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A Time Truncated Two Stage Group Acceptance Sampling Plan For Compound Rayleigh Distribution

Research Article

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Abstract: In this paper, two stage group sampling plan is developed assuming that life time of the test units follow Compound Rayleigh Distribution and the life tests is terminated at a prefixed time. The minimum number of groups required for ensuring the specified mean life at specified consumer's confidence level have been determined. The Operating characteristic values for various quality levels are obtained and the results are discussed with the help of tables and examples. The minimum mean ratios are also obtained for a specified level of producer's risk.

Keywords: Acceptance sampling, Compound Rayleigh distribution, Producer's risk, Consumer's risk, Operating Characteristic curve, Truncated life test, Two stage Group Sampling Plan.

1. Introduction

The reputation of companies depends upon the high reliability of their products. These companies compete with each other on the basis of quality and reliability. Thus quality control has become one of the most important tools to differentiate between the competitive enterprises in a global business market. In order to control the quality of the purchased goods, two major alternatives are open to a buyer. One is the complete inspection, in which every single item in the lot is inspected and tested. This is often impractical, uneconomical or impossible. Secondly, the practical inspection in which a sample of item is taken which is inspected and tested and the whole lot is accepted or rejected depending on whether few or many defective items are found in the sample.

Quality control is the use of techniques and activities to achieve, sustain, and improve the quality of a product or service. It involves integrating the following related techniques and activities such as specifications of what is needed, design of the product or service to meet the specifications, production or installation to meet the full intent of the specifications, inspection to determine conformance to specifications and review of usage to provide information for the revision of specifications, if needed.

Statistical quality control is the procedure for the control of quality by the application of theory of probability to the results of the inspection of samples of the population. Sampling plans are used in the area of quality and reliability analysis. When

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the quality of product is related to its lifetime, it is called as life test.

Acceptance sampling is a methodology commonly used in quality control and improvement. The aim is to make an inference about the quality of a lot of product from a sample. Depending on the number of defectives found in the sample, the whole lot is then either accepted or rejected, and the rejected lots can then be scraped or reworked. Thus a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria is an acceptance sampling plan.

Two stage group sampling plan was first introduced by Aslam, Jun, Rasool and Ahmad [2] for Weibull distribution, in which they calculated the number of groups for different acceptance level. Abbur Razzaque Mughal, Muhammad Hanif, Azhar Ali Imran, Muhammad Rafi and Munir Ahmad [1] developed an economic reliability two stage group acceptance sampling plan based on Weibull distribution. In this type of tests, determining the sample size is equivalent to determining the number of groups. Priyah Anburajan and Sudamani Ramaswamy [5] introduced Two Stage Group Acceptance Sampling plans on Truncated life tests for Marshall-Olkin Extended Distributions.

A Two stage Group Acceptance sampling plan based on Truncated life tests for a Exponentiated Frechet Distribution was developed by Srinivasa Rao, Rosaiah, Sridhar Babu and Siva Kumar [7]. Priyah Anburajan and Sudamani Ramaswamy [6] developed two stage group acceptance sampling plans on Truncated life tests using Log-Logistic and Gamma distribution. Muhammad Azam, Muhammad Aslam, Saminathan Balamurali, Affifa Javaid [4] developed Two stage group acceptance sampling plan for half normal percentiles.

2. Compound Rayleigh Distribution

The Rayleigh distribution plays an important role in modelling the lifetime of random phenomenon. It arises in many areas of applications, including reliability, life testing and survival analysis. Bhupendra Singh, Sharma and Dushyant Tyagi [3] have developed a reliability single sampling plan assuming that lifetimes of the test units follow Compound Rayleigh distribution and the life test is terminated at a prefixed time. This type of sampling plan is used to save the test time in practical situations. Let X denotes a random variable arising from a Rayleigh distribution with p.d.f.

$$f(t;\theta) = 2\theta t e^{-\theta t^2} \tag{1}$$

where $t > 0$ is the lifetime, and $\theta > 0$. The corresponding hazard function is $h(t) = 2\theta\ t$, $t > 0$. The mean survival time and the cumulative distribution function of the Rayleigh model are given by

$$E(t) = \frac{1}{2}\sqrt{\frac{\pi}{\theta}} \tag{2}$$

$$F(t) = 1 - e^{-\frac{t^2}{\theta}} \tag{3}$$

In life testing experiments, it is expected that the environmental conditions can not be remained the same during the testing time. Therefore, it seems logical to treat the parameters involved in the life time model as random variables. In view of this, if the parameter θ is itself a random variable, then the distribution of lifetime of each item is a Compound Rayleigh distribution. The particular form of θ, which is considered here, is the gamma p.d.f.

$$g(\theta,\ B,\ \delta) = \frac{B^{\delta}\theta^{\delta-1}e^{-B\theta}}{\Gamma\delta},\ \ \theta,\ B,\ \delta > 0. \tag{4}$$

The parameters β and δ are scale and shape parameters, respectively. The resulting Compound distribution has p.d.f.

$$f(t,\ \alpha,\ B) = \int_0^\infty 2\theta\, t\, e^{-\theta\ t^2} \frac{B^\delta \theta^{\delta-1} e^{-B\theta}}{\Gamma\delta} d\theta$$
$$= 2\delta B^\delta t \left(B + t^2\right)^{-(\delta+1)} \tag{5}$$

The mean survival time and the cumulative distribution function of the Compound Rayleigh model are given by

$$\mu = E(t) = \frac{\sqrt{B\pi}\Gamma\left(\delta - \frac{1}{2}\right)}{2\Gamma\delta} \text{ and} \tag{6}$$
$$F(t,\ B,\ \delta) = 1 - B^\delta \left(B + t^2\right)^{-\delta} \quad ,t > 0 \tag{7}$$

3. Operating Procedure for Two Stage Group Acceptance Sampling Plan for Truncated Life Test

The following is the procedure presented by Aslam, Jun, Rasool and Ahmad [2] for the two stage group acceptance sampling plan for truncated life test.

1. First stage-Draw the first random sample size n_1 from a lot, allocate r items to each of g_1 groups (or testers) so that $n_1 = rg_1$ and put them on test for the duration of t_0. Accept the lot if the number of failures from each group is c_1 or less. Truncate the test and reject the lot as soon as the number of failures in any group is larger than c_2 before t_0. Otherwise, go to the second stage.

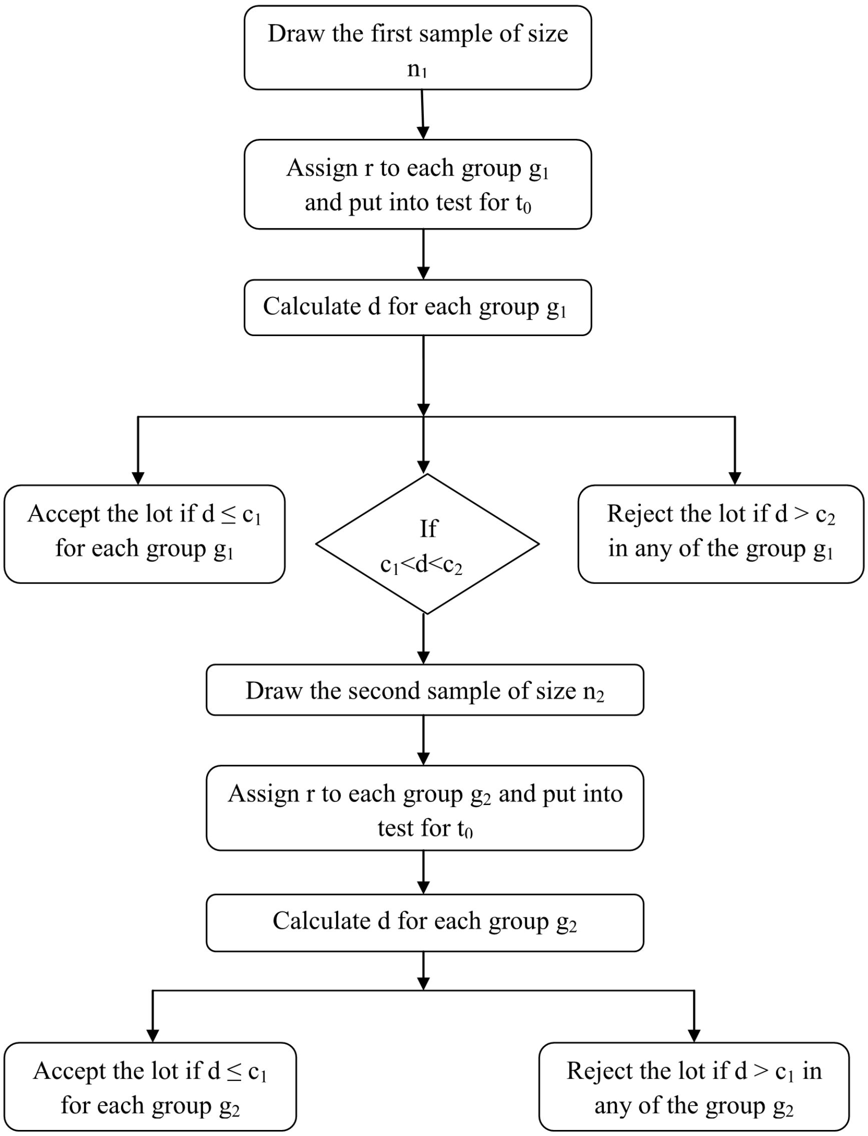
2. Second stage-Draw the second random sample of size n_2 from a lot, allocate r items to each of g_2 groups (or testers) so that $n_2 = rg_2$ and put them on test for the duration of t_0. Accept the lot if the number of failures from each group is c_1 or less. Truncate the test and reject the lot if the number of failures in any group is larger than c_1 before t_0.

3.1. Notations

n_1	Size of the first sample
n_2	Size of the second sample
g_1	Number of groups in first stage
g_2	Number of groups in second stage
r	Number of items in a group
c_1	Acceptance number of the first sample
c_2	Acceptance number of the second sample
d	Number of defectives
α	Producer's risk
β	Consumer's risk
t	Termination time
δ	Shape parameter
B	Scale parameter
p	Probability of failure before time t
p_a	Probability of acceptance of lot
μ_0	Specified mean lief

The following is the operating procedure for two stage group acceptance sampling plan for life test in the form of a flow chart.

3.2. Flow chart



3.3. Design of the sampling plan

The main objective of this plan is to set a lower consumer's risk β, on the products mean lifetime μ and to test whether the lifetime of the product is longer than our expectation. The two stage group acceptance sampling plan constitute the design parameters of g_1, g_2, c_1 and c_2 when the number of testers r and both risks are specified. The probability of acceptance of the lot at the first stage can be evaluated as,

$$P_a^1 = \left[\sum_{i=0}^{c_1} \binom{r}{i} p^i \, (1 - p)^{r-i} \right]^{g_1} \tag{8}$$

The probability of rejection of the lot at the first stage is given by,

$$P_r^1 \;=\; 1 \;-\; \left[\sum_{i\,=0}^{c_2}\begin{pmatrix} r \\ i \end{pmatrix} p^i\;(1\;-\;p)^{r\,-\,i}\right]^{g_1} \tag{9}$$

where the probability of acceptance of the lot at the second stage is

$$P_a^2 \;=\; \left[1\;-\;\left(P_a^1\;+\;P_r^1\right)\right]\left[\sum_{i\,=0}^{c_1}\begin{pmatrix} r \\ i \end{pmatrix} p^i\;(1\;-\;p)^{r\,-\,i}\right]^{g_2} \tag{10}$$

Therefore the probability of acceptance of the lot for the proposed two stage group acceptance sampling plan is given by,

$$L\,(p)\;=\;P_a^1\;+\;P_a^2 \tag{11}$$

Consider p_1, p_2 be the probability of failure corresponding to the consumer's risk and producer's risk respectively. The plan parameters can be obtained from the solution of the following inequalities.

$$\begin{aligned} L\,(p_1) &= P_a^1\;+\;P_a^2\;\leq\;\beta \\ L\,(p_2) &= P_a^1\;+\;P_a^2\;\geq\;1\;-\;\alpha \\ 1\;&\leq\;g_2\;\leq\;g_1 \\ 0\;&\leq\;c_1\;\leq\;c_2 \end{aligned} \tag{12}$$

According to Bhupendra Singh et.al, (2013) the value of p_0 is given for the compound Rayleigh distribution as

$$\begin{aligned} p_0 &= 1 - B^{\delta}(B+t^2)^{-\delta} \\ &= 1 - \frac{1}{(1+\frac{t^2}{B})^{\delta}},\quad \text{where} \end{aligned} \tag{13}$$

$$B = \left(\frac{2\mu\Gamma\delta}{\sqrt{\pi}\Gamma\left(\delta-\frac{1}{2}\right)}\right)^{\frac{1}{2}} \tag{14}$$

Substituting the value of B and $t = a\mu_0$, we get

$$p_0 = 1 - \frac{1}{1+\left(\frac{a\sqrt{\pi}\Gamma\left(\delta-\frac{1}{2}\right)}{2\frac{\mu}{\mu_0}\Gamma\delta}\right)^2} \tag{15}$$

The minimum number of groups satisfying equation (12) are obtained and presented in Table 1 for various values of β and t/μ_0. The shape parameter is fixed as 1, and if some other parameters are involved then they are assumed to be known.

3.4. Operating Characteristic (OC) Curve

The OC function of the sampling plan is the probability of accepting a lot and is given by

$$L\,(p)\;=\;P_a^1\;+\;P_a^2$$

where $p = F(t,\;\beta,\;\delta)$ is treated as a function of lot quality parameter β. The OC values for different combinations of the values of consumer's risk are computed and presented in Table 2. For a given value of the producer's risk α, the minimum value of μ/μ_0 is determined, such that it satisfies the following inequality

$$L\,(p)\;=\;P_a^1\;+\;P_a^2\;\geq\;1\;-\;\alpha$$

and are presented in Table 3.

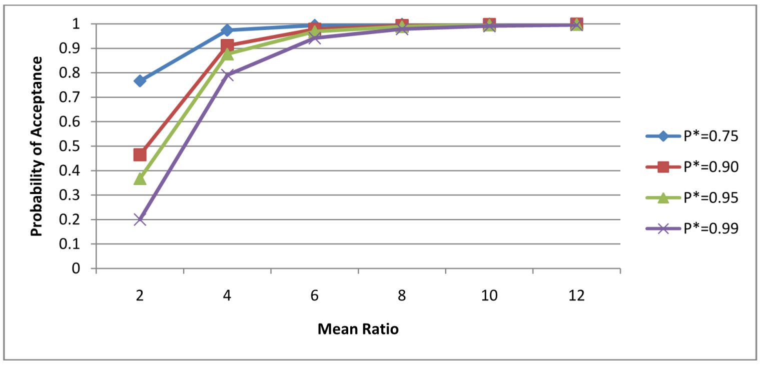


Figure 1. **OC values vs. mean ratio with experiment time ratio a=0.628**

4. Example

Suppose that the lifetime of a product follows Compound Rayleigh distribution with shape parameter $\delta = 1$. Suppose that an electrical circuit producing company wants to test their product that if the mean life time is greater than 1,000 hours based on a testing time of 628 hours and using testers equipped with 3 items each. It is assumed that $c_1 = 0$, $c_2 = 2$ and $\beta = 0.25$. Based on the consumer's risk values and the test termination time multiplier, the number of groups g_1 and g_2 are obtained as $g_1 = 1$ and $g_2 = 1$. We implement the above sampling plan as, draw the first sample of size $n_1 = 3$ items $(n_1 = rg_1)$, if no failure occur during 628 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs. Otherwise, if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size $n_2 = 3$ $(n_2 = rg_2)$ is chosen and tested. For the above conditions, the probability of acceptance will be 0.766344.

5. Conclusion

It is observed from Table 2 and from Figure 1, that the Operating Characteristic values of Compound Rayleigh distribution increases and it tends to unity when the mean ratio μ/μ_0 increases. The minimum ratio and the group size increases, with an increase in confidence level. For various experiment time ratio, the minimum number of groups required to make a decision increases with an increase in the confidence level. It is concluded that this group sampling plan would be beneficial in terms of test time and cost since a group of items can be tested simultaneously.

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β	r	$a = t/\mu_0$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	2	2 2	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	3	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	4	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	5	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	6	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
0.10	2	3 2	2 2	1 1	1 1	1 1	1 1	1 1	1 1
	3	2 2	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	4	2 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	5	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	6	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
0.05	2	3 3	3 2	2 1	1 1	1 1	1 1	1 1	1 1
	3	3 2	2 1	1 1	1 1	1 1	1 1	1 1	1 1
	4	2 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	5	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	6	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
0.01	2	4 4	4 2	2 2	2 2	1 1	1 1	1 1	1 1
	3	4 3	3 1	2 1	1 1	1 1	1 1	1 1	1 1
	4	2 2	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	5	2 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1
	6	2 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1

Table 1. Minimum number of groups required for two-stage group sampling plan when the life time of the items follows Compound Rayleigh distribution with $\delta = 1$, $c_1 = 0$, $c_2 = 2$

β	$a = t/\mu_0$	g_1	g_2	μ/μ_0					
				2	4	6	8	10	12
0.25	0.628	1	1	0.766344	0.973538	0.994080	0.998045	0.999181	0.999601
	0.942	1	1	0.455455	0.896965	0.973538	0.990767	0.996041	0.998045
	1.257	1	1	0.227606	0.765871	0.92898	0.973464	0.988227	0.994062
	1.571	1	1	0.107236	0.608056	0.858233	0.942642	0.97348	0.98631
	2.356	1	1	0.017661	0.273387	0.608222	0.802651	0.896831	0.94268
	3.141	1	1	0.003846	0.107379	0.36607	0.608311	0.766153	0.85836
	3.927	1	1	0.001082	0.042245	0.201327	0.420124	0.608161	0.740676
	4.712	1	1	0.000381	0.017661	0.107326	0.273387	0.455096	0.608222
0.10	0.628	2	2	0.464654	0.910894	0.978139	0.992521	0.996819	0.998435
	0.942	1	1	0.455455	0.896965	0.973538	0.990767	0.996041	0.998045
	1.257	1	1	0.227606	0.765871	0.92898	0.973464	0.988227	0.994062
	1.571	1	1	0.107236	0.608056	0.858233	0.942642	0.97348	0.98631
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	4.712	1	1	0.000381	0.017661	0.107326	0.273387	0.455096	0.608222
0.05	0.628	3	2	0.367937	0.87683	0.968449	0.989027	0.995296	0.997676
	0.942	2	1	0.300135	0.826434	0.951342	0.982416	0.992329	0.996176
	1.257	1	1	0.227613	0.765871	0.930403	0.973464	0.988227	0.994062
	1.571	1	1	0.107236	0.608056	0.858233	0.942642	0.97348	0.98631
	2.356	1	1	0.017661	0.273387	0.608222	0.802651	0.896831	0.94268
	3.141	1	1	0.003846	0.107379	0.36607	0.608311	0.766153	0.85836

β	$a=t/\mu_0$	g_1	g_2	μ/μ_0					
				2	4	6	8	10	12
	3.927	1	1	0.001082	0.042245	0.201327	0.420124	0.608161	0.740676
	4.712	1	1	0.000381	0.017661	0.107326	0.273387	0.455096	0.608222
0.01	0.628	4	3	0.200091	0.790605	0.941592	0.979003	0.990858	0.995443
	0.942	3	1	0.250346	0.778043	0.932721	0.974863	0.988851	0.994389
	1.257	2	1	0.014757	0.642243	0.876615	0.951212	0.977723	0.988579
	1.571	1	1	0.096539	0.608056	0.858233	0.942642	0.97348	0.98631
	2.356	1	1	0.017827	0.273387	0.608222	0.802651	0.896831	0.94268
	3.141	1	1	0.003846	0.107379	0.36607	0.608311	0.766153	0.85836
	3.927	1	1	0.001082	0.042245	0.201327	0.420124	0.608161	0.740676
	4.712	1	1	0.000381	0.017661	0.107326	0.273387	0.455096	0.608222

Table 2. Probability of acceptance for two-stage group sampling plan when the life time of the items follows Compound Rayleigh distribution with $\delta = 1$, $c_1 = 0$, $c_2 = 2$ and $r = 3$

β	r	$a=t/\mu_0$							
		0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.25	2	3.858	4.027	5.374	6.716	10.071	13.427	16.787	20.142
	3	3.332	4.997	6.668	8.334	12.498	16.662	20.832	24.995
	4	3.874	5.810	7.753	9.689	14.531	19.372	24.22	29.061
	5	4.349	6.523	8.704	10.879	16.314	21.750	27.192	32.628
	6	4.778	7.166	9.562	11.951	17.922	23.894	29.873	35.844
0.10	2	4.276	5.787	5.374	6.716	10.071	13.427	16.787	20.143
	3	4.751	4.997	6.668	8.334	12.498	16.662	20.831	24.995
	4	4.606	5.810	7.753	9.689	14.531	19.372	24.220	29.061
	5	4.349	6.523	8.704	10.879	16.314	21.750	27.193	32.628
	6	4.778	7.166	9.562	11.951	17.922	23.893	29.873	35.845
0.05	2	4.751	6.414	6.424	6.716	10.071	13.427	16.787	20.142
	3	5.263	5.953	6.668	8.334	12.498	16.662	20.831	24.995
	4	4.606	5.810	7.753	9.689	14.531	19.372	24.220	29.061
	5	4.349	6.523	8.704	10.879	16.314	21.750	27.192	32.628
	6	4.778	7.166	9.562	11.951	17.922	23.893	29.873	35.843
0.01	2	5.500	6.886	7.722	9.651	10.071	13.427	16.787	20.142
	3	6.277	6.562	7.943	8.334	12.498	16.662	20.831	24.995
	4	5.506	5.810	7.753	9.689	14.531	19.372	24.220	29.061
	5	5.165	6.523	8.704	10.879	16.314	21.750	27.192	32.628
	6	5.670	7.166	9.562	11.951	17.922	23.893	29.873	35.843

Table 3. Minimum ratio of true value μ to specified μ_0 for the acceptability of a lot with producer' risk 0.05 when the life time of the units follows Compound Rayleigh distribution