



Magnetohydrodynamic Flow of Viscous Incompressible Fluid Surrounded by a Porous Media Between Two Non-Coincident Rotating Disks

Research Article

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Abstract: Analytical investigations of the Navier-Stokes Equations examine the magnetohydrodynamic flow of a viscous incompressible fluid between two non-coincident disks rotating with the same angular velocity Ω . The governing equation of motions are coupled non linear partial differential equations together with the boundary conditions, are reformed into the linear ordinary differential equations that has been studied with the assumptions. The results have been obtained for the velocities, shearing stress and torque for several values of rotational parameter, magnetic parameter and porosity parameter. The effects of the several parameters on the flow field are presented and explained graphically.

Keywords: Magnetohydrodynamic Flow, Non-Coincident Disk, Porous Medium, Magnetic Parameter, Rotation Parameter.

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1. Introduction

Magnetohydrodynamic flow of viscous incompressible fluid has attracted the attention of various researchers due to its large applications in the field of engineering, geophysical applications, petroleum technology, chemical engineering, agriculture engineering etc. MHD flow of viscous incompressible fluid due to rotation of non-coincident disks and the fluid at infinity has been analyses and studied by many researchers. MHD flow of an oldroyd-B fluid between eccentric rotating disks was studied by Ersoy [1], Flow of viscoelastic fluids between rotating disks was studied by Rajagopal [2]. Hydromagnetic flow between two rotating disks with non-coincident parallel axes of rotation was studied by Mohanty [3]. Kanch and Jana [4] have insvestigate the Hall Effects on hydromagnetic flow between two disks with non-coincident parallel axes of rotation. Maji, Ghara, Jana and Das have investigated the Unsteady MHD flow between two eccentric rotating disks in [5]. Magnetohydrodynamic flow with reference to non-coaxial rotation of a porous disk and a fluid at infinity was studied by Guria, Das, Jana and Ghosh [6]. Das, Maji, Guria and Jana [7] have study the Hall Effects on unsteady MHD flow between two disks with non-coincident parallel axes of rotation. Jana and Ghosh have investigate the Hydrodynamic flow between two non coincident rotating disks embedded in porous media in [8].

The present research deals with the magnetohydrodynamic flow of viscous incompressible fluid embedded in porous medium between two non-coincident rotating disks. This paper is the study of combined effect of rotation, magnetic field and porosity on the fluid. Here the disks at l distance apart are rotating with the same angular velocity. The present problem attentively states that the motion in the mid plane appears as symmetric between the two disks.

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2. Mathematical Formulation and Solutions

Here the steady viscous incompressible fluid flow is considered to be symmetric. The fluid is contained between two non-coincident parallel uniformly rotating disk (Ω angular velocity) at l distance apart embedded in a porous medium. We choose the system of cylindrical polar co-ordinates (r, θ, z) with z -axis perpendicular to the plane. The upper disk at $z = h$ is rotating at a constant angular velocity (Ω) with the axis of rotation lies to the right of the z -axis and while the axis of rotation lies to the left of the z -axis of the lower disk at $z = 0$ with same angular velocity. A uniform magnetic field B_0 is applied perpendicular to the disks. The Navier-Stokes equation of motion and equation of continuity with velocity vector q , fluid pressure p , density ρ , viscosity μ , permeability k^* and conductivity σ as follows:

$$\rho (q \cdot \nabla) q = -\nabla p + \mu \nabla^2 q - (\mu/k^*) q - (\sigma B_0^2/\rho) q \quad (1)$$

$$\nabla \cdot q = 0 \quad (2)$$

The boundary conditions are

$$\left. \begin{aligned} q_r &= -\frac{1}{2}\Omega l \cos \theta, \quad q_\theta = \Omega \left(r + \frac{1}{2}l \sin \theta \right), \quad q_z = 0; \quad \text{at } z = 0 \\ q_r &= \frac{1}{2}\Omega l \cos \theta, \quad q_\theta = \Omega \left(r - \frac{1}{2}l \sin \theta \right), \quad q_z = 0; \quad \text{at } z = h \end{aligned} \right\} \quad (3)$$

In the reference of above boundary conditions the velocity components are suggested as follows

$$\left. \begin{aligned} q_r &= f(z) \cos \theta + g(z) \sin \theta \\ q_\theta &= \Omega r + g(z) \cos \theta - f(z) \sin \theta \\ q_z &= 0 \end{aligned} \right\} \quad (4)$$

Using equation (4) we have equation of motion from equation (1)

$$-\Omega (\Omega r + g \cos \theta - f \sin \theta) = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} \left(\frac{\partial^2 f}{\partial z^2} \cos \theta + \frac{\partial^2 g}{\partial z^2} \sin \theta \right) - \frac{\mu}{\rho k^*} (f \cos \theta + g \sin \theta) - \frac{\sigma B_0^2}{\rho} (f \cos \theta + g \sin \theta) \quad (5)$$

$$\Omega (f \cos \theta + g \sin \theta) = -\frac{1}{\rho} \frac{\partial p}{\partial \theta} + \frac{\mu}{\rho} \left(\frac{\partial^2 g}{\partial z^2} \cos \theta - \frac{\partial^2 f}{\partial z^2} \sin \theta \right) - \frac{\mu}{\rho k^*} (\Omega r + g \cos \theta - f \sin \theta) - \frac{\sigma B_0^2}{\rho} (\Omega r + g \cos \theta - f \sin \theta) \quad (6)$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \quad (7)$$

After eliminating p from equation (5)-(7), we obtain

$$\Omega \frac{\partial f}{\partial z} = \frac{\mu}{\rho} \frac{\partial^3 g}{\partial z^3} - \frac{\mu}{\rho k^*} \frac{\partial g}{\partial z} - \frac{\sigma B_0^2}{\rho} \frac{\partial g}{\partial z} \quad (8)$$

$$-\Omega \frac{\partial g}{\partial z} = \frac{\mu}{\rho} \frac{\partial^3 f}{\partial z^3} - \frac{\mu}{\rho k^*} \frac{\partial f}{\partial z} - \frac{\sigma B_0^2}{\rho} \frac{\partial f}{\partial z} \quad (9)$$

Or

$$\Omega f + C_1 = \frac{\mu}{\rho} \frac{\partial^2 g}{\partial z^2} - \frac{\mu}{\rho k^*} g - \frac{\sigma B_0^2}{\rho} g \quad (10)$$

$$-\Omega g + C_2 = \frac{\mu}{\rho} \frac{\partial^2 f}{\partial z^2} - \frac{\mu}{\rho k^*} f - \frac{\sigma B_0^2}{\rho} f \quad (11)$$

Here C_1 and C_2 are unknown constants and the corresponding the boundary conditions are

$$\left. \begin{aligned} f(0) &= -\frac{1}{2}\Omega l, \quad g(0) = 0; \quad \text{at } z = 0 \\ f(h) &= \frac{1}{2}\Omega l, \quad g(h) = 0; \quad \text{at } z = h \end{aligned} \right\} \quad (12)$$

Combining the Equations (10) and (11), we have

$$i\Omega F + C = \frac{\mu}{\rho} \frac{\partial^2 F}{\partial z^2} - \frac{\mu}{\rho k^*} F - \frac{\sigma B_0^2}{\rho} F \quad (13)$$

Where

$$F = f + ig, \quad C = C_1 + iC_2 \quad (14)$$

Due to the symmetric flow C can be chosen as zero then the Equation (13) and the boundary conditions reduces to

$$i\Omega F = \frac{\mu}{\rho} \frac{\partial^2 F}{\partial z^2} - \frac{\mu}{\rho k^*} F - \frac{\sigma B_0^2}{\rho} F \quad (15)$$

$$F(0) = -\frac{1}{2}\Omega l, \quad F(h) = \frac{1}{2}\Omega l \quad (16)$$

Now let us introduce dimensionless variables

$$F_1 = \frac{F}{l\Omega}, \quad \eta = \frac{z}{h}, \quad K^2 = \frac{\rho\Omega h^2}{\mu}, \quad M^2 = \frac{\sigma B_0^2 h^2}{\mu}, \quad \sigma = \frac{k^*}{h^2} \quad (17)$$

Now the non dimensional form of the governing equation and the boundary conditions are

$$\frac{d^2 F_1}{d\eta^2} - \left(\frac{1}{\sigma} + M^2 + iK^2 \right) F_1 = 0 \quad (18)$$

$$F_1(0) = -\frac{1}{2}, \quad F_1(1) = \frac{1}{2} \quad (19)$$

The solution of the Equation (18) subject to the following form

$$F_1 = \frac{1}{2} \left[\frac{\sinh(a\eta) - \sinh(1-\eta)a}{\sinh a} \right] \quad (20)$$

Where $a = \left(\frac{1}{\sigma} + M^2 + iK^2 \right)^{1/2}$. Separating the real and imaginary part, we have

$$f(\eta) = \frac{1}{2(\cosh^2 \alpha \sin^2 \beta + \sinh^2 \alpha \cos^2 \beta)} [\{\cos \beta \eta \sinh \alpha \eta - \cos \beta (1-\eta) \sinh \alpha (1-\eta)\} \cos \beta \sinh \alpha + \{\sin \beta \eta \cosh \alpha \eta - \sin \beta (1-\eta) \cosh \alpha (1-\eta)\} \sin \beta \cosh \alpha] \quad (21)$$

$$g(\eta) = \frac{1}{2(\cosh^2 \alpha \sin^2 \beta + \sinh^2 \alpha \cos^2 \beta)} [\{\sin \beta \eta \cosh \alpha \eta - \sin \beta (1-\eta) \cosh \alpha (1-\eta)\} \cos \beta \sinh \alpha - \{\cos \beta \eta \sinh \alpha \eta - \cos \beta (1-\eta) \sinh \alpha (1-\eta)\} \sin \beta \cosh \alpha] \quad (22)$$

Where

$$\alpha = \frac{1}{\sqrt{2}} \left[\left(\frac{1}{\sigma} + M^2 \right)^2 + \left(\frac{1}{\sigma} + M^2 \right) + K^2 \right]^{1/2} \quad (23)$$

$$\beta = \frac{1}{\sqrt{2}} \left[\left(\frac{1}{\sigma} + M^2 \right)^2 - \left(\frac{1}{\sigma} + M^2 \right) + K^2 \right]^{1/2} \quad (24)$$

In the absence of porosity the above results reduces as

$$f(\eta) = \frac{1}{2(\cosh^2 A \sin^2 B + \sinh^2 A \cos^2 B)} [\{\cos B \eta \sinh A \eta - \cos B (1-\eta) \sinh A (1-\eta)\} \cos B \sinh A + \{\sin B \eta \cosh A \eta - \sin B (1-\eta) \cosh A (1-\eta)\} \sin B \cosh A] \quad (25)$$

$$g(\eta) = \frac{1}{2(\cosh^2 A \sin^2 B + \sinh^2 A \cos^2 B)} [\{\sin B \eta \cosh A \eta - \sin B (1-\eta) \cosh A (1-\eta)\} \cos B \sinh A - \{\cos B \eta \sinh A \eta - \cos B (1-\eta) \sinh A (1-\eta)\} \sin B \cosh A] \quad (26)$$

Where

$$A = \frac{1}{\sqrt{2}}[M^4 + M^2 + K^2]^{1/2} \quad (27)$$

$$B = \frac{1}{\sqrt{2}}[M^4 - M^2 + K^2]^{1/2} \quad (28)$$

3. Results and Discussions

To investigate the effect of magnetic field, rotation and porosity on the flow between two disks surrounded by the porous medium with non-coincident parallel axes of rotation, the primary and secondary velocities are analysed graphically with respect to the various values of magnetic parameter, rotation parameter porosity parameter. Figure 1 and figure 4 shows the effect of the Hartmann number (M) on primary velocity. It is observed that the primary velocity as well as the secondary velocity increases on the left of the axis of rotation with the increase of M and the results is reversed on the right of axis of rotation. Figure 2 and Figure 5 shows that the primary velocity as well as the secondary velocity increases on the left of axis of rotation with increase of rotation parameter and the results observed as reversed on the right. Consequently Figure 3 and 6 shows that the primary velocity as well as the secondary velocity increases on the left of axis of rotation with increase of porosity parameter and the results observed as reversed on the right.

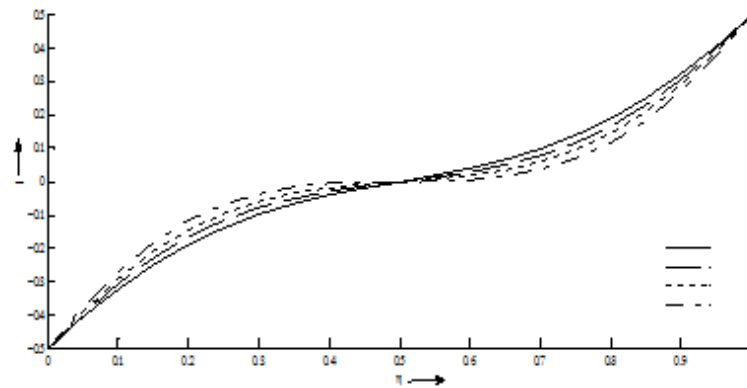


Figure 1: Variation of primary velocity f for $K^2 = 3$ and $\sigma = 0.2$.

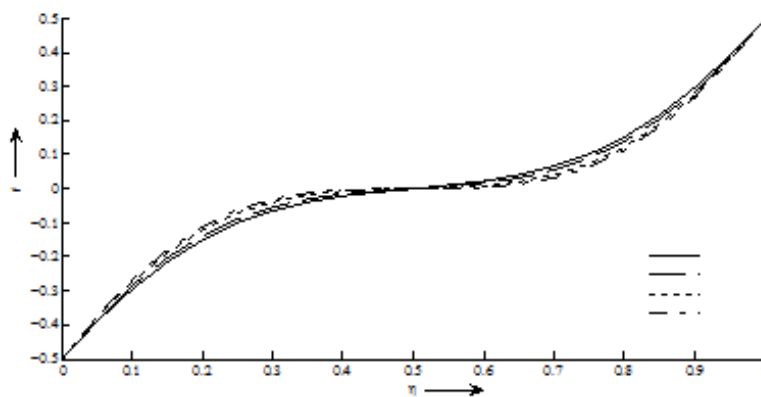


Figure 2: Variation of primary velocity f for $M^2 = 1$ and $\sigma = 0.2$.

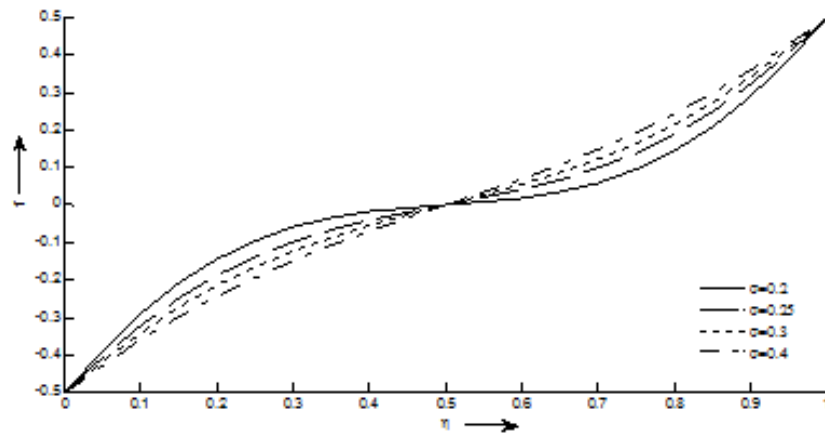


Figure 3: Variation of primary velocity f for $M^2 = 1$ and $K^2 = 1$.

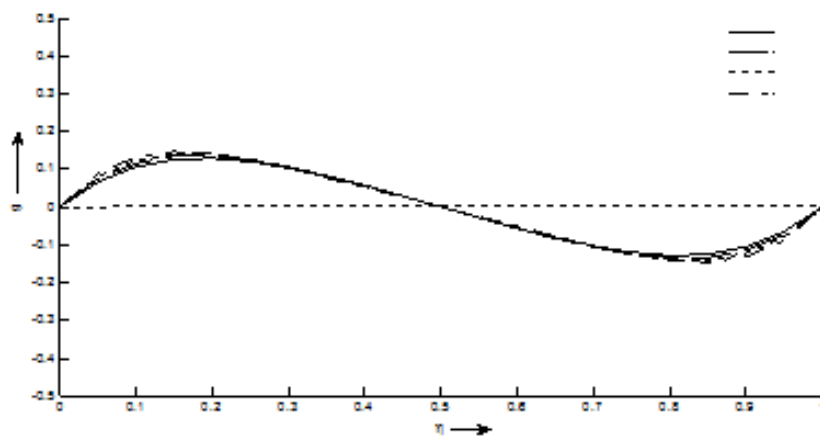


Figure 4: Variation of secondary velocity f for $K^2 = 3$ and $\sigma = 0.2$.

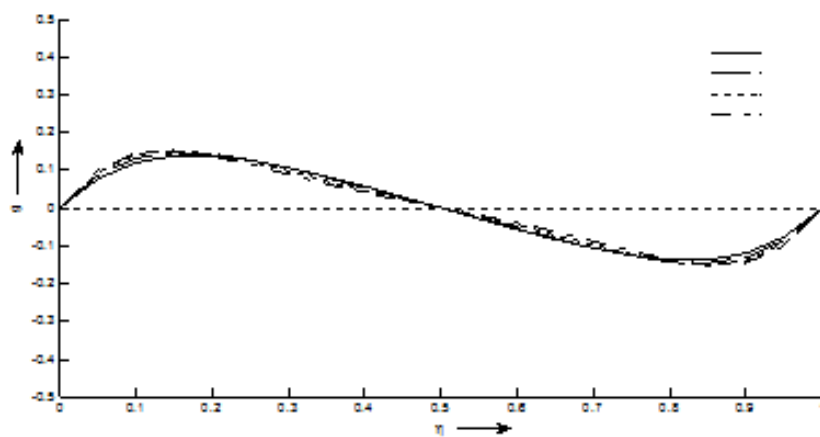


Figure 5: Variation of secondary velocity g for $M^2 = 1$ and $\sigma = 0.2$.

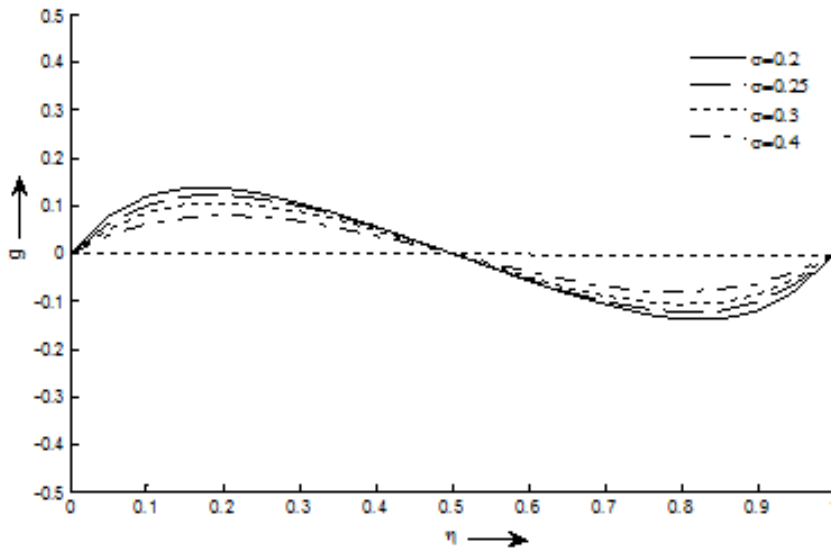


Figure 6: Variation of secondary velocity g for $M^2 = 1$ and $K^2 = 1$.

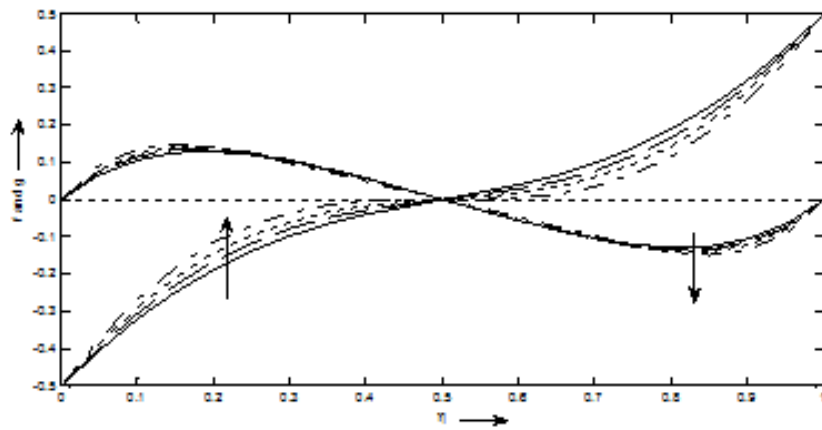


Figure 7: Variation of primary and secondary velocity for $K^2 = 3$ and $\sigma = 0.2$.

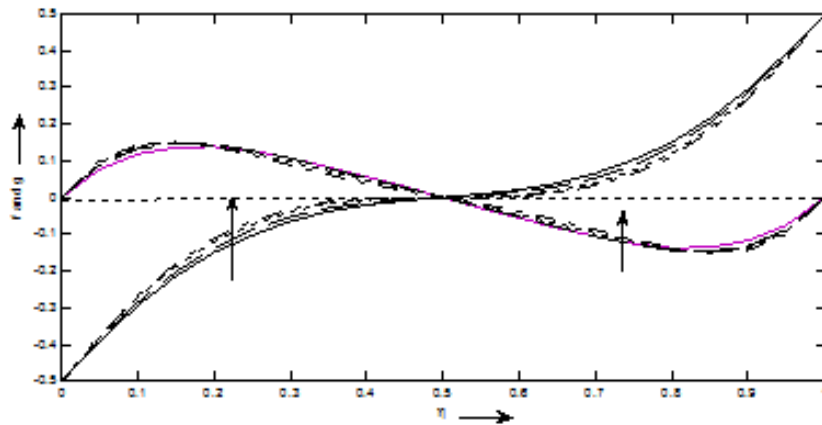


Figure 8: Variation of primary and secondary velocity for $M^2 = 1$ and $\sigma = 0.2$.

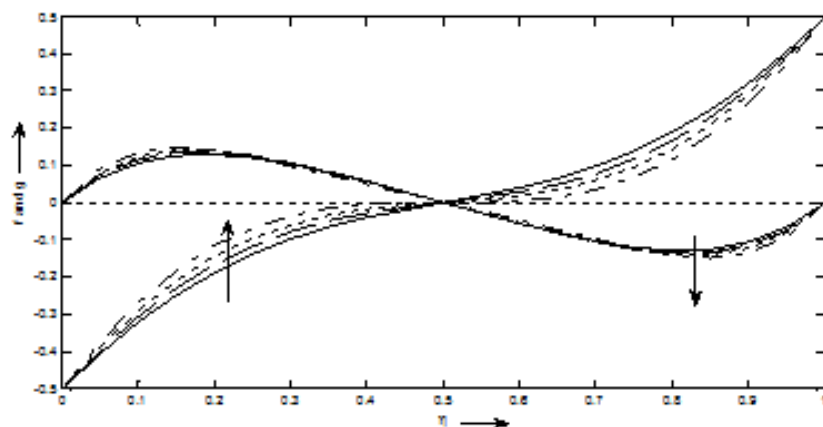


Figure 9: Variation of primary and secondary velocity for $M^2 = 1$ and $K^2 = 1$.

4. Conclusions

Magnetohydrodynamic flow of a viscous incompressible fluid embedded in porous media is investigated and the exact solution of Navier- Stokes equation has been obtained. It is observed that the both primary and secondary velocities increases as the porosity parameter, magnetic parameter and rotation parameter increases on the left side of rotation axis and the results are reversed to the right of the axis.

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