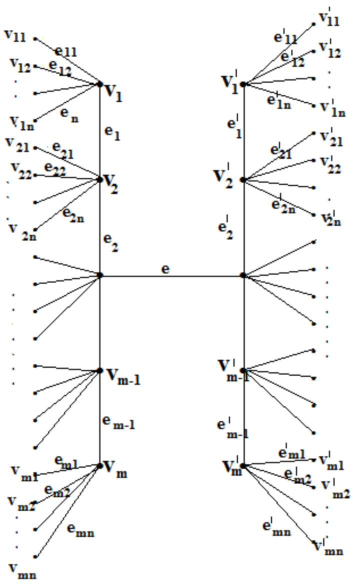

Figure 3. **Ordinary labeling of $H_m \Theta nK_1$**

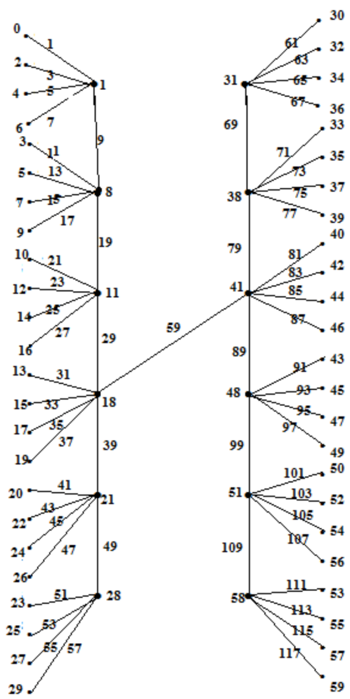

Figure 4. **Odd sum labeling of $H_6 \Theta 4K_1$**

□

Definition 2.4. *A twig is a tree obtained from a path by attaching exactly two pendent edges to each internal vertex of the path.*

Theorem 2.5. *The twig graph $TW(n)(n \geq 4)$ is a odd sum graph.*

Proof. Let $\{u_i, 1 \leq i \leq n, v_i, v_i', 1 \leq i \leq n-2\}$ be the vertices and $\{a_i, a_i', \ 1 \leq i \leq n-2; e_i, 1 \leq i \leq n-1\}$ be the edges which are as in Figure 5. First we label the vertices as follows

Define $f : V \rightarrow \{0, 1, 2, ..., q\}$ by

For $1 \leq i \leq n$

$$f(u_i) = \begin{cases} 3(i-1) & i \text{ is odd} \\ 3i-5 & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 3i+1 & i \text{ is odd} \\ 3i-1 & i \text{ is even} \end{cases}$$

$$f(v_i') = \begin{cases} 3i-1 & i \text{ is odd} \\ 3(i-1) & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n-1$ $\qquad f^*(e_i) = 6i - 5$

For $1 \leq i \leq n-2$ $\qquad f^*(a_i) = 6i - 3, \ f^*(a_i') = 6i - 1.$

Therefore $f^*(E) = \{1, 3, 5, ..., 2q-1\}$. So f is a odd sum labeling and hence, the graph $TW(n)(n \geq 4)$ is a odd sum graph. Odd sum of the graph $TW(5)$ is shown in Figure 6.

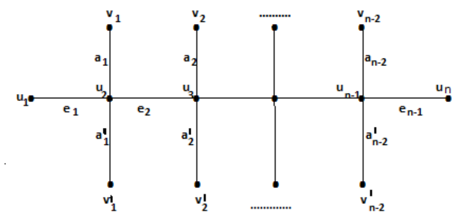

Figure 5. **Ordinary labeling of TW(n).**

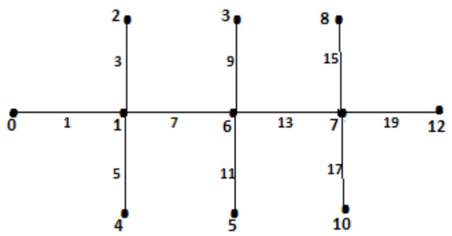

Figure 6. **Odd sum labeling of TW(5).**

□

Definition 2.6. *Let A' be the collection of paths P_n^i where n is odd and $P_n^i = u_1', u_2', ..., u_n'$, $(1 \leq i \leq m)$. Let G be the graph obtained from A with $V(G) = \bigcup_{i=1}^{n} V(P_n^i)$ and $E(G) = \bigcup_{i=1}^{n} E(P_n^i) \bigcup \left\{ u^i_{\frac{n+1}{2}} \ u^{i+1}_{\frac{n+1}{2}} \ : \ 1 \leq i \leq m-1 \right.$ the graph G denoted by $P(m, n)$.*

Theorem 2.7. *The graph $P(m, n)$ $(m \geq 2 \,\&\, n \geq 3)$ is a odd sum graph.*

Proof. Let $\{u_{ij}, \ 1 \leq i \leq m, \ 1 \leq j \leq n\}$ be the vertices and $\{e_{ij}, \ 1 \leq i \leq m, \ 1 \leq i \leq n-1 e_i, \ 1 \leq i \leq m-1\}$ be the edges which are denoted as in Figure 7. First we label the vertices as follows

Define $f : V \rightarrow \{0, 1, 2, ..., q\}$

For $1 \leq i \leq m; 1 \leq i \leq n$

$$f(u_{ij}) = m(j-1) + (i-1)$$

Then the induced edge labels are:

For $1 \leq i \leq m$; $1 \leq j \leq n-1$

$$f^*(e_{ij}) = 2m(j-1) + 2i - 1$$

Therefore $f^*(E) = \{1, 3, 5, ..., 2q-1\}$. So f is a odd sum labeling and hence, the graph $P(m, n)$ $(m \geq 2$ & $n \geq 3)$ is a odd sum graph. Odd sum of the graph $P(4, 2)$ is shown in Figure 8.

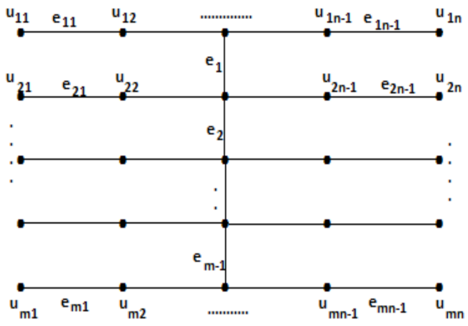

Figure 7. **Ordinary labeling of P(m, n).**

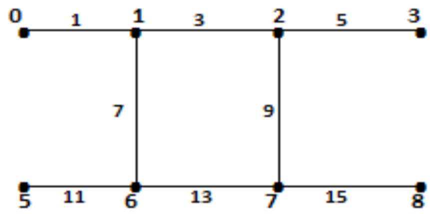

Figure 8. **Odd sum labeling of P(4,2).**

□

Theorem 2.8. *The graph (P_m, S_n) $(m \geq 4\&n \geq 2)$ is a odd sum graph.*

Proof. Let $\{u_i, u'_i, 1 \leq i \leq m, u_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertices and $\{a_i, a'_i, 1 \leq i \leq m-1, e_{ij}, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the edges which are denoted as in Figure 9. First we label the vertices as follows

Define $f : V \to \{0, 1, 2, ..., q\}$ by

For $1 \leq i \leq m$; $1 \leq j \leq n$

$$f(u_{ij}) = \begin{cases} 2(j-1) + (n+2)(i-1) & j \text{ is odd} \\ 2j + 3 + (n+2)(i-2) & j \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} 2n + (n+2)(i-1) & i \text{ is odd} \\ (n+2)(i-2) + 3 & i \text{ is even} \end{cases}$$

$$f(u'_i) = \begin{cases} (n+2)(i-1) + 1 & i \text{ is odd} \\ 2n + 2 + (n+2)(i-2) & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n$; $1 \leq j \leq m$

$$f^*(e_{ij}) = \begin{cases} 2(n+2)(i-1) + 2j - 1 & i \text{ is odd} \\ 2(n+2)(i-2) + 2n + 2j + 5 & i \text{ is even} \end{cases}$$

For $1 \leq i \leq m-1$ $\quad f^*(a_i) = 2(n+2)(i-1) + 2n + 3$

$$f^*(a'_i) = \begin{cases} (2n+3) & i \text{ is odd} \\ 2(n+2)(i-1) + 2n + 5 & i \text{ is even} \end{cases}$$

Therefore $f^*(E) = \{1, 3, 5, ..., 2q-1\}$. So f is a odd sum labeling and hence, the graph (P_m, S_n) is a odd sum graph. Odd sum of the graph $(P_4, S_3)$is shown in Figure 10.

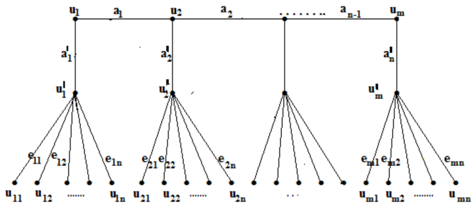

Figure 9. **Ordinary labeling of** (P_m, S_n)**.**

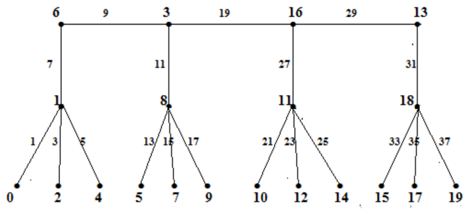

Figure 10. **Odd sum labeling of** (P_4, S_3)**.**

□

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