International Journal of Mathematics And its Applications
Volume 4, Issue 4 (2016), 101–110.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Performance Analysis of Two Echelon Perishable Inventory System with Joint Ordering Policy

Research Article

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Abstract: Perishable inventory system has received increasing attentions in the past years. An optimal ordering policy for deteriorating inventory system has become more and more important. This paper develops a model to determine the optimal order quantity that minimizes the total expected cost for perishable items. In addition, the solution methodologies as well as an analysis of results are presented. In this study, we create a scenario where two types of products are considered with two locations (Distributor and Retailer) in such a way that there exists a joint ordering policy. A continuous review perishable inventory model with independent Poisson demand for items at retailer and a direct Poisson demand at distributor is assumed. The items are supplied to the retailers from the distribution center (DC) administrated with exponential lead time. The joint probability disruption of the inventory levels of two products at retailer and the supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

Keywords: Perishable inventory, Two-echelon, Joint ordering policy, Optimization.

1. Introduction

An increasing attention in determining an optimal ordering policy for perishable inventory systems has added a new level of complexity to the task of managing inventory. The traditional inventory management problem works well under deterministic demand assumption. However, when the demand is assumed to be a random variable and items are assumed to be perishable to reflect the real life situations, the classical approach leads to a poor performance and unsatisfactory management. Nahmias [16] has done a comprehensive review of previous research on perishable inventory systems.

He has classified perishable products into two categories: random lifetime and fixed lifetime. He also mentioned in his review that the first analysis of optimal policies for a fixed life perishable commodity was begun by Van Zyl. This topic was extended later by Fries [15], Nahmias [17–20]. Most of the previous studies such as those of Nahmias [20], have concentrated on the periodic review and multi-period lifetime problem with zero lead time. Generally, goods having finite lifetimes are subject to the perishables. Hence, a perishable inventory, such as fashion garments, blood, and drugs, is one in which all the units of one material item in stock will be outdated if not being used before the expiration date, resulting in an additional outdating cost of perished items. Therefore, it is required that the outdating issue is taken into account to reflect the real-life situations. In this work, the focus is placed on the random life time and continuous review (s_i, Q_i) perishable two

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echelon inventory systems with a positive lead time. The objective of this research is to determine the optimal joint ordering quantity that minimizes the expected cost.

The role of inventory management is to coordinate the actions of sales, marketing, production, and purchasing to ensure that the correct level of stocks is held to satisfy customer demand at the lowest possible costs; hence, aiming to balance out supply and demand by regulating the supply of goods in such a way that they match demand conditions as close as possible. Some demand conditions require the application of multiple replenishment orders in continuous-review inventory systems. V. S. S. Yadavalli and J. W. Jourbet [27] highlighted that in such a study the concept of common demand for some products arises for example, when a customer arrives at a shop that sells two brands of soft drinks, the customer would be satisfied by a particular brand with a certain probability. Also, when a supplier is the same for several products under consideration, the retailer will prefer to have simultaneous replenishment for all products due to several reasons like cost considerations and so forth.

Researchers like E. Mohebbi and M. J. M. Posner [13] allowed shortages and assumed that the lost sale policy could be applied when studying such kind of demand conditions. Their demand was not deterministic but stochastic, following a Poisson distribution. Later, E. Mehdizadeh and M. J. Tarokh [14] developed optimal coordination policies for manufacturing and supply divisions. The authors considered a problem where the responsibility of the manufacturing division was to provide a sufficient amount of raw materials so that the required production level could be achieved. V. S. S. Yadavalli, C. de w van Schoor, and S. Udayabaskaran [28] studied such demand conditions and it was under continuous review of multiproduct inventory systems.

Like Mohebi and Posner [13], they also assumed that demand follows a Poisson distribution and that lead time has a probability distribution function. The results obtained were placements of reorders, lost demands, and replenished units after analyzing the imbedded renewal process describing the system. A. A. Taleizadeh et. al. [24] took the EOQ model and extended it to joint replenishment policy for purchasing expensive raw materials. Their cost elements were holding cost, purchasing cost under incremental discount, clearance and fixed order cost, transportation cost, and costs for the next order. Earlier on, a similar problem of joint coordination had been studied by [1]. Instead, the problem was between manufacturing and supplying divisions in a short-life cycle multiproduct environment. In their study, they did not consider a continuous review multiproduct inventory system on which our study focuses. Recently, B. Roushdy, et. al. [21] extended the study on a joint replenishment problem to Pareto analysis by grouping the items under consideration to an ABC classification. They concluded that a continuous review can be carried on the sum of the demands for products in the A classification.

In addition to the field of echelon inventory modeling, in this paper, we attempt to develop a joint ordering model on two products in two-echelon with independent Poisson demands but in a continuous review period. The model becomes a joint ordering policy in the retailer in the sense that the first product cannot be ordered without considering the inventory levels of the second product. In addition, we try to explore some limitations that may be encountered on implementing joint ordering policy.

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [26]. Sivazlian [23] discussed the stationary characteristics of a multi commodity single period inventory system. The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain) when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw-materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris [8]. Clark and Scarf [6] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968. Hadley, G and Whitin, T. M., [7], Naddor.E [14] analyses various inventory Systems. HP's (Hawlett Packard) Strategic Planning and Modeling(SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [16] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [9] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthy et.al., [10] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [11]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

All these papers deal with repairable items with batch ordering. A Complete review was provided by Benito M. Beamon [4]. Sven Axsater [3] proposed an approximate model of inventory structure in SC. He assumed $(S-1, S)$ polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [1] have analyzed two commodity inventory system under various ordering policies. Yadavalli et. al., [27] have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et. al., [28] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhagan et. al. [2] considered analysis of two commodity inventory system with compliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location (Lower echelon). We extend the same in to multi-echelon structure (Supply Chain)with joint ordering policy. The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model

2.1. The Problem Description

The inventory control system considered in this paper is defined as follows. We assume that two perishable products are supplied from warehouse to distribution centre (DC) which adopts (0, M) replenishment policy then the product is supplied to retailer (R) who adopts (s_i, Q_i) policy. The demands at retailer node follows independent Poisson distribution with rate λ_i (i = 1, 2). The items are supplied to the retailers in packs of Q ($Q = Q_1 + Q_2$) where $Q_i (= S_i - s_i)$ from the distribution center (DC) administrated with exponential lead time having parameter μ (>0). The direct demand at distributor node follows Poisson distribution with rate λ_D. It is also assumed that both items are perishes only at retailer node with common exponential perishable rate $\gamma > 0$. The replenishment of items in terms of pockets is made from WH to DC is instantaneous. Demands that occur during the stock out periods are assumed to be lost sales. In this model the maximum inventory level at retailer node S_i is fixed and he reorder level is fixed as s_i for the i-th commodity and the ordering policy is to place order for $Q_i(= S_i - s_i)$ items (i = 1, 2) when both the inventory levels are less than or equal to their respective reorder levels. The maximum inventory level at DC is M ($M = nQ$). The joint probability distribution for both commodities is obtained in steady state cases. The optimization criterion is to minimize the total cost rate incurred at all the location subject to the performance level constrains. According to the assumptions the on hand inventory levels at all the nodes follows a random process.

2.2. Analysis

Let $I_i(t)$, (i = 1, 2, 3) denote the on-hand inventory levels for commodity-1, commodity-2 at retailer node and Distribution Centre (DC) respectively at time t+. From the assumptions on the input and output processes, $I(t) = \{I_i(t); t \geq 0\}$ (i = 1, 2, 3) is a Markov process with state space

$$E = \left\{ \begin{array}{l} (i,k,m)\ /i = S_1, (S_1-1),\ \dots, s_1, (s_1-1),\ \dots, 2, 1, 0.\ , \\ \text{k} \ = S_2, (S_2-1),\ \dots, s_2, (s_2-1),\ \dots, 2, 1, 0. \\ \text{m} \ = \ \text{nQ}, (\text{n}-1)\text{Q},\ \ \dots, \text{Q} \end{array} \right\}$$

Since E is finite and all its states are recurrent non-null, $I(t) = \{I_i(t); t \geq 0\}$ is an irreducible Markov process with state space E and it is an ergodic process. Hence the limiting distribution exists and is independent of the initial state. The infinitesimal generator of this process

$$R = (a(i,k,m : j,l,n))_{(i,k,m),(j,l,n)\in E}$$

can be obtained from the following arguments.

1. The arrival of a demand or perish of an item for commodity-1 at retailer node makes a state transition in the Markov process from (i,k,m) to $(i-1,k,m)$ with intensity of transition $\lambda_1 + i\gamma$.

2. The arrival of a demand or perish of an item for commodity-2 at retailer node makes a state transition in the Markov process from (i,k,m) to $(i,k-1m)$ with intensity of transition $\lambda_2 + k\gamma$.

3. The arrival of a demand at distributor node makes a state transition in the Markov process from (i,k,m) to $(i,km-Q)$ with intensity of transition λ_D.

4. Joint Replenishment of inventory for commodity-1 and commodity-2 at retailer node makes a state transition in the Markov process from (i,k,m) to $(i+Q_1, k+Q_2, m-Q)$ with intensity of transition μ, where $Q = Q_1 + Q_2$.

The infinitesimal generator R is given by

$$R = \begin{pmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ 0 & 0 & A & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \end{pmatrix}$$

The entities of matrix R are given by

$$[R]_{pxq} = \begin{cases} A \;\; if \;\; p=q & ;\ \mathrm{p} \ = \ \mathrm{nQ}\ ,(\mathrm{n}-1)\mathrm{Q},\ \ldots\ ,\ 3\mathrm{Q},\ 2\mathrm{Q}. \\ B \;\; if \;\; p=q+Q & ;\ \mathrm{p} \ = \ \mathrm{nQ}\ ,(\mathrm{n}-1)\mathrm{Q},\ \ldots\ ,\ 3\mathrm{Q},\ 2\mathrm{Q}. \\ B \;\; if \;\; p=q-(n-1)Q & ;\ \mathrm{p} \ = \ \mathrm{Q}. \\ 0 \quad \text{otherwise} \end{cases}$$

The sub matrices of matrix R are given by

$$[A]_{pxq} = \begin{cases} A_1 \;\; if \;\; p=q & ;\ \mathrm{p} \ = S_2,(S_2-1),(S_2-2),\ \ldots\ ,(s_2+1). \\ A_2 \;\; if \;\; p=q+Q & ;\ \mathrm{p} \ = S_2,(S_2-1),(S_2-2),\ \ldots\ ,1. \\ A3 \;\; \text{if} \;\; \mathrm{p}\ =\ \mathrm{q} & ;\ \mathrm{p}\ =\ \mathrm{s}_2,\ (\mathrm{s}_2-1),((\mathrm{s}_2-2),\ \ldots\ 1. \\ A4 \;\; \text{if} \;\; \mathrm{p}\ =\ \mathrm{q} & ;\ \mathrm{p}\ =\ 0 \\ 0 \quad \text{otherwise} \end{cases}$$

$$[B] = \begin{cases} \mu \;\; \text{if p} \ = \ \text{q} \ + \ \mathrm{Q}_2\ ;\ \mathrm{q}\ = \mathrm{s}_2\ ,\ (\mathrm{s}_2-1),\ \ldots\ ,1,0 \\ 0 \quad \text{otherwise} \end{cases}$$

The sub matrices of matrix A are given by

$$A_1 = \begin{cases} \lambda_1+i\gamma \quad \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\lambda_1+i\gamma+\lambda_2+k\gamma+\lambda_D)\ \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\lambda_2+k\gamma+\lambda_D) \quad \text{if p} \ = \ \mathrm{q}\ ;\ \mathrm{q}\ =\ 0 \\ 0 \quad \text{otherwise} \end{cases}$$

$$A_2 = \begin{cases} \lambda_2+k\gamma \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ 0 \quad \text{otherwise} \end{cases}$$

$$A_3 = \begin{cases} \lambda_1+i\gamma \quad \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\lambda_1+i\gamma+\lambda_2+k\gamma+\mu)\ \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\lambda_2+k\gamma+\lambda_D+\mu) \quad \text{if p} \ = \ \mathrm{q}\ ;\ \mathrm{q}\ =\ 0 \\ 0 \quad \text{otherwise} \end{cases}$$

$$A_4 = \begin{cases} \lambda_1+i\gamma \quad \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\lambda_1+i\gamma+\lambda_2+k\gamma+\lambda_D+\mu)\ \text{if p} \ = \ \mathrm{q}-1\ ;\ \mathrm{q}\ =\ 1,2,\ \ldots\ ,\ \mathrm{S}_1 \\ -(\mu+\lambda_D) \quad \text{if p} \ = \ \mathrm{q}\ ;\ \mathrm{q}\ =\ 0 \\ 0 \quad \text{otherwise} \end{cases}$$

2.3. Transient Analysis

Define the transient probability function

$$p_{(i,k,m)}(j,l,n:t) = p_r\{(I_1(t),I_2(t),I_3(t)) = (j,l,n)j(I_1(0),I_2(0),I_3(0)) = (i,k,m)\}.$$

The transient matrix for $t \geq 0$ is of the form $P(t) = (p_{(i,k,m)}(j,l,n:t))_{(i,k,m)(j,l,n)\in E}$ satisfies the Kolmogorov- forward equation $P'(t) = P(t).R$, where R is the infinitesimal generator of the process $\{I(t),t \geq 0\}$. From the above equation, together with initial condition $P(0) = I$, the solution can be express of the form $P(t) = P(0)e^{Rt} = e^{Rt}$, where the matrix expansion in power series form is

$$e^{Rt} = I + \sum_{n=1}^{\infty} \frac{R^n t^n}{\mathrm{n!}}.$$

2.4. Steady State Analysis

The structure of the infinitesimal matrix R reveals that the state space E of the Markov process $\{I(t);\ t \geq 0\}$ is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$^{m}\Pi_{i}^{k} = \lim_{t\to\infty} \Pr\left\{(I_1(t), I_2(t), I_D(t)) = (i,k,m)\right\},$$

where $^{m}\Pi_{i}^{k}$ is the steady state probability that the system be in state (i,k,m), (Cinlar [5]). Let $\Pi = ({}^{nQ}\Pi,\ {}^{(n-1)Q}\Pi,\ {}^{(n-2)Q}\Pi,\ {}^{2Q}\Pi,\ {}^{Q}\Pi)$. Denote the steady state probability distribution, where ${}^{jQ}\Pi = \left(\Pi_S^k,\ \Pi_{S-1}^k, ...,\ \Pi_1^k,\ \Pi_0^k\right)$ for $j = 1,2,...,n$ and $k = 1,2,\ldots,S$ for the system under consideration. For each (i,k,m), $^{m}\Pi_{i}^{k}$ can be obtained by solving the matrix equation $\Pi R = 0$. Since the state space is finite and R is irreducible, the stationary probability vector ? for the generator R always exists and satisfies $\Pi R = 0$ and$\Pi e = 1$. The vector Π can be represented by

$$\Pi = (\Pi^{<Q1,Q2>},\ \Pi^{<2Q1,2Q2>},\ \Pi^{<3Q1,3Q2>}, ...,\ \Pi^{<n1Q1,n2Q2>})$$

Now the structure of R shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the Gaver algorithm for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

2.5. Algorithm

1. Determine recursively the matrix Dn, $0 \leq n \leq N$ by using $D0 = A_0$ & $Dn = A_n + B_n(-D_{n-1}^{-1})\,C,\quad n = 1,2,...,K.$
2. Solve the system $\Pi^{<N>} D_N = 0$.
3. Compute recursively the vector $\Pi^{<n>},\ \ n = N-1,...,0$ using $\Pi^{<n>} = \Pi^{<n+1>} B_{n+1}(-D_n^{-1}),\ \ n = n-1,...,0.$
4. Re-normalize the vector Π, using $\Pi e = 1$.

2.6. Operating Characteristics

In this section, we derive some important system performance measures.

(a) Mean Inventory Level

Let IL_{R1} and IL_{R2} denote the expected inventory level in the steady state at retailer node for the commodity-1 and commodity-2, and IL_D denote the expected inventory level at distribution centre. They are defined as

$$IL_{R1} = \sum_{m=Q}^{nQ} * \sum_{k=0}^{S_2} \sum_{i=0}^{S_1} i\Pi^{<<<i,k,m,o>>>}$$

$$IL_{R2} = \sum_{m=Q}^{nQ} * \sum_{k=0}^{S_2} \sum_{i=0}^{S_1} k\Pi^{<<<i,k,m,o>>>}$$

$$IL_{D} = \sum_{m=Q}^{nQ} * \sum_{k=0}^{S_2} \sum_{i=0}^{S_1} m\Pi^{<<<i,k,m,o>>>};\ Q = Q_1 + Q_2$$

(b) Mean Reorder Rate

Consider the reorder events β_R for both commodities at retailer node and β_D of at distribution centre. It is observe that β_D event occur whenever the inventory level at DC node reaches 0 whereas the event β_R occurs whenever the inventory level at retailer node drops to either (s_1, s_2) or $(s_1, j), j < s_2$ or (i, s_2), $i < s_1$. The mean reorder rate at retailer node and distribution centre are given by

$$B_R = \sum_{m=Q}^{nQ}\left(\sum_{k=0}^{s_2}(\lambda_1 + (s_1+1)\gamma)\Pi^{<<<<s_1+1,k,m>>>>+}\sum_{i=0}^{s_1}(\lambda_2 + (s_2+1)\gamma)\Pi^{<<<i,s_2+1,m>>>}\right)$$

$$B_D = \sum_{k=0}^{s_2}\sum_{i=0}^{s_1}(\mu + \lambda_D)\Pi^{<<<<i,k,Q>>>>}$$

(c) Mean Shortage Rate

Shortage occurs only at retailer node and the mean shortage rate at retailer is denoted by α_R which is given by

$$\alpha_R = \sum_{m=Q}^{nQ} * \left(\sum_{k=0}^{S_2}(\lambda_1)\Pi <<< 0, k, m >>> + \sum_{k=0}^{S_1}(\lambda_2)\Pi <<< i, 0, m >>>\right)$$

2.7. Cost Analysis

In this section we analyze the cost structure for the proposed models by considering the minimization of the steady state total expected cost per time. The long run expected cost rate for the model is defined to be

$$C(s, Q) = h_{R1}\, IL_{R1} + h_{R2}\, IL_{R2} + h_D\, IL_D + k_r\, B_R + k_D\, B_D + g_R\, \alpha_R,$$

where

1. h_{R1}, h_{R2} and h_D are the holding cost per unit of item of Commodity-1, Commodity-2 at retailer node and holding cost per unit items (Q items, $Q = Q_1 + Q_2$) at distribution centre respectively per unit time.

2. IL_{R1}, IL_{R2} and IL_D are the inventory level of Commodity- 1, Commodity- 2 at retailer node and inventory level at distribution centre respectively per unit time.

3. K_r, and K_D are the Fixed ordering cost at retailer node distribution centre respectively.

4. B_R and B_D are the mean reorder rate at retailer node and distribution centre.

5. α_R is the mean shortage rate at retail node.

6. g_R is the shortage cost per unit shortage at retailer node.

Although, we have not proved analytically the convexity of the cost function $\mathbf{TC}(s_1, s_2, \mathbf{Q})$, our experience with considerable number of numerical examples indicates that $\mathbf{TC}(s_1, s_2, \mathbf{Q})$ for fixed Q to be convex in s. In some cases, it turned out to be an increasing function of s. Hence, we adopted the numerical search procedure to determine the optimal values s*, consequently, we obtain optimal n*. For large number of parameters, our calculation of $\mathbf{TC}(s_1, s_2, \mathbf{Q})$ revealed a convex structure for the same.

3. Numerical Illustration

In the section the problem of minimizing the long run total expected cost per unit time under the following cost structure is considered for discussion. The optimum values of the system parameters s_1 and s_2 are obtained and the sensitive analysis is also done for the system. The results we obtained in the steady state case may be illustrated through the following numerical example, $S_1 = 20$, $S_2 = 25$, $M = 200$, $\lambda_1 = 3$, $\lambda_2 = 4$, $\lambda_D = 2$, $\mu = 3$, $\gamma = 2$, $h_R = 1.1$, $h_D = 1.2$, $k_R = 1.5$, $k_d = 1.3$, $g_R = 2.1$, $g_D = 2.2$, $C_o = 2.3$. The cost for different reorder level are given by

s_1	2	3	4	5*	6	7	8
s_2	3	4	5	6*	7	8	9
$Q = (Q_1, +Q_2)$	40	38	36	34*	32	30	28
$TC(s_1, s_2Q)$	118.06031	107.943	92.3616	89.6588*	96.1757	112.6	126.14

Table 1. Total expected cost rate as a function s_1, s_2 and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost **TC(s_1, s_2, Q)** are indicated by the symbol *. The Convexity of the cost function is given in the graph.

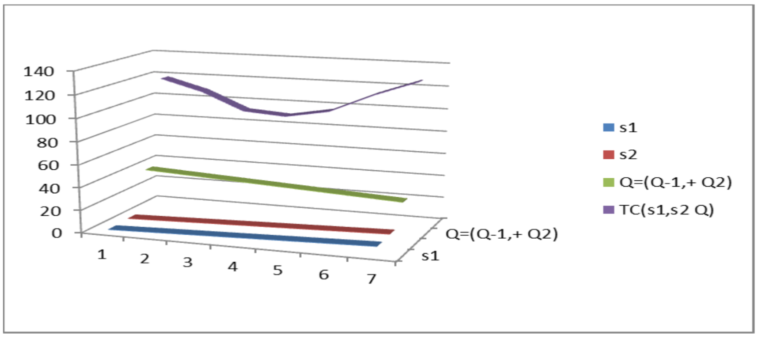

Figure 1. Total expected cost rate as a function TC(s_1, s_2, Q), s_1, s_2 and Q

3.1. Sensitivity Analysis

The effect of changes in Demand rate at retailer node for product 1 and 2.

	$\lambda_2 = 8$	$\lambda_2 = 10$	$\lambda_2 = 12$	$\lambda_2 = 14$	$\lambda_2 = 16$
$\lambda_1 = 12$	45.975	47.348	48.159	48.5817	48.7414
$\lambda_1 = 14$	47.197	48.57	49.3813	49.804	49.9637
$\lambda_1 = 16$	48.079	49.452	50.2632	50.686	50.8457
$\lambda_1 = 18$	48.749	50.122	50.933	51.3557	51.5154
$\lambda_1 = 20$	49.292	50.665	51.4764	51.8991	52.0588

Table 2. Total expected cost rate as a function when demand increases

The graph of the demand rate variation is given below and it describes, if the demand rate increases then the total cost also increases.

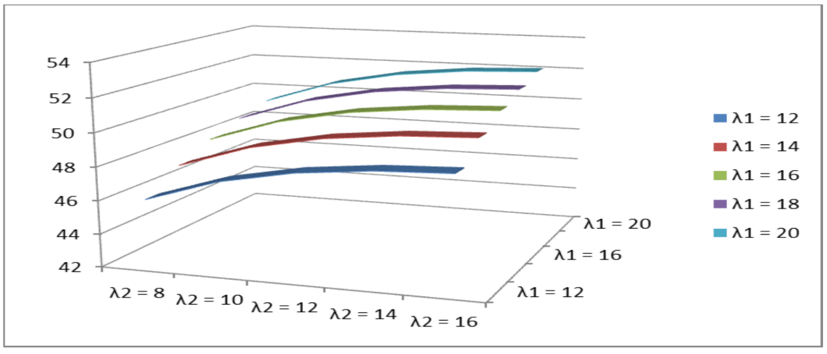

Figure 2. TC(s_1, s_2, Q) for different demand rates

	$S_2 = 30$	$S_2 = 35$	$S_2 = 40$	$S_2 = 45$	$S_2 = 50$
$S_1 = 20$	193.66	182.88	168.948	155.345	144.043
$S_1 = 25$	198.91	189.86	176.729	162.829	150.811
$S_1 = 30$	203.56	196.29	184.335	170.443	157.756
$S_1 = 35$	207.69	202.13	191.637	178.118	164.896
$S_1 = 40$	211.38	207.35	198.515	185.757	172.224

Table 3. **Total expected cost rate as a function when S_1 and S_2 increases**

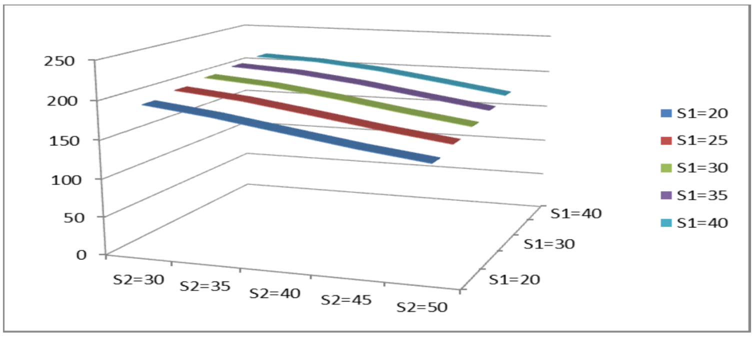

Figure 3. **TC(s_1, s_2, Q) for different S_1 and S_2 values**

From the graph it is identified that the total cost decrease when S_2 increases and increases when S_1 increases.

4. Conclusion

This paper has presented a stochastic model to determine the optimal joint ordering quantity. With numerical examples, It is shown that the cost function to be minimized is convex in ordering quantity. Direct computation of an optimal ordering policy for a perishable product is accomplished. By assuming a suitable cost structure on the perishable inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times using techniques from renewal theory and semi-regenerative processes. Once this is done, the general model can be used to generate various special eases

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