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Balanced Mean Cordial Labeling and Graph Operations

Research Article

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Abstract: Balanced mean cordial labeling is a mean cordial labeling f with $|v_f(i) - v_f(j)| = 0$, $|e_f(i) - e_f(j)| = 0$, $\forall\, i, j \in \{0, 1, 2\}$. In this paper, we investigate mean cordial labeling for P$(t \cdot H)$, where H be any graph and $t \equiv 0 (\text{mod } 3)$. We also investigate balanced mean cordial labeling for TP$(t \cdot H)$, G*, P$_t \times G$, C$_t \times G$, where H and G both are balanced cordial graphs.

Keywords: Path Union, Mean Cordial, Balanced mean cordial.

1. Introduction

Mean cordial labeling of a graph defined by Ponraj etal.[4] and they investigated mean cordial labeling for P$_n$, C$_n$, K$_{1,n}$, K$_n$, W$_n$, P$_n$ ∪ P$_m$, P_n^2 and triangular snake. Balanced mean cordial graph, which is a mean cordial graph with additional condition $|v_f(i) - v_f(j)| = |e_f(i) - e_f(j)| = 0\ \forall\, i, j \in \{0, 1, 2\}$. Path union of a graph G obtained by t copies G$^{(1)}$, G$^{(2)}$,..., G$^{(t)}$ of the graph G and it is denoted by P$(t \cdot G)$. It is obtained by joining a vertex v of G$^{(i)}$ with same vertex of G$^{(i+1)}$ by an edge, $\forall\, i = 1, 2, \ldots, t-1$. It is obvious that P$(t \cdot G)$ can be obtained by $|V(G)|$ different ways and P$(t \cdot k_1)$=P_t. Star of a graph G is denoted by G^* and is obtain by p+1 copies G$^{(0)}$, G$^{(1)}$,..., G$^{(p)}$ of a graph G with V(G)= $\{v_1, v_2, \cdots v_p\}$. It is obtained by joining each vertex v_i of $G^{(0)}$ with the corresponding vertex v_i of $G^{(i)}$, $\forall\, i = 1, 2, \ldots, p$. We call $G^{(0)}$ as central copy of G^*.

It is obvious that K_1^*=K_2, K_2^*=P_6. Let G be a graph and G$^{(1)}$, G$^{(2)}$,..., G$^{(t)}$ (t≥ 2) be t copies of G. Then the graph obtained by joining a triplet of distinct vertices say u, v, w of $G^{(i)}$ with same vertices of the graph $G^{(i+1)}$ by three distinct edges, $\forall\ i = 1, 2, \ldots, t-1$ is called triple path union of t copies of the graph G, such graph we obtain by ${}^pC_3$ different ways, where p=$|V(G)|$. We denote such graph by TP$(t \cdot G)$ and it is obvious that TP$(t \cdot P_3)$= $P_t \times P_3$, TP$(t \cdot C_3)$= $P_t \times C_3$.

All graphs in this paper are finite, undirected. The vertex set of graph G and the edge set of G are denoted by V(G) and E(G) respectively. Take p= $|V(G)|$ and q= $|E(G)|$. Terminology not defined here are used in the sense of Harary [2]. Double path union of a graph G defined by kaneria, Teraiya and meera [3] and proved that $D(n \cdot K_{r,s})$, $D(n \cdot P_m)$, $D(n \cdot C_m)$ ($m \equiv 0 (\text{mod } 4)$) are α-graceful graphs.

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2. Main Results

Theorem 2.1. *Path union of t copies of a graph H is a mean cordial, where $t \equiv 0 (mod\ 3)$.*

Proof. Let H be a graph with $p = |V(H)|$, $q = |E(H)|$. Let $t = 3s$ for some s∈ N. Let V(H) = {v_1, v_2, ... v_p} and G=P($t \cdot G$). Take V(G)={$v_{i,1}$,$v_{i,2}$,...,$v_{i,p}$ / i=1,2,... ,t}. For each i=1,2,.., $t-1$, join $v_{i,k}$ with $v_{i+1,k}$ by an edge to form P($t \cdot G$), for some k∈{1,2,...,p}. Define vertex labeling function f:V(G)⟶{0,1,2} as follows.

f($v_{i,j}$) = 2 ; if 1≤ i ≤ s

= 1 ; if s+1≤ i ≤ 2s

= 0 ; if 2s+1≤ i ≤ 3s,∀ j=1,2,· · · ,p.

It is obvious to that $v_f(2)$=$v_f(1)$=$v_f(0)$=sp, $e_f(2)$=s(q+1)=$e_f(1)$, $e_f(0)$=sq+s-1 in P($t \cdot H$). Because for each ($v_{i,k}$, $v_{i+1,k}$) ∈ E(G),

$f^*((v_{i,k})$+f$(v_{i+1,k}))$=$\lceil \frac{f(v_{i,k})+f(v_{i+1,k})}{2} \rceil$

= 2 ; if 1≤ i ≤ s

= 1 ; if s+1≤ i ≤ 2s

= 0 ; if 2s+1≤ i ≤ $3s-1$

Thus, above defined labeling function f give rise a mean cordial labeling to the graph G and so, it is a mean cordial graph. □

Theorem 2.2. *Let H be a balanced mean cordial graph and f:V(H)⟶ {0, 1, 2} be a balanced mean cordial labeling for H. Let u, v, w ∈ V(H) be such that f(u)=2, f(v)=1 and f(w)=0. Then the triple path union of t copies of H obtained by joining the triplet u, v, w in H is also balanced mean cordial graph.*

Proof. Let V(H)={v_1, v_2, ... v_p}, where $p = |V(H)|$. Let f:V(H)⟶ {0, 1, 2} be a balanced mean cordial labeling for H. Let u, v, w ∈ V(H) be such that f(u)=2, f(v)=1, f(w)=0. Let G=TP($t \cdot H$) and V($H^{(i)}$)={$v_j^{(i)}$/ 1 ≤ j ≤ p} the set of vertices of i^{th} copy of H in TP($t \cdot H$). Now join vertices $u^{(i)}$, $v^{(i)}$, $w^{(i)}$ of $H^{(i)}$ with $u^{(i+1)}$, $v^{(i+1)}$, $w^{(i+1)}$ respectively by three distinct edges , ∀ i=1, 2. ... $t-1$. It is obvious that V(G)= $\bigcup_{i=1}^{t}$ V($H^{(i)}$) and E(G)= $\bigcup_{i=1}^{t}$ E($H^{(i)}$)∪{($u^{(i)}$,$u^{(i+1)}$),($v^{(i)}$,$v^{(i+1)}$), ($w^{(i)}$,$w^{(i+1)}$)/i=1,2,... t-1}. Define a vertex labeling function g:V(G)⟶ {0, 1, 2} by $g(v_j^{(i)})$=f(v_j), ∀ j=1,2,... p, ∀ i=1,2,... t. It is observed that $v_f(2)$=$v_f(1)$=$v_f(0)$=$\frac{p}{3}$, $e_f(2)$=$e_f(1)$=$e_f(0)$=$\frac{q}{3}$, as f is a balanced cordial labeling function for H . Therefore, $v_g(2)$=$v_g(1)$=$v_g(0)$=$\frac{tp}{3}$, $e_g(2)$=$e_g(1)$=$e_g(0)$= $\frac{tq}{3} + (t-1)$. Because for each i=1, 2, ..., t−1. g(($u^{(i)}$,$u^{(i+1)}$)=2,g(($v^{(i)}$,$v^{(i+1)}$)=1, g(($w^{(i)}$, $w^{(i+1)}$)=0. Thus, above defined labeling pattern g give rise a balanced mean cordial labeling to the graph TP($t \cdot H$) and so, it is balanced mean cordial. □

Theorem 2.3. *Let G be a balanced mean cordial graph and f:V(G)⟶ {0, 1, 2} be a balanced mean cordial labeling for G. Then G^* is also a balanced mean cordial graph.*

Proof. Let V(G)={v_1, v_2, ... v_p} and f:V(G)⟶ {0, 1, 2} be a balanced mean cordial labeling for G. V($G^{(i)}$)={$v_j^{(i)}$/ 1 ≤ j ≤p} be the vertex set of i^{th} copy of G in G^*, 0≤i≤p. Now join each vertex $v_i^{(0)}$ of $G_i^{(0)}$ with the corresponding vertex $v_i^{(i)}$ of $G_i^{(i)}$ by an edge, ∀ i=1,2,.., p to form G^*. It is obvious that V(G^*)= $\bigcup_{i=0}^{t} V(G^{(i)}) = \{v_j^{(i)}/\ 0 \leq j \leq p,\ 1 \leq i \leq p\}$ and E(G^*)= $\bigcup_{i=0}^{p} E(G) \cup \{(u_i^{(0)},u_i^{(i)})$/i=1, 2, ..., p}. Define a vertex labeling function $g : V(G^*) \longrightarrow \{0,1,2\}$ by $g(v_j^{(i)}) = f(v_j)$, $\forall\ j = 1,2,\dots,p$, $\forall\ i = 1,2,\dots,p$. Above defined labeling pattern g give rise $v_g(2)$=$v_g(1)$=$v_g(0)$=$\frac{(p+1)p}{3}$ and $e_g(2)$=$e_g(1)$= $e_g(0)$= $\frac{(p+1)q}{3}+\frac{p}{3}$, as for each i=1,2, ..., p, $g^*((u_i^{(0)},u_i^{(i)}))$=$f(u_i)$. Thus, g^* is a balanced mean cordial graph. □

Theorem 2.4. *Let G be a balanced mean cordial graph and f be a balanced mean cordial labeling for G. Then $P_t \times G$ is also a balanced mean cordial graph.*

Proof. Let G, V(G) and f are same as discussed in the Theorem 2.3. Now join each vertex $v_j^{(i)}$ of $G^{(i)}$ with corresponding vertex $v_j^{(i+1)}$ of $G^{(i+1)}$ by an edge, $\forall\ i = 1, 2, \ldots, t-1$, $\forall\ j = 1, 2, \ldots, p$ to form $P_t \times G$, where $G^{(i)}$ is the i^{th} copy of graph G. Take V(P_t × G)= $\bigcup_{i=1}^{t} V(G^{(i)}) = \{v_j^{(i)}\ /\ 1 \leq j \leq p,\ 1 \leq i \leq t\}$ and E(P_t × G)=$\bigcup_{i=1}^{t} E(G^{(i)})$ ∪{($v_j^{(i)}$,$v_j^{(i+1)}$)/i=1, 2, …, $t-1$, $j = 1, 2, \ldots, p\}$. Define a vertex labeling function g:V(P_t × G)⟶{0, 1, 2} by g($v_j^{(i)}$)=f(v_j), ∀ j= 1, 2, …, p,∀ i=1, 2, …, t. Above defined labeling pattern g give rise to $v_g(2)=v_g(1)=v_g(0)=\frac{tp}{3}$ and $e_g(2)= e_g(1)= e_g(0)= \frac{tq}{3}+\frac{(t-1)p}{3}$, as for each i=1, 2, …, $t-1$, j=1, 2, …, p, $g^*((v_j^{(i)},v_j^{(i+1)}))$=f($v_j$), Thus, P_t × G is a balanced mean cordial. □

Illustration 2.5. *A balanced mean cordial graph G, and its balanced mean cordial labeling and balanced mean cordial labeling for P_4 × G are shown in*

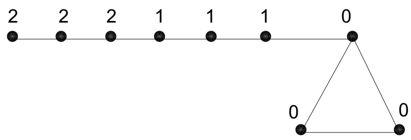

Figure 1. **A balanced mean cordial labeling of graph G**

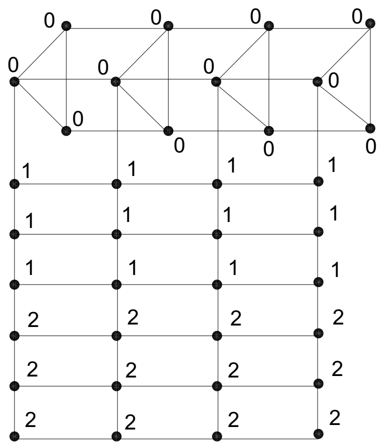

Figure 2. **Balanced mean cordial labeling for p_4 × G**

Theorem 2.6. *If G is a balanced mean cordial graph and f is a balanced mean cordial labeling for G, then so is C_t × G.*

Proof. Let G, V(G), f, V($G^{(i)}$) are same as discussed in the Theorem 2.3. It is obvious that V(C_t × G)=V(P_t × G), E(C_t × G)=E(P_t × G) ∪{($v_j^{(t)}$,$v_j^{(1)}$)/j=1, 2, …, p}. If we define a vertex labeling function on V(C_t × G), as it is already defined in Theorem 2.4 on V(P_t × G), then such labeling function g give rise to $v_g(2)=v_g(1)=v_g(0)=\frac{tp}{3}$ and $e_g(2)=e_g(1)=e_g(0)=\frac{t(p+q)}{3}$. Thus, C_t × G is a balanced mean cordial graph. □

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