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Generalization of Common Coupled Fixed Point Theorems in Complex Valued b-Metric Spaces

Research Article

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Abstract: Recently, Azam et al. [1] introduced the complex valued metric space and obtained sufficient conditions for the existence of common fixed points. Rao et al. [20] introduce the notion of complex valued b-metric spaces. In this paper, some common coupled fixed point theorems have been established for a pair of mappings in a complete complex valued b-metric space in view of diverse contractive conditions. Our results extend and improve several fixed point theorems in the literature.

MSC: 47H10, 54H25.

Keywords: Common fixed point, Coupled fixed point, Complete complex valued b-metric space, Complex-valued metric spaces.

1. Introduction and Preliminaries

The Banach fixed point theorem [6] is the first important result in fixed point theory. There are a lot of generalizations of the Banach contraction mapping principle in the literature. The concept of b-metric space was introduced by Bakhtin [5] and Czerwik [9]. They proved the contraction mapping principle in b-metric spaces that generalized the famous Banach contraction mapping principle in metric spaces.

Azam et al. [1] first introduced the concept of complex valued metric spaces and proved some common fixed point theorems for a pair of contractive type mappings satisfying a rational inequality. Many authors have been studied several fixed point and common fixed point results for two maps satisfying rational inequality in the context of complex valued-metric spaces [7, 13, 19, 21, 22].

In 2013, Rao et al. [20] introduced the notion of complex valued b-metric space which was more general than the well known complex valued metric spaces [1]. In sequel, AA.Mukheimer [16] proved some common fixed point theorems of two self mappings satisfying some contraction condition on complex valued b-metric spaces.

In [8], Bhaskar and Lakshmikantham introduced the concept of coupled fixed points for a given partially ordered set X. Subsequently, Samet et al. [23] proved the most of the coupled fixed point theorems on ordered metric spaces. The purpose of the present paper is to extend and generalize the results of Kutbi et al. [13] and prove the existence and

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uniqueness of the common coupled fixed point in complete complex valued b-metric space in view of diverse contractive conditions. The results given in this paper substantially extend and strengthen the results given in [1, 10, 11, 13, 16, 20].

The following definitions and results will be needed in the sequel. Let $\mathbb{C}$ be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1 \precsim z_2 \text{ if and only if } Re(z_1) \leq Re(z_2), Im(z_1) \leq Im(z_2).$$

Consequently, one can infer that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(1). $Re(z_1) = Re(z_2), Im(z_1) < Im(z_2)$,

(2). $Re(z_1) < Re(z_2), Im(z_1) = Im(z_2)$,

(3). $Re(z_1) < Re(z_2), Im(z_1) < Im(z_2)$,

(4). $Re(z_1) = Re(z_2), Im(z_1) = Im(z_2)$.

In particular, we write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (1), (2) and (3) is satisfied and we write $z_1 \prec z_2$ if only (3) is satisfied. Notice that

(1). $0 \precsim z_1 \precnsim z_2 \Rightarrow |z_1| < |z_2|$,

(2). $z_1 \precsim z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3$.

Definition 1.1 ([20]). *Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{C}$ is called a complex valued b-metric on X if for all $x, y, z \in X$ the following conditions are satisfied:*

(1). $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;

(2). $d(x, y) = d(y, x)$;

(3). $d(x, y) \precsim s[d(x, z) + d(z, y)]$.

The pair (X, d) is called a complex valued b-metric space.

Example 1.2 ([20]). *Let $X = [0, 1]$. Define the mapping $d : X \times X \to \mathbb{C}$ by $d(x, y) = |x - y|^2 + i|x - y|^2$ for all $x, y \in X$. Then (X, d) is a complex valued b-metric space with $s = 2$.*

Definition 1.3 ([20]). *Let (X, d) be a complex valued b-metric space.*

(1). A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x, r) = \{y \in X : d(x, y) \prec r\} \subseteq A$, where $B(x, r)$ is an open ball. Then $\overline{B(x, r)} = \{y \in X : d(x, y) \precsim r\}$ is a closed ball.

(2). A point $x \in X$ is called a limit point of a set A whenever for every $0 \prec r \in \mathbb{C}, B(x, r) \cap (A - \{x\}) \neq \phi$.

(3). A subset $A \subseteq X$ is called open set whenever each element of A is an interior point of A.

(4). A subset $B \subseteq X$ is called closed set whenever each limit point of B belongs to B. The family $F = \{B(x, r) : x \in X, 0 \prec r\}$ is a sub-basis for a Hausdorff topology τ on X.

Definition 1.4 ([20]). *Let (X, d) be a complex valued b-metric space, $\{x_n\}$ be a sequence in X and $x \in X$.*

(1). If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$converges to x and x is the limit point of $\{x_n\}$. We denote this by $lim_{n\to\infty}x_n = x$ or $\{x_n\} \to x\, as\, n \to \infty$.

(2). If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $N \in \mathbb{N}$ such that for all $n > N$, $d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$, then $\{x_n\}$is said to be Cauchy sequence.

(3). If every Cauchy sequence in X is convergent, then (X, d) is said to be a complete complex valued b-metric space.

Lemma 1.5 ([20]). *Let (X, d) be a complex valued b-metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 1.6 ([20]). *Let (X, d) be a complex valued b-metric space and let $\{x_n\}$be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$, where $m \in \mathbb{N}$.*

2. Common Coupled Fixed Point Theorems

Theorem 2.1. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $S, T : X \times X \to X$ are mappings satisfying:*

$$d(S(x,y),T(u,v)) \precsim \frac{\propto (d(x,u)+d(y,v))}{2} + \frac{(\beta d(x,S(x,y))d(u,T(u,v)) + \gamma d(u,S(x,y))d(x,T(u,v)))}{(1+d(x,u)+d(y,v))} \tag{1}$$

for all $x, y, u, v \in X$ and $\propto, \beta$ and γ are nonnegative reals with $s \propto +\beta < 1$ and $\alpha + \gamma < 1$. Then S and T have a unique common coupled fixed point.

Proof. Let x_0 and y_0 be arbitrary points in X. Define

$$x_{2n+1} = S(x_{2n}, y_{2n}), y_{2n+1} = S(y_{2n}, x_{2n}) \text{ and}$$
$$x_{2n+2} = T(x_{2n+1}, y_{2n+1}), y_{2n+2} = T(y_{2n+1}, x_{2n+1}) \tag{2}$$

for $n = 0, 1, 2, \ldots$. Now, we show that the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X Then,

$$\begin{aligned}
d(x_{2n+1}, x_{2n+2}) &= d(S(x_{2n}, y_{2n}), T(x_{2n+1}, y_{2n+1})) \\
&\precsim \frac{\propto (d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}))}{2} \\
&\quad + \frac{(\beta d(x_{2n}, S(x_{2n}, y_{2n}))d(x_{2n+1}, T(x_{2n+1}, y_{2n+1})) + \gamma d(x_{2n+1}, S(x_{2n}, y_{2n}))d(x_{2n}, T(x_{2n+1}, y_{2n+1})))}{1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})} \\
&\precsim \frac{\propto (d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}))}{2} + \frac{\beta d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})}{1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})} \\
&\quad + \frac{\gamma d(x_{2n+1}, x_{2n+1})d(x_{2n}, x_{2n+2})}{1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})} \\
&\precsim \frac{\propto (d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}))}{2} + \frac{\beta d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})}{1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})}
\end{aligned} \tag{3}$$

which implies that

$$|d(x_{2n+1}, x_{2n+2})| \leq \frac{\propto |d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})|}{2} + \frac{\beta |d(x_{2n}, x_{2n+1})d(x_{2n+1}, x_{2n+2})|}{|1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})|}. \tag{4}$$

Since $|1 + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})| > |d(x_{2n}, x_{2n+1})|$, so we get

$$|d(x_{2n+1}, x_{2n+2})| \leq \frac{\propto |d(x_{2n}, x_{2n+1})| + \propto |d(y_{2n}, y_{2n+1})|}{2} + \beta |d(x_{2n+1}, x_{2n+2})| \tag{5}$$

$$|d(x_{2n+1}, x_{2n+2})| \leq \frac{1}{2}(\frac{\propto}{1-\beta})|d(x_{2n}, x_{2n+1})| + \frac{1}{2}(\frac{\propto}{1-\beta})|d(y_{2n}, y_{2n+1})|. \tag{6}$$

Similarly, one can show that

$$|d(y_{2n+1}, y_{2n+2})| \leq \frac{1}{2}(\frac{\propto}{1-\beta})|d(y_{2n}, y_{2n+1})| + \frac{1}{2}(\frac{\propto}{1-\beta})|d(x_{2n}, x_{2n+1})|. \tag{7}$$

Also, $d(x_{2n+2}, x_{2n+3})$

$$\begin{aligned}
&= d(T(x_{2n+1}, y_{2n+1}), S(x_{2n+2}, y_{2n+2})) \\
&\precsim \frac{\propto (d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2}))}{2} \\
&+ \frac{(\beta d(x_{2n+1}, T(x_{2n+1}, y_{2n+1}))d(x_{2n+2}, S(x_{2n+2}, y_{2n+2})) + \gamma d(x_{2n+2}, T(x_{2n+1}, y_{2n+1}))d(x_{2n+1}, S(x_{2n+2}, y_{2n+2})))}{1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})} \\
&\precsim \frac{\propto (d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2}))}{2} + \frac{\beta d(x_{2n+1}, x_{2n+2})d(x_{2n+2}, x_{2n+3})}{1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})} \\
&+ \frac{\gamma d(x_{2n+2}, x_{2n+2})d(x_{2n+1}, x_{2n+3})}{1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})} \\
&\precsim \frac{\propto (d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2}))}{2} + \frac{\beta d(x_{2n+1}, x_{2n+2})d(x_{2n+2}, x_{2n+3})}{1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})} \qquad (8)
\end{aligned}$$

so that

$$|d(x_{2n+2}, x_{2n+3})| \leq \frac{\propto |d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})|}{2} + \frac{\beta |d(x_{2n+1}, x_{2n+2})||d(x_{2n+2}, x_{2n+3})|}{|1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})|}. \tag{9}$$

As $|1 + d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2})| > |d(x_{2n+1}, x_{2n+2})|$, therefore,

$$|d(x_{2n+2}, x_{2n+3})| \leq \frac{1}{2}(\frac{\propto}{1-\beta})|d(x_{2n+1}, x_{2n+2})| + \frac{1}{2}(\frac{\propto}{1-\beta})|d(y_{2n+1}, y_{2n+2})|. \tag{10}$$

Similarly, one can show that

$$|d(y_{2n+2}, y_{2n+3})| \leq \frac{1}{2}(\frac{\propto}{1-\beta})|d(y_{2n+1}, y_{2n+2})| + \frac{1}{2}(\frac{\propto}{1-\beta})|d(x_{2n+1}, x_{2n+2})|. \tag{11}$$

Adding up (6) & (7) and (10) & (11) we get

$$|d(x_{2n+1}, x_{2n+2})| + |d(y_{2n+1}, y_{2n+2})| \leq \frac{\propto}{1-\beta}|d(x_{2n}, x_{2n+1})| + \frac{\propto}{1-\beta}|d(y_{2n}, y_{2n+1})| \tag{12}$$

$$|d(x_{2n+2}, x_{2n+3})| + |d(y_{2n+2}, y_{2n+3})| \leq \frac{\propto}{1-\beta}|d(x_{2n+1}, x_{2n+2})| + \frac{\propto}{1-\beta}|d(y_{2n+1}, y_{2n+2})|. \tag{13}$$

Since $s \propto +\beta < 1$ and $s \geq 1$ we get $\propto +\beta < 1$. Therefore with $h = \frac{\propto}{1-\beta} < 1$, and for all $n \geq 0$ and consequently, we have

$$\begin{aligned}
|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})| &\leq h(|d(x_{n-1}, x_n)| + |d(y_{n-1}, y_n)|) \\
&\leq \cdots \leq h^n(|d(x_0, x_1)| + |d(y_0, y_1)|). \qquad (14)
\end{aligned}$$

Now if $|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})| = \delta_n$ then

$$\delta_n \leq h\delta_{n-1} \leq \cdots \leq h^n \delta_0. \tag{15}$$

Without loss of generality, we take $m > n, m, n \in \mathbb{N}$, and since $0 \leq h < 1$, so we get

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq s(|d(x_n, x_{n+1})| + |d(x_{n+1}, x_m)|) + s(|d(y_n, y_{n+1})| + |d(y_{n+1}, y_m)|) \\
&\leq s(|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})|) + s^2(|d(x_{n+1}, x_{n+2})| + |d(x_{n+2}, x_m)|) \\
&\quad + s^2(|d(y_{n+1}, y_{n+2})| + |d(y_{n+2}, y_m)|) \\
&\leq s(|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})|) + s^2(|d(x_{n+1}, x_{n+2})| + |d(y_{n+1}, y_{n+2})|) \\
&\quad + s^3(|d(x_{n+2}, x_{n+3})| + |d(x_{n+3}, x_m)|) + s^3(|d(y_{n+2}, y_{n+3})| + |d(y_{n+3}, y_m)|) \\
&\cdots \\
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq s(|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})|) + s^2(|d(x_{n+1}, x_{n+2})| + |d(y_{n+1}, y_{n+2})|) \\
&\quad + s^{m-n-1}(|d(x_{m-2}, x_{m-1})| + |d(y_{m-2}, y_{m-1})|) + s^{m-n}(|d(x_{m-1}, x_m)| + |d(y_{m-1}, y_m)|).
\end{aligned}$$

By using (15), we get

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq sh^n(|d(x_0, x_1)| + |d(y_0, y_1)|) + s^2h^{n+1}(|d(x_0, x_1)| + |d(y_0, y_1)|) \\
&\cdots \\
&\quad + s^{m-n-1}h^{m-2}(|d(x_0, x_1)| + |d(y_0, y_1)|) + s^{m-n}h^{m-1}(|d(x_0, x_1)| + |d(y_0, y_1)|) \\
&= sh^n\delta_0 + s^2h^{n+1}\delta_0 + \cdots + s^{m-n-1}h^{m-2}\delta_0 + s^{m-n}h^{m-1}\delta_0 \\
&= \sum_{i=1}^{m-n} s^i h^{i+n-1}\delta_0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq \sum_{i=1}^{m-n} s^{i+n-1}h^{i+n-1}\delta_0 \\
&= \sum_{t=n}^{m-1} s^t h^t \delta_0 \\
&\leq \sum_{t=n}^{\infty} (sh)^t \delta_0 = \frac{(sh)^n}{1 - sh}\delta_0
\end{aligned}$$

and hence

$$|d(x_n, x_m)| + |d(y_n, y_m)| \leq \frac{(sh)^n}{1 - sh}\delta_0 \to 0 \text{ as } m, n \to +\infty.$$

This implies that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. Since X is complete, there exists $x, y \in X$ such that $x_n \to x$ and $y_n \to y$ as $n \to +\infty$. We now show that $x = S(x, y)$and $y = S(y, x)$. We assume on the contrary that $x \neq S(x, y)$ and $y \neq S(y, x)$ so that $0 \prec d(x, S(x, y)) = u_1$ and $0 \prec d(y, S(y, x)) = u_2$; then we have

$$\begin{aligned}
u_1 = d(x, S(x, y)) &\precsim sd(x, x_{2n+2}) + sd(x_{2n+2}, S(x, y)) \qquad (16) \\
&\precsim sd(x, x_{2n+2}) + sd(T(x_{2n+1}, y_{2n+1}), S(x, y)) \\
&\precsim sd(x, x_{2n+2}) + \frac{s \propto (d(x_{2n+1}, x) + d(y_{2n+1}, y))}{2} + \frac{s\beta d(x_{2n+1}, T(x_{2n+1}, y_{2n+1}))d(x, S(x, y))}{1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)} \\
&\quad + \frac{s\gamma d(x, T(x_{2n+1}, y_{2n+1}))d(x_{2n+1}, S(x, y))}{1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)} \\
&= sd(x, x_{2n+2}) + \frac{s \propto (d(x_{2n+1}, x) + d(y_{2n+1}, y))}{2} \\
&\quad + \frac{s\beta d(x_{2n+1}, x_{2n+2})d(x, S(x, y))}{1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)} + \frac{s\gamma d(x, x_{2n+2})d(x_{2n+1}, S(x, y))}{1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)}
\end{aligned}$$

which implies that

$$|u_1| \leq s|d(x, x_{2n+2})| + \frac{s \propto |d(x_{2n+1}, x) + d(y_{2n+1}, y)|}{2} + \frac{s\beta|d(x_{2n+1}, x_{2n+2})||d(x, S(x, y))|}{|1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)|}$$
$$+ \frac{s\gamma|d(x, x_{2n+2})||d(x_{2n+1}, S(x, y))|}{|1 + d(x_{2n+1}, x) + d(y_{2n+1}, y)|}. \tag{17}$$

Taking the limit of (17) as $n \to +\infty$, we obtain $|d(x, S(x, y))| = 0$, which is a contradiction so that $x = S(x, y)$. Similarly, one can prove that $y = S(y, x)$. It follows similarly that $x = T(x, y)\, and\, y = T(y, x)$. Hence (x, y) is a common coupled fixed point of S and T. Now, we show that $S\, and\, T$ have a unique common coupled fixed point. For this, assume that $(x^\star, y^\star) \in X$ is another common coupled fixed point of S and T. Then

$$\begin{aligned} d(x, x^\star) &= d(S(x, y), T(x^\star, y^\star)) \\ &\precsim \frac{\propto (d(x, x^\star) + d(y, y^\star))}{2} + \frac{\beta d(x, S(x, y))d(x^\star, T(x^\star, y^\star))}{1 + d(x, x^\star) + d(y, y^\star)} + \frac{\gamma d(x, T(x^\star, y^\star))d(x^\star, S(x, y))}{1 + d(x, x^\star) + d(y, y^\star)}. \\ &\precsim \frac{\propto (d(x, x^\star) + d(y, y^\star))}{2} + \frac{\beta d(x, x)d(x^\star, x^\star)}{1 + d(x, x^\star) + d(y, y^\star)} + \frac{\gamma d(x, x^\star)d(x^\star, x)}{1 + d(x, x^\star) + d(y, y^\star)}. \end{aligned}$$

So that

$$|d(x, x^\star)| \leq \frac{\propto |d(x, x^\star) + d(y, y^\star)|}{2} + \frac{\gamma|d(x, x^\star)||d(x^\star, x)|}{|1 + d(x, x^\star) + d(y, y^\star)|}. \tag{18}$$

Since $|1 + d(x, x^\star) + d(y, y^\star)| > |d(x, x^\star)|$, so we get

$$|d(x, x^\star)| \leq \frac{\propto |d(x, x^\star) + d(y, y^\star)|}{2} + \gamma|d(x, x^\star)| = (\frac{\propto}{2 - \propto - 2\gamma})|d(y, y^\star)|. \tag{19}$$

Similarly, one can easily prove that

$$|d(y, y^\star)| \leq (\frac{\propto}{2 - \propto - 2\gamma})|d(x, x^\star)|. \tag{20}$$

Adding up (19) and (20), we get

$$|d(x, x^\star)| + |d(y, y^\star)| \leq (\frac{\propto}{2 - \propto - 2\gamma})(|d(x, x^\star)| + |d(y, y^\star)|)$$

which is a contradiction because $\propto + \gamma < 1$. So $x^\star = x$ and $y = y^\star$ which proves the uniqueness of common coupled fixed point of S and T. This completes the proof. □

Corollary 2.2. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \times X \to X$ satisfy*

$$d(T(x, y), T(u, v)) \precsim \frac{\propto (d(x, u) + d(y, v))}{2} + \frac{\beta d(x, T(x, y))d(u, T(u, v))}{1 + d(x, u) + d(y, v)} + \frac{\gamma d(u, T(x, y))d(x, T(u, v))}{1 + d(x, u) + d(y, v)} \tag{21}$$

for all $x, y, u, v \in X$, where $\propto, \beta$ and γ are nonnegative reals with $s \propto + \beta < 1$ and $\propto + \gamma < 1$. Then T has a unique coupled fixed point.

Proof. We can prove this result by applying Theorem 2.1 by setting $S = T$. □

Corollary 2.3. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $T : X \times X \to X$ satisfy*

$$d(T^n(x, y), T^n(u, v)) \precsim \frac{\propto (d(x, u) + d(y, v))}{2} + \frac{\beta d(x, T^n(x, y))d(u, T^n(u, v))}{1 + d(x, u) + d(y, v)} + \frac{\gamma d(u, T^n(x, y))d(x, T^n(u, v))}{1 + d(x, u) + d(y, v)} \tag{22}$$

for all $x, y, u, v \in X$, where $\propto, \beta$ and γ are nonnegative reals with $s \propto + \beta < 1$ and $\propto + \gamma < 1$. Then T has a unique coupled fixed point.

Proof. From Corollary 2.2, we obtain $(x, y) \in X \times X$ such that $T^n(x, y) = x$. The uniqueness follows from

$$\begin{aligned}
d(T(x,y),x) &= d(T(T^n(x,y),y),T^n(x,y)) \\
&= d(T^n(T(x,y),y),T^n(x,y)) \\
&\precsim \frac{\propto (d(T(x,y),x)+d(y,y))}{2} + \frac{\beta d(T(x,y),T^n(T(x,y)))d(x,T^n(x,y))}{1+d(T(x,y),x)+d(y,y)} + \frac{\gamma d(x,T^n(T(x,y),y))d(T(x,y)T^n(x,y))}{1+d(T(x,y),x)+d(y,y)} \\
&\precsim \frac{\propto (d(T(x,y),x))}{2} + \frac{\gamma d(x,T(T^n(x,y),y))d(T(x,y),x)}{1+d(T(x,y),x)} \\
&= \frac{\propto (d(T(x,y),x))}{2} + \frac{\gamma d(x,T(x,y))d(x,T(x,y))}{1+d(x,T(x,y))}. \qquad (23)
\end{aligned}$$

By taking modulus of (23) we get

$$|d(T(x,y),x)| \leq \frac{\propto |d(T(x,y),x)|}{2} + \frac{\gamma |d(x,T(x,y))||d(x,T(x,y))|}{|1+d(x,T(x,y))|}.$$

Since $|1 + d(x, T(x, y))| > |d(x, T(x, y))|$

$$\begin{aligned}
|d(T(x,y),x)| &\leq (\frac{\propto}{2} + \gamma)|d(T(x,y),x)| \\
&< |d(T(x,y),x)|, \quad \text{a contradiction.}
\end{aligned}$$

So, $T(x, y) = x$. Hence $T(x, y) = T^n(x, y) = x$. Similarly, it can be proved,

$$T(y,x) = T^n(y,x) = y.$$

Therefore, the coupled fixed of T is unique. This completes the proof. □

Theorem 2.4. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $S, T : X \times X \to X$ are mappings satisfying:*

$$d(S(x,y),T(u,v)) \precsim \begin{cases} \frac{\propto (d(x,u)+d(y,v))}{2} + \frac{\beta d(x,S(x,y))d(u,T(u,v))}{d(x,T(u,v))+d(u,S(x,y))+d(x,u)+d(y,v)} & if\, D \neq 0 \\ 0 & if\, D = 0 \end{cases} \qquad (24)$$

for all $x, y, u, v \in X$, where $D = d(x, T(u, v)) + d(u, S(x, y)) + d(x, u) + d(y, v)$ and $\propto, \beta$ are nonnegative reals with $s(\propto +\beta) < 1$. Then S and T have a unique common coupled fixed point.

Proof. Let x_0 and y_0 be arbitrary points in X. Define

$$\begin{aligned}
&x_{2n+1} = S(x_{2n}, y_{2n}), y_{2n+1} = S(y_{2n}, x_{2n}) \quad \text{and} \\
&x_{2n+2} = T(x_{2n+1}, y_{2n+1}), y_{2n+2} = T(y_{2n+1}, x_{2n+1}), \quad \text{for} \quad n = 0, 1, 2, \qquad (25)
\end{aligned}$$

Now, we assume that

$$\begin{aligned}
D_S(x_{2n}, y_{2n}) &= d(S(x_{2n}, y_{2n}), T(x_{2n+1}, y_{2n+1})) \\
&= d(x_{2n}, T(x_{2n+1}, y_{2n+1})) + d(x_{2n+1}, S(x_{2n}, y_{2n})) + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) \\
&= d(x_{2n}, x_{2n+2}) + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) \neq 0, \\
D_S(y_{2n}, x_{2n}) &= d(S(y_{2n}, x_{2n}), T(y_{2n+1}, x_{2n+1})) \\
&= d(y_{2n}, T(y_{2n+1}, x_{2n+1})) + d(y_{2n+1}, S(y_{2n}, x_{2n})) + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) \\
&= d(y_{2n}, y_{2n+2}) + d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) \neq 0.
\end{aligned}$$

Then

$$\begin{aligned} d(x_{2n+1},x_{2n+2}) &= d(S(x_{2n},y_{2n}),T(x_{2n+1},y_{2n+1})) \\ &\precsim \frac{\propto (d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1}))}{2} + \frac{\beta d(x_{2n},S(x_{2n},y_{2n}))d(x_{2n+1},T(x_{2n+1},y_{2n+1}))}{D_S(x_{2n},y_{2n})} \\ &= \frac{\propto (d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1}))}{2} + \frac{\beta d(x_{2n},x_{2n+1})d(x_{2n+1},x_{2n+2})}{d(x_{2n},x_{2n+2})+d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1})}. \end{aligned} \tag{26}$$

Taking modulus of (26), we get

$$\begin{aligned} |d(x_{2n+1},x_{2n+2})| &\leq \frac{\propto |d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1})|}{2} + \frac{\beta|d(x_{2n},x_{2n+1})||d(x_{2n+1},x_{2n+2})|}{|d(x_{2n},x_{2n+2})+d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1})|} \\ &\leq \frac{\propto |d(x_{2n},x_{2n+1})+d(y_{2n},y_{2n+1})|}{2} + \beta|d(x_{2n},x_{2n+1})|. \end{aligned}$$

As $|d(x_{2n+1},x_{2n+2})| \leq |d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n})+d(y_{2n},y_{2n+1})|$. Therefore

$$|d(x_{2n+1},x_{2n+2})| \leq \frac{(\propto +2\beta)}{2}|d(x_{2n},x_{2n+1})| + \frac{\propto}{2}|d(y_{2n},y_{2n+1})|. \tag{27}$$

Similarly, it can be easily proved,

$$|d(y_{2n+1},y_{2n+2})| \leq \frac{(\propto +2\beta)}{2}|d(y_{2n},y_{2n+1})| + \frac{\propto}{2}|d(x_{2n},x_{2n+1})|. \tag{28}$$

Now if

$$\begin{aligned} D_T(x_{2n+1},y_{2n+1}) &= d(T(x_{2n+1},y_{2n+1}),S(x_{2n+2},y_{2n+2})) \\ &= d(x_{2n+2},T(x_{2n+1},y_{2n+1}))+d(x_{2n+1},S(x_{2n+2},y_{2n+2}))+d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1}) \\ &= d(x_{2n+1},x_{2n+3})+d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1}) \neq 0. \end{aligned}$$

Then,

$$\begin{aligned} d(x_{2n+2},x_{2n+3}) &= d(T(x_{2n+1},y_{2n+1}),S(x_{2n+2},y_{2n+2})) \\ &\precsim \frac{\propto (d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1}))}{2} + \frac{\beta d(x_{2n+2},S(x_{2n+2},y_{2n+2}))d(x_{2n+1},T(x_{2n+1},y_{2n+1}))}{D_T(x_{2n+1},y_{2n+1})} \\ &= \frac{\propto (d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1}))}{2} + \frac{\beta d(x_{2n+2},x_{2n+3})d(x_{2n+1},x_{2n+2})}{d(x_{2n+1},x_{2n+3})+d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1})}. \end{aligned} \tag{29}$$

Taking modulus of (29), we get

$$\begin{aligned} |d(x_{2n+2},x_{2n+3})| &\leq \frac{\propto |d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1})|}{2} + \frac{\beta|d(x_{2n+2},x_{2n+3})||d(x_{2n+1},x_{2n+2})|}{|d(x_{2n+1},x_{2n+3})|+|d(x_{2n+2},x_{2n+1})|+|d(y_{2n+2},y_{2n+1})|} \\ &\leq \frac{\propto |d(x_{2n+2},x_{2n+1})+d(y_{2n+2},y_{2n+1})|}{2} + \beta|d(x_{2n+1},x_{2n+2})|. \end{aligned}$$

As $|d(x_{2n+2},x_{2n+3})| \leq |d(x_{2n+2},x_{2n+1})|+|d(x_{2n+1},x_{2n+3})|+|d(y_{2n+2},y_{2n+1})|$. Therefore

$$|d(x_{2n+2},x_{2n+3})| \leq \frac{(\propto +2\beta)}{2}|d(x_{2n+2},x_{2n+1})| + \frac{\propto}{2}|d(y_{2n+1},y_{2n+2})|. \tag{30}$$

Similarly, if $D_T(y_{2n+1},x_{2n+1}) \neq 0$ one can easily prove that

$$|d(y_{2n+2},y_{2n+3})| \leq \frac{(\propto +2\beta)}{2}|d(y_{2n+1},y_{2n+2})| + \frac{\propto}{2}|d(x_{2n+1},x_{2n+2})|. \tag{31}$$

Adding up the inequalities (27) with (28) and (30) with (31), we get

$$|d(x_{2n+1}, x_{2n+2})| + |d(y_{2n+1}, y_{2n+2})| \leq (\propto +\beta)(|d(x_{2n}, x_{2n+1})| + |d(y_{2n}, y_{2n+1})|) \tag{32}$$

$$|d(x_{2n+2}, x_{2n+3})| + |d(y_{2n+2}, y_{2n+3})| \leq (\propto +\beta)(|d(x_{2n+1}, x_{2n+2})| + |d(y_{2n+1}, y_{2n+2})|). \tag{33}$$

Since $s(\propto +\beta) < 1$ and $s \geq 1$, we get $\propto +\beta < 1$. Therefore with $h = (\propto +\beta) < 1$ and for all $n \geq 0$ and consequently, we get

$$|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})| \leq h(|d(x_{n-1}, x_n)| + |d(y_{n-1}, y_n)|) \leq \cdots \leq h^n(|d(x_0, x_1)| + |d(y_0, y_1)|). \tag{34}$$

Now if $|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})| = \delta_n$, then

$$\delta_n \leq h\delta_{n-1} \leq \cdots \leq h^n\delta_0. \tag{35}$$

Without loss of generality, we take $m > n, m, n \in \mathbb{N}$ and since $0 \leq h < 1$, so we get

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq s(|d(x_n, x_{n+1})| + |d(x_{n+1}, x_m)|) + s(|d(y_n, y_{n+1})| + |d(y_{n+1}, y_m)|)\\
&\leq s(|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})|) + s^2(|d(x_{n+1}, x_{n+2})| + |d(x_{n+2}, x_m)|)\\
&+ s^2(|d(y_{n+1}, y_{n+2})| + |d(y_{n+2}, y_m)|)\\
&\ldots\\
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq s(|d(x_n, x_{n+1})| + |d(y_n, y_{n+1})|) + s^2(|d(x_{n+1}, x_{n+2})| + |d(y_{n+1}, y_{n+2})|)\\
&+ \cdots + s^{m-n-1}(|d(x_{m-2}, x_{m-1})| + |d(y_{m-2}, y_{m-1})|) + s^{m-n}(|d(x_{m-1}, x_m)| + |d(y_{m-1}, y_m)|).
\end{aligned}$$

By using (35), we get

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq sh^n(|d(x_0, x_1)| + |d(y_0, y_1)|) + s^2h^{n+1}(|d(x_0, x_1)| + |d(y_0, y_1)|)\\
&+ \cdots + s^{m-n-1}h^{m-2}(|d(x_0, x_1)| + |d(y_0, y_1)|) + s^{m-n}h^{m-1}(|d(x_0, x_1)| + |d(y_0, y_1)|)\\
&= sh^n\delta_0 + s^2h^{n+1}\delta_0 + \cdots + s^{m-n-1}h^{m-2}\delta_0 + s^{m-n}h^{m-1}\delta_0\\
&= \sum_{i=1}^{m-n} s^i h^{i+n-1}\delta_0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
|d(x_n, x_m)| + |d(y_n, y_m)| &\leq \sum_{i=1}^{m-n} s^{i+n-1}h^{i+n-1}\delta_0\\
&= \sum_{t=n}^{m-1} s^t h^t \delta_0 \leq \sum_{t=n}^{\infty} (sh)^t\delta_0\\
&= \frac{(sh)^n}{1 - sh}\delta_0
\end{aligned}$$

and hence $|d(x_n, x_m)| + |d(y_n, y_m)| \leq \frac{(sh)^n}{1-sh}\delta_0 \to 0 \, as \, m, n \to +\infty$. This implies that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. Since X is complete, so there exists $x, y \in X$ such that $x_n \to x$ and $y_n \to y$ as $n \to +\infty$. We now show that $x = S(x, y)$and $y = S(y, x)$. We suppose on the contrary that $x \neq S(x, y)$ and $y \neq S(y, x)$ so that

$$0 \prec d(x, S(x, y)) = u_1 \text{ and}$$

$$0 \prec d(y, S(y, x)) = u_2 \tag{36}$$

then we have

$$\begin{aligned}
u_1 = d(x,S(x,y)) &\precsim sd(x,x_{2n+2}) + sd(x_{2n+2},S(x,y)) \\
&\precsim sd(x,x_{2n+2}) + sd(T(x_{2n+1},y_{2n+1}),S(x,y)) \\
&\precsim sd(x,x_{2n+2}) + \frac{s\propto(d(x_{2n+1},x)+d(y_{2n+1},y))}{2} + \frac{s\beta d(x_{2n+1},T(x_{2n+1},y_{2n+1}))d(x,S(x,y))}{d(x,T(x_{2n+1},y_{2n+1}))+d(x_{2n+1},S(x,y))+d(x_{2n+1},x)+d(y_{2n+1},y)} \\
&\precsim sd(x,x_{2n+2}) + \frac{s\propto(d(x_{2n+1},x)+d(y_{2n+1},y))}{2} + \frac{s\beta u_1 d(x_{2n+1},x_{2n+2})}{d(x,x_{2n+2})+d(x_{2n+1},S(x,y))+d(x_{2n+1},x)+d(y_{2n+1},y)}
\end{aligned}$$

which implies that

$$|u_1| \le s|d(x,x_{2n+2})| + \frac{s\propto|d(x_{2n+1},x)+d(y_{2n+1},y)|}{2} + \frac{s\beta|u_1||d(x_{2n+1},x_{2n+2})|}{|d(x,x_{2n+2})+d(x_{2n+1},S(x,y))+d(x_{2n+1},x)+d(y_{2n+1},y)|}. \quad (37)$$

By taking $n\to+\infty$, we get $|d(x,S(x,y))|=0$, which is contradiction so that $x=S(x,y)$. Now

$$\begin{aligned}
u_2 = d(y,S(y,x)) &\precsim sd(y,y_{2n+2}) + sd(y_{2n+2},S(y,x)) \\
&\precsim sd(y,y_{2n+2}) + sd(T(y_{2n+1},x_{2n+1}),S(y,x)) \\
&\precsim sd(y,y_{2n+2}) + \frac{s\propto(d(y_{2n+1},y)+d(x_{2n+1},x))}{2} + \frac{s\beta d(y_{2n+1},T(y_{2n+1},x_{2n+1}))d(y,S(y,x))}{d(y_{2n+1},S(y,x))+d(y,T(y_{2n+1},x_{2n+1}))+d(y_{2n+1},y)+d(x_{2n+1},x)} \\
&\precsim sd(y,y_{2n+2}) + \frac{s\propto(d(y_{2n+1},y)+d(x_{2n+1},x))}{2} + \frac{s\beta u_2 d(y_{2n+1},y_{2n+2})}{d(y_{2n+1},S(y,x))+d(y,y_{2n+2})+d(y_{2n+1},y)+d(x_{2n+1},x)}
\end{aligned}$$

which implies that

$$|u_2| \le s|d(y,y_{2n+2})| + \frac{s\propto|d(y_{2n+1},y)+d(x_{2n+1},x)|}{2} + \frac{s\beta|u_2||d(y_{2n+1},y_{2n+2})|}{|d(y_{2n+1},S(y,x))+d(y,y_{2n+2})+d(y_{2n+1},y)+d(x_{2n+1},x)|}. \quad (38)$$

By taking $n\to+\infty$, gives us $|d(y,S(y,x))|=0$ which is a contradiction so that $y=S(y,x)$. It follows similarly that $x=T(x,y)\, and\, y=T(y,x)$. Hence (x,y) is a common coupled fixed point of S and T. As in Theorem 2.1, the uniqueness of common coupled fixed point remains a consequence of contraction condition (24). We have obtained the existence and uniqueness of a unique common coupled fixed point of

$$D_S(x_{2n},y_{2n}), D_S(y_{2n},x_{2n}), D_T(x_{2n+1},y_{2n+1}), D_T(y_{2n+1},x_{2n+1}) \neq 0 \quad (39)$$

for all $n\in\mathbb{N}$. Now assume that $D_S(x_{2n},y_{2n})=0$ for some $n\in\mathbb{N}$. From

$$d(x_{2n},{}_{2n+2}) + d(x_{2n},x_{2n+1}) + d(y_{2n},y_{2n+1}) = 0. \quad (40)$$

We get $x_{2n}=x_{2n+1}=x_{2n+2}$ and $y_{2n}=y_{2n+1}$. If $D_S(y_{2n},x_{2n})\neq 0$, using (1), we deduce

$$d(y_{2n+1},y_{2n+2}) = d(S(y_{2n},x_{2n}),T(y_{2n+1},x_{2n+1})) = 0. \quad (41)$$

That is $y_{2n+1}=y_{2n+2}$ (this equality holds also if $D_S(y_{2n},x_{2n})=0$). The equalities

$$x_{2n}=x_{2n+1}=x_{2n+2}, y_{2n}=y_{2n+1}=y_{2n+2}. \quad (42)$$

This ensure that (x_{2n+1},y_{2n+1}) is a unique common coupled fixed point of S and T. The same holds if either $D_S(y_{2n},x_{2n})=0, D_T(x_{2n+1},y_{2n+1})=0$ or $D_T(y_{2n+1},x_{2n+1})=0$. □

Corollary 2.5. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let $S, T : X \times X \to X$ are mappings satisfying:*

$$d(S(x,y),T(u,v)) \precsim \begin{cases} \frac{\beta d(x,S(x,y))d(u,T(u,v))}{d(x,T(u,v))+d(u,S(x,y))+d(x,u)+d(y,v)} & if\ D \neq 0 \\ 0 & if\ D = 0, \end{cases} \tag{43}$$

for all $x, y, u, v \in X$, where $D = d(x, T(u, v)) + d(u, S(x, y)) + d(x, u) + d(y, v)$ and β is a nonnegative real such that $0 < s\beta < 1$. Then S and T have a unique common coupled fixed point.

Proof. We can prove this result by applying Theorem 2.4 by setting $\propto = 0$. □

Corollary 2.6. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let the mapping $T : X \times X \to X$ satisfy*

$$d(T(x,y),T(u,v)) \precsim \begin{cases} \frac{\propto(d(x,u)+d(y,v))}{2} + \frac{\beta d(x,T(x,y))d(u,T(u,v))}{d(x,T(u,v))+d(u,T(x,y))+d(x,u)+d(y,v)} & if\ D \neq 0 \\ 0 & if\ D = 0 \end{cases} \tag{44}$$

for all $x, y, u, v \in X$, where $D = d(x, T(u, v)) + d(u, T(x, y)) + d(x, u) + d(y, v)$ and $\propto \beta$ are nonnegative reals with $s(\propto + \beta) < 1$. Then T has a unique coupled fixed point.

Proof. We can prove this result by applying Theorem 2.4 by setting $S = T$. □

Corollary 2.7. *Let (X, d) be a complete complex valued b-metric space with the coefficient $s \geq 1$ and let the mapping $T : X \times X \to X$ satisfy:*

$$d(T^n(x,y),T^n(u,v)) \precsim \begin{cases} \frac{\propto(d(x,u)+d(y,v))}{2} + \frac{\beta d(x,T^n(x,y))d(u,T^n(u,v))}{d(x,T^n(u,v))+d(u,T^n(x,y))+d(x,u)+d(y,v)} & if\ D \neq 0 \\ 0 & if\ D = 0 \end{cases} \tag{45}$$

for all $x, y, u, v \in X$, where $D = d(x, T^n(u, v)) + d(u, T^n(x, y)) + d(x, u) + d(y, v)$ and $\propto \beta$ are nonnegative reals with $s(\propto + \beta) < 1$. Then T has a unique coupled fixed point.

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