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Analysis of Successive Occurrence of Digit 0 in Prime Numbers till 1 Trillion

Research Article

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Abstract: All primes less than 1 trillion are analyzed for successive occurrence of digit 0 in them. Multiple successive occurrences of 0's are examined. The first and last instances of successive occurrence of all possible repetitions of 0's are determined within initial 12 ranges of increasing powers of 10.

MSC: 11Y35, 11Y60, 11Y99

Keywords: Prime numbers, digit 0, successive occurrences

1. Introduction

A prime number is positive integer greater than 1 having only two positive divisors, viz., 1 and itself. The spread of prime numbers amongst integers is being studied theoretically [1] as well as for high ranges [10]-[23]. Owing to immense importance of 0 in a number system [2], recent works has analyzed occurrence of digit 0 in natural numbers [24], [25] and in prime numbers [26] in similar high ranges. Here we examine successive occurrences of 0's in prime numbers.

We consider prime numbers p in ranges $1 < p < 10^n$, $1 \leq n \leq 12$. The number 10^n contains more than n digits and hence is not included in each such range. All prime numbers in this huge range could be obtained efficiently owing to choice of better prime generating algorithm coming out from their comparisons in [3] to [9]. As insignificant 0's won't matter, they are ignored.

2. Occurrence of Single Successive Digit 0 in Prime Numbers

Single occurrence of 0 is trivially to be considered as successive. So, values determined in [26] for occurrence of single 0 in prime numbers are also valid as occurrence of single successive 0!. This is outcome of extensive execution of a computer program written in Java Language on many computer systems. The percentage of number of primes containing single successive 0 in each of aforementioned ranges calculated with base as all integers containing single successive 0 [25] in the respective ranges is declining.

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Table 1. **Number of Prime Numbers in Various Ranges with Single 0 in Their Digits [26]**

Sr. No.	Range	Number of Primes with Single Successive(!) 0
1	$1-10^1$	0
2	$1-10^2$	0
3	$1-10^3$	15
4	$1-10^4$	206
5	$1-10^5$	2,223
6	$1-10^6$	22,281
7	$1-10^7$	214,893
8	$1-10^8$	2,022,364
9	$1-10^9$	18,800,595
10	$1-10^{10}$	173,468,013
11	$1-10^{11}$	1,592,608,078
12	$1-10^{12}$	14,567,387,959

3. Occurrence of Multiple Successive 0's in Prime Numbers

Successively has real meaning when more units are under consideration. Earlier, count of all numbers containing double, triple and higher number of successive digit 0's in them within the ranges $1-10^n$, $1 \leq n \leq 12$ is determined [25]. Now the number of primes in these ranges containing multiple number of successive digit 0's is found out.

Table 2. **Number of Prime Numbers in Various Ranges with Multiple Successive 0's in Their Digits**

Sr. No.	Number Range <	Number of Primes with 2 Successive 0's	Number of Primes with 3 Successive 0's	Number of Primes with 4 Successive 0's
1	10^4	13	0	0
2	10^5	163	8	0
3	10^6	1,856	122	9
4	10^7	18,866	1,557	121
5	10^8	186,756	16,579	1,392
6	10^9	1,785,280	164,610	14,598
7	10^{10}	16,820,059	1,596,930	147,230
8	10^{11}	157,000,248	15,225,056	1,445,490
9	10^{12}	1,454,675,413	143,408,647	13,904,117

Sr. No.	Number Range <	Number of Primes with 5 Successive 0's	Number of Primes with 6 Successive 0's	Number of Primes with 7 Successive 0's
1	10^7	5	0	0
2	10^8	100	3	0
3	10^9	1,191	89	8
4	10^{10}	12,936	1,080	70
5	10^{11}	133,321	11,847	949
6	10^{12}	1,320,722	121,627	10,831

Sr. No.	Number Range <	Number of Primes with 8 Successive 0's	Number of Primes with 9 Successive 0's	Number of Primes with 10 Successive 0's
4	10^{10}	8	0	0
5	10^{11}	66	2	0
6	10^{12}	880	61	2

The count of occurrence of multiple successive 0 digits in primes in various ranges of powers of 10 is graphically plotted with vertical axis in on logarithmic scale, showing near parallel exponential curves.

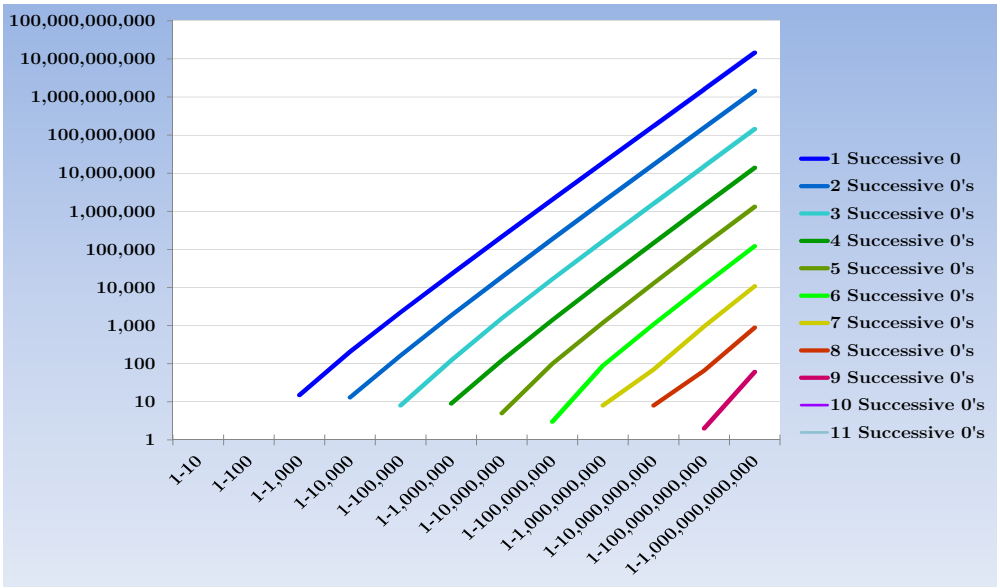

Figure 1. **Number of Primes in Various Ranges with Multiple Successive 0's in Their Digits**

Their percentage with respect to number of integers with equal number of successive 0's in respective ranges varies as follows.

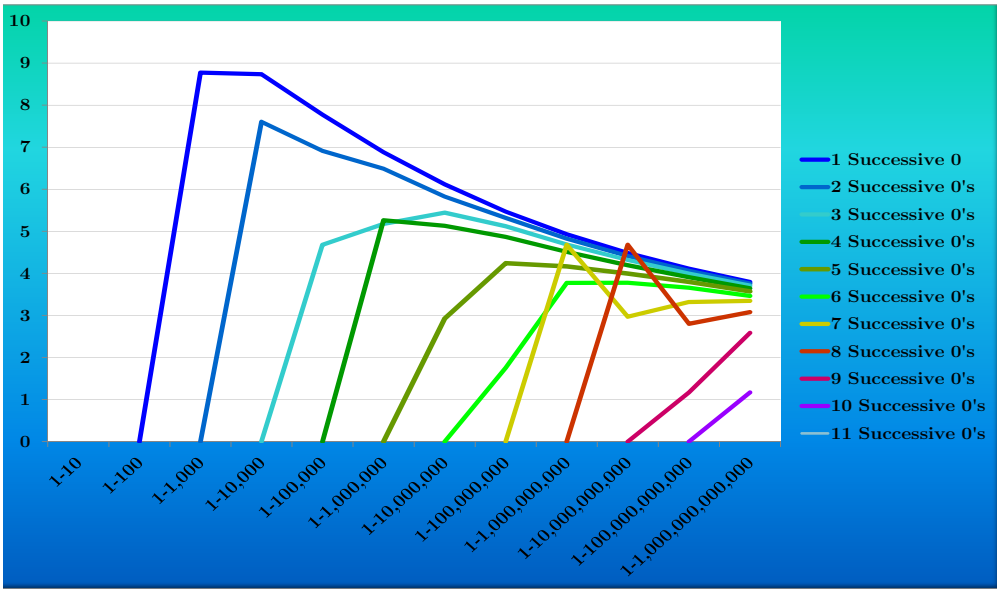

Figure 2. Percentage of Primes in Various Ranges with Multiple Successive 0's in Their Digits With Respect to All Such Integers in Respective Ranges

4. First Occurrence of Successive Digits 0's in Prime Numbers

For all natural numbers, the first number containing 0 is clearly 10. For higher ranges, first occurrence of 2 0's is in 100, that of 3 is in 1000 and so on. In fact, first occurrence of multiple 0's also happens to be of successive 0's as given in

Formula 4.1 ([25]). *If n and r are natural numbers, then the first occurrence of successive r zeros in integers in range*

$1 \leq m < 10^n$ *is*

$$f = \begin{cases} -, \text{if} r \geq n \\ 10^r, \text{if} r < n \end{cases}$$

No formula being currently available for such occurrences in prime numbers, we have individually determined these.

Table 3. **First Prime Numbers in Various Ranges with Multiple Successive 0's in Their Digits**

Sr. No.	Range	First Prime Number in Range with					
		1 Successive 0	2 Successive 0's	3 Successive 0's	4 Successive 0's	5 Successive 0's	6 Successive 0's
1	$1-10^1$	-	-	-	-	-	-
2	$1-10^2$	-	-	-	-	-	-
3	$1-10^3$	101	-	-	-	-	-
4	$1-10^4$	101	1,009	-	-	-	-
5	$1-10^5$	101	1,009	10,007	-	-	-
6	$1-10^6$	101	1,009	10,007	100,003	-	-
7	$1-10^7$	101	1,009	10,007	100,003	1,000,003	-
8	$1-10^8$	101	1,009	10,007	100,003	1,000,003	20,000,003
9	$1-10^9$	101	1,009	10,007	100,003	1,000,003	20,000,003
10	$1-10^{10}$	101	1,009	10,007	100,003	1,000,003	20,000,003
11	$1-10^{11}$	101	1,009	10,007	100,003	1,000,003	20,000,003
12	$1-10^{12}$	101	1,009	10,007	100,003	1,000,003	20,000,003

Sr. No.	Range	First Prime Number in Range with			
		7 Successive 0's	8 Successive 0's	9 Successive 0's	10 Successive 0's
1	$1-10^1$	-	-	-	-
2	$1-10^2$	-	-	-	-
3	$1-10^3$	-	-	-	-
4	$1-10^4$	-	-	-	-
5	$1-10^5$	-	-	-	-
6	$1-10^6$	-	-	-	-
7	$1-10^7$	-	-	-	-
8	$1-10^8$	-	-	-	-
9	$1-10^9$	100,000,007	-	-	-
10	$1-10^{10}$	100,000,007	1,000,000,007	-	-
11	$1-10^{11}$	100,000,007	1,000,000,007	30,000,000,001	-
12	$1-10^{12}$	100,000,007	1,000,000,007	30,000,000,001	100,000,000,003

There is no surprise that these occurrences are first occurrences of 0's as well as first occurrences of successive 0's in primes.

5. Last Occurrence of Successive Digits 0's in Prime Numbers

The formula for last occurrence of r number of successive 0's in all integers in ranges $1-10^n$, $1 \leq n \leq 12$, is

Formula 5.1 ([25]). *If n and r are natural numbers, then the last occurrence of r successive 0's in numbers in range $1 \leq m < 10^n$ is*

$$l = \begin{cases} - \quad , \text{if} r \geq n \\ 10^n - 10^r, \text{if} r < n \end{cases}$$

The last occurrences of prime numbers with r number of successive 0's in them in these ranges have been found out.

Table 4. **Last Prime Numbers in Various Ranges with Multiple Successive 0's in Their Digits**

Sr. No.	No. of Successive 0's	Last Prime Number in Range 1 -							
		10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8
1	1	-	-	907	9,907	99,907	999,907	9,999,907	99,999,703
2	2	-	-	-	9,007	98,009	999,007	9,997,007	99,994,009
3	3	-	-	-	-	90,007	990,001	9,970,001	99,990,001
4	4	-	-	-	-	-	900,007	9,800,009	99,900,001
5	5	-	-	-	-	-	-	8,000,009	99,000,007
6	6	-	-	-	-	-	-	-	40,000,003
7	7	-	-	-	-	-	-	-	-
8	8	-	-	-	-	-	-	-	-
9	9	-	-	-	-	-	-	-	-
10	10	-	-	-	-	-	-	-	-
11	11	-	-	-	-	-	-	-	-

Sr. No.	No. of Successive 0's	Last Prime Number in Range 1 -		
		10^9	10^{10}	10^{11}
1	1	999,999,607	9,999,999,707	99,999,999,907
2	2	999,999,001	9,999,999,001	99,999,998,003
3	3	999,970,009	9,999,980,009	99,999,970,001
4	4	999,200,009	9,999,800,003	99,999,400,009
5	5	997,000,007	9,999,000,001	99,999,000,001
6	6	970,000,007	9,990,000,001	99,990,000,007
7	7	700,000,001	9,900,000,007	99,600,000,001
8	8	-	9,000,000,001	93,000,000,001
9	9	-	-	40,000,000,003
10	10	-	-	-
11	11	-	-	-

The remark for general occurrences of 0's also applies to successive occurrences.

Remark 5.2. *The maximum number of successive* 0*'s in any prime number in the range* $1 - 10^n$*, for* $n > 2$*, is at most* $n - 2$.

All numbers of this work give integer sequences which deserve independent treatment in their own right.

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References

[1] Fine Benjamin and Rosenberger Gerhard, *Number Theory: An Introduction via the Distribution of Primes*, Birkhauser, (2007).

[2] Neeraj Anant Pande, *Numeral Systems of Great Ancient Human Civilizations*, Journal of Science and Arts, 10(2)(2010), 209-222.

[3] Neeraj Anant Pande, *Evolution of Algorithms: A Case Study of Three Prime Generating Sieves*, Journal of Science and Arts, 24(3)(2013), 267-276.

[4] Neeraj Anant Pande, *Algorithms of Three Prime Generating Sieves Improvised Through Nonprimality of Even Numbers (Except* 2*)*, International Journal of Emerging Technologies in Computational and Applied Sciences, 4(6)(2013), 274-279.

[5] Neeraj Anant Pande, *Algorithms of Three Prime Generating Sieves Improvised by Skipping Even Divisors (Except* 2*)*, International Journal of Research in Formal, Applied & Natural Sciences, 1(4)(2013), 22-27.

[6] Neeraj Anant Pande, *Prime Generating Algorithms through Nonprimality of Even Numbers (Except* 2*) and by Skipping Even Divisors (Except* 2*)*, Journal of Natural Sciences, 2(1)(2014), 107-116.

[7] Neeraj Anant Pande, *Prime Generating Algorithms by Skipping Composite Divisors*, International Journal of Computer Science & Engineering Technology, 5(09)(2014), 935-940.

[8] Neeraj Anant Pande, *Improved Prime Generating Algorithms by Skipping Composite Divisors and Even Numbers (Other Than* 2*)*, Journal of Science and Arts, 31(2)(2015), 135-142.

[9] Neeraj Anant Pande, *Refinement of Prime Generating Algorithms*, International Journal of Innovative Science, Engineering & Technology, 2(6)(2015), 21-24.

[10] Neeraj Anant Pande, *Analysis of Primes Less Than a Trillion*, International Journal of Computer Science and Engineering Technology, 6(6)(2015), 332-341.

[11] Neeraj Anant Pande, *Analysis of Primes in Arithmetical Progressions* $3n + k$ *up to a Trillion*, IOSR Journal of Mathematics, 11(3)(2015), 72-85.

[12] Neeraj Anant Pande, *Analysis of Primes in Arithmetical Progressions* $4n + k$ *up to a Trillion*, International Journal of Mathematics and Computer Applications Research, 5(4)(2015), 1-18.

[13] Neeraj Anant Pande, *Analysis of Primes in Arithmetical Progressions* $5n + k$ *up to a Trillion*, Journal of Research in Applied Mathematics, 2(5)(2015), 14-29.

[14] Neeraj Anant Pande, *Analysis of Primes in Arithmetical Progressions* $6n + k$ *up to a Trillion*, International Journal of Mathematics and Computer Research, 3(6)(2015), 1037-1053.

[15] Neeraj Anant Pande, *Analysis of Primes in Arithmetical Progressions* $7n + k$ *up to a Trillion*, International Journal of Mathematics and Its Applications, 4(2-A)(2016), 13-30.

[16] Neeraj Anant Pande, *Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions* $8n + k$, IOSR Journal of Mathematics, 12(3-V)(2016), 79-87.

[17] Neeraj Anant Pande, *Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions* $8n+k$, American International Journal of Research in Science, Technology, Engineering and Mathematics, 15(1)(2016), 1-7.

[18] Neeraj Anant Pande, *Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions* $9n + k$, International Journal of Advances in Mathematics and Statistics, 1(2)(2016), 13-23.

[19] Neeraj Anant Pande, *Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions* $9n + k$, International Journal of Mathematics and Statistics Invention, 4(5)(2016), 13-20.

[20] Neeraj Anant Pande, *Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions* $10n+k$, Journal of Research in Applied Mathematics, 2(9)(2016), 10-18.

[21] Neeraj Anant Pande, *Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical Progressions* $10n + k$, International Journal of Engineering Maths and Computer Science, 4(7)(2016).

[22] Neeraj Anant Pande, *Block-wise Distribution of Primes less than a Trillion in Arithmetical Progressions* $11n + k$, International Journal of Recent Research in Mathematics, Computer Science and Information Technology, 3(1)(2016), 70-81.

[23] Neeraj Anant Pande, *Spacings Between and Units & Tens Place Digits in Primes till One Trillion in Arithmetical*

Progressions $11n + k$, International Journal of Mathematics and Computer Research, 4(6)(2016), 1493-1501.

[24] Neeraj Anant Pande, *Analysis of Occurrence of Digit 0 in Natural Numbers Less Than* 10^n, American International Journal of Research in Formal, Applied and Natural Sciences, Communicated, (2016).

[25] Neeraj Anant Pande, *Analysis of Successive Occurrence of Digit 0 in Natural Numbers Less Than* 10^n, IOSR-Journal of Mathematics, Communicated, (2016).

[26] Neeraj Anant Pande, *Analysis of Occurrence of Digit* 0 *in Prime Numbers till* 1 *Trillion*, Journal of Research in Applied Mathematics, Communicated, (2016).