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# New Class of $\alpha g^*p$ -Continuous Functions in Topological Spaces

**Research Article** 

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**Abstract:** The aim of this paper is to introduce and study the new class of functions namely  $\alpha g^*p$ -totally continuous functions and totally  $\alpha g^*p$ -continuous functions. Further the relationship between this new class with other classes of existing functions are established. Also strongly  $(\alpha g^*p)^*$ -continuous functions in topological spaces are introduced and studied.

**MSC:** 54C10.

Keywords:  $\alpha g^*p$ -totally continuous function, totally  $\alpha g^*p$ -continuous function, strongly  $(\alpha g^*p)^*$ -continuous function,  $\alpha g^*p$ -co-Hausdorff space.

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## 1. Introduction

In 1980, Jain [2] introduced the concept of totally continuous functions. T. M. Nour [9] introduced the concept of totally semi-continuous functions as a generalization of totally continuous functions and several properties of totally semi-continuous functions were obtained. Benchalli et al. [1] introduced and studied semi-totally continuous and semi-totally open functions. Veerakumar [14] introduced totally pre-continuous and strongly pre-continuous functions as alternative stronger forms of totally continuous functions and strongly continuous functions respectively. In this paper, we introduce a new class of continuous functions called  $\alpha g^*p$ -totally continuous functions and totally  $\alpha g^*p$ -continuous functions and investigate some of their fundamental properties. Also we define strongly ( $\alpha g^*p$ )\*-continuous functions in topological spaces.

Throughout this paper, the spaces X ,Y and Z always mean topological spaces  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  respectively. For a subset A of X, the closure and the interior of A in X are denoted by cl(A) and int(A) respectively. The union of all preopen sets of X contained in A is called pre-interior of A and it is denoted by pint(A). The intersection of all preclosed sets of X containing A is called pre-closure of A and it is denoted by pcl(A). Also the collection of all  $\alpha g^*p$ -open subsets of X is denoted by  $\alpha g^*p$ -O(X).

### 2. Preliminaries

We recall the following definitions and notations, which are useful in the sequel.

**Definition 2.1.** A subset A of a topological space  $(X, \tau)$  is called

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- (1). preopen [7] if  $A \subseteq int$  (cl (A)) and preclosed if cl (int(A))  $\subseteq A$ .
- (2).  $\alpha$ -open [8] if  $A \subseteq int$  (cl (int (A))) and  $\alpha$ -closed if  $cl(int(cl(A))) \subseteq A$ .

**Definition 2.2.** A subset A of a topological space  $(X, \tau)$  is called

- (1). generalized closed (briefly, g-closed) [4] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in X.
- (2).  $\alpha$ -generalized closed(briefly,  $\alpha$ g-closed) [5] if  $\alpha$  cl(A)  $\subseteq$  U whenever A  $\subseteq$  U and U is open in X.
- (3). generalized preclosed (briefly, gp-closed) [6] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is open in X.
- (4). generalized star preclosed (briefly,  $g^*p$ -closed set) [13] if pcl (A)  $\subseteq$  U whenever A  $\subseteq$  U and U is g-open in X.
- (5). generalized pre star closed (briefly  $gp^*$ -closed set) [3] if cl  $(A) \subseteq U$  whenever  $A \subseteq U$  and U is gp-open in X.
- (6).  $\alpha$ -generalized star closed (briefly,  $\alpha g^*$ -closed set) if  $\alpha cl (A) \subseteq U$  whenever  $A \subseteq U$  and U is  $\alpha g$ -open in X.

**Definition 2.3** ([12]). A subset A of a topological space  $(X, \tau)$  is called alpha generalized star preclosed set (briefly,  $\alpha g^*p$ closed) if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and U is  $\alpha g$ -open in X.

**Definition 2.4** ([10]). For a subset A of  $(X, \tau)$ , the intersection of all  $\alpha g^*p$ -closed sets containing A is called the  $\alpha g^*p$ -closure of A and is denoted by  $\alpha g^*p$ -cl(A). That is,  $\alpha g^*p$ -cl(A) =  $\cap \{F : F \text{ is } \alpha g^*p$ -closed in X,  $A \subseteq F \}$ .

**Definition 2.5.** A function  $f : (X, \tau) \to (Y, \sigma)$  is called

- (1). totally continuous [2] if  $f^{-1}(V)$  is clopen in X for each open set V of Y.
- (2). totally pre-continuous [14] at each point of X if for each open subset V in Y containing f(x), there exists a pre-clopen subset U in X containing x such that  $f(U) \subset V$ .

**Definition 2.6** ([10]). A subset A of a topological space  $(X, \tau)$  is called

- (1).  $\alpha g^* p$ -continuous if  $f^{-1}(V)$  is  $\alpha g^* p$ -closed in  $(X, \tau)$  for every closed set V in  $(Y, \sigma)$ .
- (2).  $\alpha g^* p$ -irresolute if  $f^{-1}(V)$  is  $\alpha g^* p$ -closed in  $(X, \tau)$  for every  $\alpha g^* p$ -closed set V in  $(Y, \sigma)$ .
- (3).  $\alpha g^*p$ -closed if f(V) is  $\alpha g^*p$ -closed in  $(Y, \sigma)$  for every  $\alpha g^*p$ -closed set V in  $(X, \tau)$ .
- (4).  $\alpha g^*p$ -open if f(V) is  $\alpha g^*p$ -open in  $(Y, \sigma)$  for every  $\alpha g^*p$ -open set V in  $(X, \tau)$ .

**Definition 2.7** ([11]). A topological space X is said to be  $\alpha g^*p$ -connected if X cannot be written as the disjoint union of two non empty  $\alpha g^*p$ -open sets in X.

**Definition 2.8** ([10]). A space  $(X, \tau)$  is called

- (1). an  $\alpha g^*p$ -space if every  $\alpha g^*p$ -closed set is closed.
- (2). a  $T_{\alpha g*p}$ -space if every  $\alpha g*p$ -closed set is preclosed.

**Definition 2.9** ([15]). A space X is said to be

- (1).  $T_1$  if for every pair of distinct points x and y in X there exist open sets U and V such that  $x \in U$  but  $y \notin U$  and  $y \in V$  but  $x \notin V$ .
- (2).  $T_2$  if for each distinct points x, y in X, there exist two disjoint open sets U and V containing x and y respectively.

# 3. $\alpha g^* p$ -totally Continuous Functions

In this section we define  $\alpha g^*p$ -totally continuous function and investigate their fundamental properties

**Definition 3.1.** A function  $f: (X, \tau) \to (Y, \sigma)$  is said to be  $\alpha g^*p$ -totally continuous function if the inverse image of every  $\alpha g^*p$ -open set of Y is clopen in X.

**Theorem 3.2.** A bijective function  $f : (X, \tau) \to (Y, \sigma)$  is  $\alpha g^*p$ -totally continuous function if and only if the inverse image of every  $\alpha g^*p$ -closed subset of Y is clopen in X.

*Proof.* Let F be any  $\alpha g^*p$ -closed set in Y. Then Y \ F is a  $\alpha g^*p$ -open set in Y. By definition,  $f^{-1}(Y \setminus F)$  is clopen in X. That is, X \  $f^{-1}(F)$  is clopen in X. This implies  $f^{-1}(F)$  is clopen in X.

Conversely if V is  $\alpha g^*p$ -open in Y then Y \ V is  $\alpha g^*p$ -closed in Y. By assumption,  $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$  is clopen in X, which implies  $f^{-1}(V)$  is clopen in X. Therefore f is  $\alpha g^*p$ -totally continuous function.

#### Theorem 3.3.

(1). Every  $\alpha g^*p$ -totally continuous function is totally continuous.

(2). Every totally continuous function is  $\alpha g^*p$ -continuous.

#### Proof.

- (1). Let U be any open subset of Y. Since every open set is  $\alpha g^*p$ -open, U is  $\alpha g^*p$ -open in Y and  $f: (X, \tau) \to (Y, \sigma)$  is  $\alpha g^*p$ -totally continuous, it follows that  $f^{-1}(U)$  is clopen in X.
- (2). Obvious.

Remark 3.4. The converse of Theorem 3.3 is not true, which can be verified from the following examples.

**Example 3.5.** Let  $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a, b\}, \{c, d\}\}$  and  $\sigma = \{Y, \phi, \{a, b\}\}.$   $\alpha g^*p$ - $O(Y) = \{Y, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}.$  Define a function  $f : X \to Y$  by f(a) = c, f(b) = d, f(c) = a, f(d) = b. Then f is totally continuous but f is not  $\alpha g^*p$ -totally continuous , since  $f^{-1}(\{a, c\}) = \{a, c\}$  is not clopen in X.

**Example 3.6.** Let  $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \phi, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ .  $\alpha g^* p$ - $C(X) = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ . Define  $f : X \to Y$  by f(a) = c, f(b) = d, f(c) = a, f(d) = b. Then f is  $\alpha g^* p$ -continuous but f is not totally continuous, since  $f^{-1}(\{d\}) = \{b\}$  is not clopen in X.

**Theorem 3.7.** Let  $f : (X, \tau) \to (Y, \sigma)$  be a function, where X and Y are topological spaces. Then the following are equivalent:

- (1). f is  $\alpha g^*p$ -totally continuous.
- (2). For each  $x \in X$  and each  $\alpha g^*p$ -open set V in Y with  $f(x) \in V$ , there is a clopen set U in X such that  $x \in U$  and  $f(U) \subset V$ .

*Proof.* (1) $\Rightarrow$ (2) Suppose f is  $\alpha g^*p$ -totally continuous and let V be any  $\alpha g^*p$ -open set in Y containing f(x) such that  $x \in f^{-1}(V)$ . Since f is  $\alpha g^*p$ -totally continuous,  $f^{-1}(V)$  is clopen in X. Let  $U = f^{-1}(V)$  then U is a clopen set in X and  $x \in U$ . Also  $f(U) = f(f^{-1}(V)) \subset V$ . This implies  $f(U) \subset V$ .

 $(2)\Rightarrow(1)$  Let V be a  $\alpha g^*p$ -open set in Y. Let  $x \in f^{-1}(V)$  be any arbitrary point. This implies  $f(x) \in V$ . Therefore by (2) there is a clopen set  $G_x$  containing x such that  $f(G_x) \subset V$ , which implies  $G_x \subset f^{-1}(V)$  is a clopen neighbourhood of x. Since x is arbitrary, it implies  $f^{-1}(V)$  is a clopen neighbourhood of each of its points. Hence it is a clopen set in X. Therefore f is  $\alpha g^*p$ -totally continuous.

**Definition 3.8.** For a function  $f : (X, \tau) \to (Y, \sigma)$ , the subset  $\{(x, f(x)) : x \in X\} \subset X \times Y$  is called the graph of f and is denoted by G(f).

**Theorem 3.9.** A function  $f: (X, \tau) \to (Y, \sigma)$  is  $\alpha g^*p$ -totally continuous, if its graph function is  $\alpha g^*p$ -totally continuous.

*Proof.* Let  $g: X \to X \times Y$  be a graph function of  $f: X \to Y$ . Suppose g is  $\alpha g^*p$ -totally continuous and F be  $\alpha g^*p$ -open in Y, then  $X \times F$  is a  $\alpha g^*p$ -open set of  $X \times Y$ . Since f is  $\alpha g^*p$ -totally continuous,  $g^{-1}(X \times F) = f^{-1}(F)$  is clopen in X. Thus the inverse image of every  $\alpha g^*p$ -open set in Y is clopen in X. Therefore g is  $\alpha g^*p$ -totally continuous.

**Theorem 3.10.** If  $f : (X, \tau) \to (Y, \sigma)$  is  $\alpha g^*p$ -totally continuous surjection and X is connected then Y is  $\alpha g^*p$ -connected.

*Proof.* Suppose Y is not  $\alpha g^*p$ -connected, let A and B form a disconnection of Y. Then A and B are  $\alpha g^*p$ -open sets in Y and  $Y = A \cup B$  where  $A \cap B = \phi$ . Also  $f^{-1}(Y) = X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are non empty clopen sets in X, because f is  $\alpha g^*p$ -totally continuous. Further,  $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(\phi) = \phi$ . This implies X is not connected, which is a contradiction. Hence Y is  $\alpha g^*p$ -connected.

**Theorem 3.11.** If a function  $f : (X, \tau) \to (Y, \sigma)$  is totally continuous and Y is a  $\alpha g^*p$ -space then f is  $\alpha g^*p$ -totally continuous.

*Proof.* Let V be  $\alpha g^*p$ -open in Y. Since Y is a  $\alpha g^*p$ -space, V is open in Y. Also as f is totally continuous,  $f^{-1}(V)$  is open and closed in X. Hence  $f^{-1}(V)$  is clopen in X. Therefore f is  $\alpha g^*p$ -totally continuous.

#### Theorem 3.12.

(1). If  $f: X \to Y$  and  $g: Y \to Z$  are  $\alpha g^*p$ -totally continuous, then  $g \circ f: X \to Z$  is also  $\alpha g^*p$ -totally continuous.

(2). If  $f: X \to Y$  is  $\alpha g^*p$ -totally continuous and  $g: Y \to Z$  is  $\alpha g^*p$ -continuous, then  $g \circ f: X \to Z$  is totally continuous.

**Theorem 3.13.** Let  $f: X \to Y$  be a  $\alpha g^*p$ -open map and  $g: Y \to Z$  be any function. If  $g \circ f: X \to Z$  is  $\alpha g^*p$ -totally continuous, then g is  $\alpha g^*p$ -irresolute.

*Proof.* Let  $g \circ f : X \to Z$  be  $\alpha g^*p$ -totally continuous. Let V be  $\alpha g^*p$ -open set in Z. Since  $g \circ f$  is  $\alpha g^*p$ -totally continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is clopen in X. Since f is  $\alpha g^*p$ -open,  $f(f^{-1}(g^{-1}(V)))$  is  $\alpha g^*p$ -open in Y. Then  $g^{-1}(V)$  is  $\alpha g^*p$ -open in Y. Hence g is  $\alpha g^*p$ -irresolute.

**Theorem 3.14.** Let  $f : X \to Y$  be  $\alpha g^*p$ -totally continuous and  $g : Y \to Z$  be any function, then  $g \circ f : X \to Z$  is  $\alpha g^*p$ -totally continuous if and only if g is  $\alpha g^*p$ -irresolute.

*Proof.* Let V be a  $\alpha g^*p$ -open subset of Z. Then  $g^{-1}(V)$  is  $\alpha g^*p$ -open in Y as g is  $\alpha g^*p$ -irresolute. Then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is clopen in X. Hence  $g \circ f : X \to Z$  is  $\alpha g^*p$ -totally continuous. Conversely, let  $g \circ f : X \to Z$  be  $\alpha g^*p$ -totally continuous. Let V be a  $\alpha g^*p$ -open set in Z, then  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is clopen in X. Since f is  $\alpha g^*p$ -totally continuous,  $g^{-1}(V)$  is  $\alpha g^*p$ -open in Y. Hence g is  $\alpha g^*p$ -irresolute.

# 4. Totally $\alpha g^*p$ -continuous Functions

In this section we define the totally  $\alpha g^*p$ -continuous and strongly  $(\alpha g^*p)^*$ -continuous functions and investigate their fundamental properties

**Definition 4.1.** A function  $f : (X, \tau) \to (Y, \sigma)$  is called

- (1). totally  $\alpha g^*p$ -continuous at a point  $x \in X$  if for each open subset V in Y containing f(x), there exists a  $\alpha g^*p$ -clopen subset U in X containing x such that  $f(U) \subset V$ .
- (2). totally  $\alpha g^*p$ -continuous if it has this property at each point of X.

**Theorem 4.2.** The following statements are equivalent for a function  $f : (X, \tau) \to (Y, \sigma)$ , whenever the class of  $\alpha g^*p$ -closed sets in  $(X, \tau)$  are closed under finite union:

- (1). f is totally  $\alpha g^*p$ -continuous.
- (2). For every open set V of Y,  $f^{-1}(V)$  is  $\alpha g^*p$ -clopen in X.

*Proof.*  $(1)\Rightarrow(2)$  Let V be an open subset of Y and let  $x \in f^{-1}(V)$ . Since  $f(x) \in V$ , by (1), there exists a  $\alpha g^*p$ -clopen set  $U_x$  in X containing x such that  $U_x \subset f^{-1}(V)$ . We obtain  $f^{-1}(V) = \bigcup \{U_x : x \in f^{-1}(V)\}$ . Thus  $f^{-1}(V)$  is  $\alpha g^*p$ -clopen in X. (2) $\Rightarrow(1)$  Straightforward.

**Definition 4.3.** A function  $f : (X, \tau) \to (Y, \sigma)$  is said to be strongly  $(\alpha g^* p)^*$ -continuous if the inverse image of every  $\alpha g^* p$ -open set of  $(Y, \sigma)$  is  $\alpha g^* p$ -clopen in  $(X, \tau)$ .

#### Theorem 4.4.

- (1). Every strongly  $(\alpha g^*p)^*$ -continuous function is totally  $\alpha g^*p$ -continuous.
- (2). Every totally  $\alpha g^*p$ -continuous function is  $\alpha g^*p$ -continuous.
- (3). Every totally continuous function is totally  $\alpha g^*p$ -continuous.
- (4). Every  $\alpha g^*p$ -totally continuous function is totally  $\alpha g^*p$ -continuous.

*Proof.* (1) Let V be an open set in Y. Then V is  $\alpha g^*p$ -open in Y. Then  $f^{-1}(V)$  is  $\alpha g^*p$ -clopen in X as f is a strongly  $(\alpha g^*p)^*$ -continuous function. Hence f is totally  $\alpha g^*p$ -continuous.

**Remark 4.5.** The converse of Theorem 4.4 is not true, which can be verified from the following examples.

**Example 4.6.** Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{X, \phi, \{a, b\}\}$  and  $\sigma = \{Y, \phi, \{a, b\}, \{c, d\}\}$ . Define  $f : X \to Y$  by f(a) = d, f(b) = a, f(c) = c, f(d) = b. Then f is totally  $\alpha g^* p$ -continuous but f is not strongly  $(\alpha g^* p)^*$ -continuous, totally continuous and  $\alpha g^* p$ -totally continuous.

**Example 4.7.** Let  $X = Y = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}, and \sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}.$   $\alpha g^* p \cdot C(X) = \{X, \phi, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}.$  Define  $f : X \to Y$  by f(a) = b, f(b) = c, f(c) = a, f(d) = d. Then f is  $\alpha g^* p$ -continuous but f is not totally  $\alpha g^* p$ -continuous.

**Theorem 4.8.** If  $f : (X, \tau) \to (Y, \sigma)$  is a totally  $\alpha g^* p$ -continuous map from a  $\alpha g^* p$ -connected space  $(X, \tau)$  onto a space  $(Y, \sigma)$ , then  $(Y, \sigma)$  is an indiscrete space.

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*Proof.* On the contrary, suppose that  $(Y, \sigma)$  is not an indiscrete space. Let A be a proper non-empty open subset of  $(Y, \sigma)$ . Since f is totally  $\alpha g^*p$ -continuous map, then  $f^{-1}(A)$  is a proper non-empty  $\alpha g^*p$ -clopen subset of X. Then  $X = f^{-1}(A) \cup (X \setminus f^{-1}(A))$  which is a contradiction to the fact that X is  $\alpha g^*p$ -connected. Therefore Y must be an indiscrete space.  $\Box$ 

**Theorem 4.9.** Let  $f: (X, \tau) \to (Y, \sigma)$  be a totally  $\alpha g^*p$ -continuous map and Y be a  $T_1$ -space. If A is a non-empty subset of a  $\alpha g^*p$ -connected space X, then f(A) is singleton.

*Proof.* Suppose if possible f(A) is not singleton, let  $f(x_1) = y_1 \in A$  and  $f(x_2) = y_2 \in A$ . Since  $y_1, y_2 \in Y$  and Y is a  $T_1$  space, then there exists an open set G in  $(Y, \sigma)$  containing  $y_1$  but not  $y_2$ . Since f is totally  $\alpha g^*p$ -continuous,  $f^{-1}(G)$  is a  $\alpha g^*p$ -clopen set containing  $x_1$  but not  $x_2$ . Now  $X = f^{-1}(G) \cup (X \setminus f^{-1}(G))$ . Thus X is a union of two non-empty  $\alpha g^*p$ -open sets which is a contradiction.

**Definition 4.10.** A space  $(X, \tau)$  is said to be

- (1).  $\alpha g^* p$ -co- $T_1$  if for each pair of disjoint points x and y of X, there exists  $\alpha g^* p$ -clopen sets U and V containing x and y, respectively such that  $x \in U$ ,  $y \notin U$  and  $x \notin V$ ,  $y \in V$ .
- (2).  $\alpha g^* p$ -co- $T_2$  if for each pair of disjoint points x and y of X, there exists  $\alpha g^* p$ -clopen sets U and V in X, respectively such that  $x \in U$  and  $y \in V$ .
- (3).  $\alpha g^*p$ -co-Hausdorff if every two distinct points of X can be separated by disjoint  $\alpha g^*p$ -clopen sets.

**Theorem 4.11.** If  $f : (X, \tau) \to (Y, \sigma)$  is totally  $\alpha g^* p$ -continuous injective function and Y is a  $T_1$  space, then X is  $\alpha g^* p$ -co- $T_1$ .

*Proof.* Since Y is  $T_1$ , for any distinct points x and y in X, there exists open sets V, W in Y such that  $f(x) \in V$ ,  $f(y) \notin V$ ,  $f(x) \notin W$  and  $f(y) \in W$ . Since f is totally  $\alpha g^*p$ -continuous,  $f^{-1}(V)$  and  $f^{-1}(W)$  are  $\alpha g^*p$ -clopen subsets of  $(X, \tau)$  such that  $x \in f^{-1}(V)$ ,  $y \notin f^{-1}(V)$ ,  $x \notin f^{-1}(W)$  and  $y \in f^{-1}(W)$ . This shows that X is  $\alpha g^*p$ -co- $T_1$ .

**Theorem 4.12.** If  $f : (X, \tau) \to (Y, \sigma)$  is totally  $\alpha g^* p$ -continuous injective function and Y is a T<sub>2</sub>-space, then X is  $\alpha g^* p$ -co-T<sub>2</sub>.

*Proof.* For any distinct points x and y in X, there exists disjoint open sets U and V in Y such that  $f(x) \in U$  and  $f(y) \in V$ and  $U \cap V = \phi$ . Since f is totally  $\alpha g^* p$ -continuous,  $f^{-1}(U)$  and  $f^{-1}(V)$  are  $\alpha g^* p$ -clopen in X containing x and y respectively. Therefore  $f^{-1}(U) \cap f^{-1}(V) = \phi$  because  $U \cap V = \phi$ . This shows that X is  $\alpha g^* p$ -co- $T_2$ .

**Theorem 4.13.** Let  $f : (X, \tau) \to (Y, \sigma)$  be a totally  $\alpha g^* p$ -continuous injective function. If Y is hausdorff, then X is  $\alpha g^* p$ -co-Hausdorff.

*Proof.* Let  $x_1$  and  $x_2$  be two distinct points of X. Since f is injective and Y is Hausdorff, there exists open sets  $V_1$  and  $V_2$  in Y such that  $f(x_1) \in V_1$ ,  $f(x_2) \in V_2$  and  $V_1 \cap V_2 = \phi$ . By previous theorem,  $x_i \in f^{-1}(V_i) \in \alpha g^*p$ -clopen (X) for i=1, 2 and  $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$ . Thus X is  $\alpha g^*p$ -co-Hausdorff.

Definition 4.14. A space X is said to be

- (1).  $\alpha g^*p$ -co-compact if every  $\alpha g^*p$ -clopen cover of X has a finite subcover.
- (2).  $\alpha g^*p$ -co-compact relative to X if every cover of a  $\alpha g^*p$ -clopen set of X has a finite subcover.
- (3). countably  $\alpha g^*p$ -co-compact if every countable cover of X by  $\alpha g^*p$ -clopen sets has a finite subcover.

(4).  $\alpha g^*p$ -co-Lindelof if every  $\alpha g^*p$ -clopen cover of X has a countable subcover.

**Theorem 4.15.** Let  $f : (X, \tau) \to (Y, \sigma)$  be a totally  $\alpha g^* p$ -continuous surjective function. Then the following statements hold.

- (1). If X is  $\alpha g^*p$ -co-Lindelof then Y is Lindelof.
- (2). If X is countably  $\alpha g^*p$ -co-compact then Y is countably compact.
- (3). If X is  $\alpha g^*p$ -co-compact then Y is compact.
- (4). If X is countably  $\alpha g^*p$ -co-compact then Y is countably compact.

*Proof.* Let  $\{V_{\alpha} : \alpha \in I\}$  be an open cover of Y. Since f is totally  $\alpha g^*p$ -continuous,  $\{f^{-1}(V_{\alpha}) : \alpha \in I\}$  is a  $\alpha g^*p$ -clopen cover of X. Since X is  $\alpha g^*p$ -co-Lindelof, there exists a countable subset  $I_0$  of I such that  $X = \bigcup \{V_{\alpha} : \alpha \in I\}$  and hence Y is Lindelof. Proof of 2 to 4 is similar.

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