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Lattice Labelling of a Graph

Research Article

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Abstract: Let $G(u, v)$ be a connected graph. Let $L = (L, \wedge, \vee)$ be a lattice under partial order τ. Let $f : v \longrightarrow L$ be a injective function. Let $f^{\star} : E \longrightarrow L'$ be the function induced by f such that $f^{\star}(xy) = [f(x) \vee f(y)] - [f(x) \wedge f(y)]$ then we say f is a τ labeling if L is injective and L' is a subset of L. A τ labeling is called lattice labeling if L' is a sub lattice of L. In this paper we give lattice labeling of certain graphs.

Keywords: Graph Labeling, Lattice labelling and τ labelling.

1. Introduction

Lots of research work is been carried out in the labeling of graphs in past few work since the first initiated by A. Rosa[1]. Due to development of information technology and solving typical algorithms of coloring problems graph labeling is extremely useful. For a large network of transmitters spread out in a planar region, the channel assignment problem is to assign a numerical channel representing frequency, to each transmitter. The channels assigned to nearby transmitters satisfy some separation constraints so as to avoid interference. The goal is to minimize the portion of the frequency spectrum that must be allocated to the problem, so that it is desire to minimize the span(the largest frequency) of the feasible assignment. labelling is an assignment of positive integers to the vertices.

A nonempty set L closed under two binary operation,$\wedge$ and $\vee$ is called a lattice $(L, \wedge, \vee)$ provided following axioms hold:Idempotent law, Commutative law, Associative law, Absorption law. Lattice is a Poset $(L, \leq)$ in which every 2-element subset{a,b} has a lub and glb.That is, poset $(L, \leq)$ is a lattice if for every $a, b \in L$,lub(a,b) and glb(a,b) exist in L,$a \vee b = LCM(a, b)$,the least common multiple of a and b ,$a \wedge b = GCD(a, b)$ the greatest common divisor of a and b .

In many settings the flow of information from one person or computer program to another is restricted via security clearances. We can use lattice model represent different information flow policies.For example, one common information flow policy is the multilevel security policy used in government and military systems. Each piece of information is assigned to a security class, and each security class is represented by a pair (A,C) where A is an authority level and C is the category. people and computer programs are then allowed access to information from a specific restricted set of security classes.

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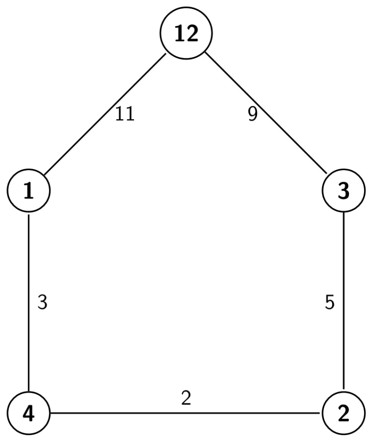
Let $L = (L, \wedge, \vee)$ be a lattice under the partial order τ. Let $f^\star : E \longrightarrow L'$ be the function induced by such that $f^\star(xy) = [f(x) \vee f(y)] - [f(x) \wedge f(y)]$ then we say f is a τ labeling of L is injective and L' is a subset of L. A τ labelling if L is injective and L' is a subset of L. a τ labelling is called lattice labelling if L' is a sub lattice of L.

Example 1.1. *Consider a lattice* $< Z_{12}, / >$ *where* $Z_{12} = \{0, 1, 2, 3...12\}$ *where the operation* $\wedge$ *and* $\vee$ *defined by* $a \wedge b = gcd(a, b)$ *and* $a \vee b = lcm(a, b)$ $\forall a, b \in Z_n$*.*

Consider the graph G, Take Z_{12} and f as shown in figure.



this f is τ-labeling but not lattice labeling

Theorem 1.2. *For any cycle* C_n $\exists$ *a bijective function* $f : V \longrightarrow \{1, 2, 3...n\}$ *such that a edge induced function* $f^* : E \longrightarrow S$, $S \subset N$ *defined by* $f^\star(ab) = lcm(f(a), f(b)) - gcd(f(a), f(b))$ *is injective.*

Proof. Let $a_1, a_2, ...a_n$ be the vertices of C_n such that $(a_i, a_j) \in E$ iff $j = i+1$. Define $f : V \longrightarrow S$ where $S = \{1, 2, 3, ...n\}$ by $f(a_i) = i$ then we show that $f^\star : E \longrightarrow S^\star \subset Z$ defined by $f^\star(ab) = lcm(f(a), f(b)) - gcd(f(a), f(b))$ is injective. Infact, Suppose let $e_1 = a_i, a_{i+1} and e_2 = a_j, a_{j+1}$ receive the same label that is

$$lcm(i, i+1) - gcd(i, i+1) = lcm(j, j+1) - gcd(j, j+1)$$

$$i(i+1) = j(j+1)$$

$$i^2 + i = j^2 + j$$

$$i^2 - j^2 + i - j = 0$$

$$(i+j)(i-j) + (i-j) = 0$$

$$(i-j)(i+j+1) = 0$$

$$i = j \quad i+j+1 = 0$$

$$i + J = -1(not \quad admissible \because i, j > 0)$$

$$i = j$$

□

Theorem 1.3. *Lattice labelling of star* $L_n[K_{1,n}] = n$

Proof. Consider a star $K_{1,n}$

To prove $L_n(K_{1,n}) = n$ we have to prove the following cases, (i) $L_n(K_{1,n}) \leq n$ (ii) $L_n(K_{1,n}) \geq n$

Case 1 : We first prove that

$$L_n(K_{1,n}) \leq n \tag{1}$$

label the central vertex by 1 and for other vertices v label using $K \leq n$. Therefore $lcm - gcd = k - 1 \leq n$

Case 2 : On the other hand

$$L_n(K_{1,n}) \geq n \tag{2}$$

$(K_n) - 1 \geq n$ or $(K_n) - 1 \geq n + 1$. If k is a central labelling then as either $k\chi_n$ or $k\chi_{n+1}$; $(kn) - 1 \geq n$ or $k(n+1) - 1 \geq n$. □

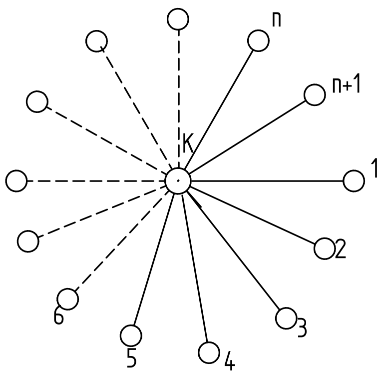


Figure 1. **Star** $K_{1,n}$

Theorem 1.4. *Every path p_n on n vertices is a lattice graph.*

Proof. If $n = 1$ Z is obvious. Every chain on n elements is isomorphic to path p_n now consider a path p_n, Now consider a path p_n whose vertices are labelled by integers 1 to n ∋: x is adjacent to $x + 1$ $\forall x$ for $1 \leq x \leq n$. Let $L = \{2^m / m \in Z_{\{}n/\{0\}\}$ then L is a chain under division which is isomorphic to c. Define $f : v(p_n) \longrightarrow L$ by $f(x) = 2^x$ $\forall x \in v(p_n)$ now consider the induced function $f^\star : E(p_n) \longrightarrow L'$ defined by

$$f^\star(ab) = f(a) \wedge f(b) - f(a) \vee f(b).$$

Since for each edge $ab \in E(P_n)$; $b = a + 1$

$$f(ab) = f(a, a+1) = 2^{a+1} - 2^a = 2^a \in L$$

$L' = R(f^\star) \subseteq L$ further $< L', / >= \{2^a / a \in Z_{\{n-1\}/\{0\}\}}$ is a chain so it is itself a lattice. Hence the proof. □

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