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Supra α-Locally Closed Sets and Supra α-Locally Continuous Functions in Supra Topological Spaces

Research Article

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Abstract: The aim of this paper is to introduce a new type of sets called supra α-locally closed sets and new type of functions called supra α-locally continuous functions. Furthermore, we obtain some of their properties.

Keywords: S-α-LC sets, S-α-LC* sets, S-α-LC** sets, S-α-L-continuous and S-α-L-irresolute.

1. Introduction

In 1965, α-sets and β-sets were defined and studied in topological spaces by Njastad [7]. In topological spaces, Gnanambalet. al. [3] introduced α-locally closed sets and discussed its properties. Gnanambal and Balachandran [4] defined the notion of β-locally closed sets in topological spaces. The supra topological spaces, S-continuous functions and S*-continuous functions were introduced by Mashhouret. al. [6]. In 2008, Devi et. al. [2] defined and investigated the concept of supra α-open sets and sα-continuous maps in supra topological spaces. Ravi et.al. [8] introduced and studied supra β-open sets and supra β-continuous maps. Dayana Mary and Nagaveni [1] defined and discussed supra β-locally closed sets and their functions. In this paper we introduce the concept of supra α-locally closed sets and study its basic properties. Also we introduce the concepts of supra α-locally continuous maps and investigate several properties for these classes of maps.

2. Preliminaries

Throughout this paper, (X,τ), (Y,σ) and (Z,η) (or simply, X, Y and Z) represent topological space on which no separation axioms are assumed, unless explicitly stated. For a subset A of (X,τ), cl(A) and int(A) represent the closure of A with respect to τ and the interior of A with respect to τ, respectively. Let P(X) be the power set of X. The complement of A is denoted by X-A or A^c. Now we recall some Definitions and results which are useful in the sequel.

Definition 2.1 ([6, 9])**.** *Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ is said to a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions. The pair (X,μ) is called a supra topological space. The elements of μ are said to be supra open in (X,μ). Complement of supra open sets are called supra closed sets.*

Definition 2.2 ([9])**.** *Let A be a subset (X,μ). Then*

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(1). The supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap\{B : \ B$ is a supra closed and $A \subseteq B\}$.

(2). The supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup\{B : \ B$ is a supra open and $B \subseteq A\}$.

Definition 2.3 ([6]). *A Let (X, τ) be a topological space and μ be a supra topology of X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.*

Definition 2.4 ([2]). *Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f : (X, \tau) \to (Y, \sigma)$ is called supra continuous, if the inverse image of each open set of Y is a supra open set in X.*

Definition 2.5 ([6, 9]). *Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be supra topologies associated with τ and σ respectively. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be supra irresolute, if $f^{-1}(A)$ is supra open set of X for every supra open set A in Y.*

Definition 2.6 ([2]). *Let (X, μ) be a supra topological space. A subset A of X is called supra α-open if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$. The complement of supra α-open set is called supra α-closed. The class of all supra α-open sets is denoted by $S\text{-}\alpha O(X)$.*

Definition 2.7 ([2]). *Let A be a subset (X, μ). Then*

(1). The supra α-closure of a set A is, denoted by $cl^{\mu}_{\alpha}(A)$, defined as $cl^{\mu}_{\alpha}(A) = \cap\{B : \ B$ is a supra $\alpha-$closed and $A \subseteq B\}$.

(2). The supra α-interior of a set A is, denoted by $int^{\mu}_{\alpha}(A)$, defined as $int^{\mu}_{\alpha}(A) = \cup\{B : \ B$ is a supra $\alpha-$open and $B \subseteq A\}$.

Definition 2.8 ([8]). *Let (X, μ) be a supra topological space. A subset A of X is called supra β-open if $A \subseteq cl^{\mu}(int(cl^{\mu}(A)))$. The complement of supra β-open set is called supra β-closed. The class of all supra β-open sets is denoted by $S\text{-}\beta O(X)$.*

Definition 2.9 ([8]). *Let A be a subset (X, μ). Then*

(1). The supra β-closure of a set A is, denoted by $cl^{\mu}_{\beta}(A)$, defined as $cl^{\mu}_{\beta}(A) = \cap\{B : \ B$ is a supra $\beta-$closed and $A \subseteq B\}$.

(2). The supra β-interior of a set A is, denoted by $int^{\mu}_{\beta}(A)$, defined as $int^{\mu}_{\beta}(A) = \cup\{B : \ B$ is a supra $\beta-$open and $B \subseteq A\}$.

Definition 2.10 ([1]). *Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra β-locally closed set (briefly supra β-LC set), if $A = U \cap V$, where U is supra β-open in (X, μ) and V is supra β-closed in (X, μ). The collection of all supra β-locally closed sets of X will be denoted by S-β-LC(X).*

Definition 2.11 ([1]). *Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra β-dense, if $cl^{\mu}_{\beta}(A) = X$.*

Definition 2.12 ([1]). *A supra topological space (X, μ) is called supra β-submaximal space, if every supra dense subset is supra β-open in X.*

3. Supra α-Locally Closed Sets

In this section, we introduce the notions of supra α-locally closed sets and discuss some of their properties.

Definition 3.1. *Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra α-locally closed set (briefly supra α-LC set), if $A = U \cap V$, where U is supra α-open in (X, μ) and V is supra α-closed in (X, μ). The collection of all supra α-locally closed sets of X will be denoted by S-α-LC(X).*

Remark 3.2. *Every supra α-closed set (resp. supra α-open set) is S-α-LC.*

Definition 3.3. *Let (X,μ) be a supra topological space. The collection of all subsets A in (X,μ) given by A=U ∩ V, where U is a supra α-open set and V is a supra closed set of (X,μ), is denoted by S-α-LC*(X,μ).*

Definition 3.4. *Let (X,μ) be a supra topological space. The collection of all subsets A in (X,μ) given by A=U ∩ V, where U is a supra open set and V is a supra α-closed set of (X,μ), is denoted by S-α-LC**(X,μ).*

Definition 3.5. *Let A, B $\subseteq (X,\mu)$. Then A and B are said to be supra α-separated if A $\cap cl^{\mu}_{\alpha}(B)$ = B $\cap cl^{\mu}_{\alpha}(A) = \phi$.*

Theorem 3.6. *Let A be a subset of (X,μ). If A∈ S-α-LC*(X,μ), then A is S-α-LC.*

Proof. Let A∈ S-α-LC*(X,μ), then A= U ∩ V, where U is supra α-open set and V is supra closed. Since every supra closed set is supra α-closed, A∈ S-α-LC(X,μ). □

Theorem 3.7. *Let A be a subset of (X,μ). If A∈ S-α-LC**(X,μ), then A is S-α-LC.*

Proof. The proof follows from the fact that, every supra open set is supra α-open set. □

Example 3.8. *Let X = $\{a, b, c, d\}$ and $\mu = \{\phi, X, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Then S-$\alpha$-LC$(X,\mu)$ = S-α-LC*(X,μ) = P(X)- $\{\{a, b\}, \{c, d\}\}$. S- α-LC**(X,μ) = P(X)-$\{\{a, b\}, \{c, d\}, \{a, c, d\}\}$.*

Theorem 3.9. *For a subset A of (X,μ), the following are equivalent:*

(i) A∈ S-α-LC(X,μ).*

(ii) A = U $\cap cl^{\mu}(A)$, for some supra α-open set U.

(iii) $cl^{\mu}(A)$ – A is supra α-closed.

(iv) A∪ [X- $cl^{\mu}(A)$] is supra α-open.

Proof. (i) ⇒ (ii): Given A ∈ S-α-LC*(X,μ). Then there exist a supra α-open subset U and a supra closed subset V such that A=U ∩V. Since A ⊂ U and A $\subset cl^{\mu}(A)$, A ⊂ U $\cap cl^{\mu}(A)$.
Conversely, $cl^{\mu}(A) \subset$ V and hence A = U ∩ V ⊃U $\cap(cl^{\mu}(A))$. Therefore, A = U $\cap cl^{\mu}(A)$
(ii) ⇒ (i): Let A = U$\cap cl^{\mu}(A)$, for some supra α-open set U. Then, $cl^{\mu}(A)$ is supra closed and hence A = U $\cap cl^{\mu}(A)\in$ S-α-LC*(X,μ).
(ii) ⇒ (iii): Let A = U $\cap cl^{\mu}(A)$, for some supra α-open set U. Then A∈ S-α-LC*(X,μ). This implies U is supra α-open and $cl^{\mu}(A)$ is supra closed. Therefore, $cl^{\mu}(A)-$ A is supra α-closed.
(iii) ⇒ (ii): Let U= X – [$cl^{\mu}(A)$- A]. By (iii), U is supra α-open in X. Then A = U $\cap cl^{\mu}(A)$ holds.
(iii) ⇒ (iv): Let P =$cl^{\mu}(A)$–A be supra α-closed. Then X-P = X -[$cl^{\mu}(A)$– A] =A ∪ [(X- $cl^{\mu}(A)$]. Since X-P is supra α-open, A ∪ [X- $cl^{\mu}(A)$] is supra α-open.
(iv) ⇒ (iii): Let U = A ∪ [(X- $cl^{\mu}(A)$]. Since X – U is supra α-closed and X - U = $cl^{\mu}(A)$ - A is supra α-closed. □

Theorem 3.10. *For a subset A of (X,μ), the following are equivalent:*

(i). A∈ S- α-LC(X,μ).

(ii). A = U $\cap cl^{\mu}_{\alpha}(A)$, for some supra α-open set U.

(iii). $cl^{\mu}_{\alpha}(A)$ - A is supra α-closed.

(iv). A∪ [X - $cl^{\mu}_{\alpha}(A)$] is supra α -open.

(v). $A \subseteq int^{\mu}_{\alpha}(A \cup [X - cl^{\mu}_{\alpha}(A)]$.

Proof. (i) $\Rightarrow$ (ii): Given A $\in$ S- α-LC(X,μ). Then there exist a supra α-open subset U and a supra α-closed subset V such that A = U $\cap$ V. Since A $\subset$ U and A $\subset cl^{\mu}_{\alpha}(A)$, A $\subset$ U $\cap cl^{\mu}_{\alpha}(A)$.

Conversely, $cl^{\mu}_{\alpha}(A) \subset$ V and hence A = U $\cap$ V $\supset$U $\cap cl^{\mu}_{\alpha}(A)$. Therefore A = U $\cap cl^{\mu}_{\alpha}(A)$.

(ii) $\Rightarrow$ (i): Let A = U $\cap cl^{\mu}_{\alpha}(A)$, for some supra α-open set U. Then, $cl^{\mu}_{\alpha}(A)$ is supra α-closed and hence A = U $\cap cl^{\mu}_{\alpha}(A) \in$ S-α-LC*(X,μ).

(ii) $\Rightarrow$ (iii): Let A = U $\cap cl^{\mu}_{\alpha}(A)$, for some supra α-open set U. Then A$\in$ S- α-LC (X,μ). This implies U is supra α-open and $cl^{\mu}_{\alpha}(A)$ is supra α-closed. Therefore, $cl^{\mu}_{\alpha}(A) -$ A is supra α-closed.

(iii) $\Rightarrow$ (ii): Let U = X – [$cl^{\mu}_{\alpha}(A)$ - A]. By (iii), U is supra α-open in X. Then A = U $\cap cl^{\mu}_{\alpha}(A)$ holds.

(iii) $\Rightarrow$ (iv): Let P = $cl^{\mu}_{\alpha}(A)$–A be supra α-closed. Then X-P = X - [$cl^{\mu}_{\alpha}(A)$ – A] =A $\cup$ [(X- $cl^{\mu}_{\alpha}(A)$]. Since X-P is supra α-open, A $\cup$ [X- $cl^{\mu}_{\alpha}(A)$] is supra α-open.

(vi) $\Rightarrow$ (iii): Let U = A $\cup$ [(X- $cl^{\mu}_{\alpha}(A)$]. Since X – U is supra α-closed and X - U = $cl^{\mu}_{\alpha}(A)$- A is supra α-closed.

(vi) $\Rightarrow$ (v): Since U = A $\cup$ [(X- $cl^{\mu}_{\alpha}(A)$] is supra-α-open, A $\subseteq int^{\mu}_{\alpha}$(A $\cup$ [(X- $cl^{\mu}_{\alpha}(A)$]).

(v) $\Rightarrow$ (iv): It is obvious. □

Theorem 3.11. *If $P \subset Q \subset X$ and Q is S-α-LC, then there exists a S-α-LC set R such that $P \subset R \subset Q$.*

Theorem 3.12. *For a subset A of (X,μ), if $A \in$ S-α-LC**(X,μ), then there exist a supra open set P such that $A = P \cap cl^{\mu}(A)$.*

Proof. Let A$\in$ S-α-LC**(X,μ). Then A = P $\cap$ V, where P is supra open set and V is supra α-closed set. Then A =P $\cap$ V $\Rightarrow$ A $\subset$ P. Obviously, A $\subset cl^{\mu}(A)$. Therefore

$$A \subset P \cap cl^{\mu}(A) \tag{1}$$

Also we have $cl^{\mu}(A) \subset V$. This implies

$$A = P \cap V \supset P \cap cl^{\mu}(A) \Rightarrow A \supset P \cap cl^{\mu}(A) \tag{2}$$

From (1) and (2), we have $A = P \cap cl^{\mu}(A)$. □

Theorem 3.13. *For a subset A of (X,μ), if $A \in$ S-α-LC**(X,μ), then there exist an supra open set P such that $A = P \cap cl^{\mu}_{\alpha}(A)$.*

Proof. Let A$\in$ S-α-LC**(X,μ).Then A=P $\cap$ V, where P is supra open set and V is supra α-closed set.Then A = P $\cap$ V $\Rightarrow$ A $\subset$ P. Then A $\subset cl^{\mu}_{\alpha}(A)$. Therefore,

$$A \subset P \cap cl^{\mu}_{\alpha}(A) \tag{3}$$

Also we have $cl^{\mu}_{\alpha}(A) \subset$ V. This implies,

$$A = P \cap V \supset P \cap cl^{\mu}_{\alpha}(A) \Rightarrow A \supset P \cap cl^{\mu}_{\alpha}(A) \tag{4}$$

From (3) and (4), we get $A = P \cap cl^{\mu}_{\alpha}(A)$. □

Theorem 3.14. *Let A be a subset of (X,μ). If $A \in$ S-α-LC**(X,μ), then $cl^{\mu}_{\alpha}(A)$-A supra α-closed and A $\cup$ [(X- $cl^{\mu}_{\alpha}(A)$] is supra α-open.*

Remark 3.15. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.16. *Let* $X = \{a, b, c, d\}$ *and* $\mu = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$. *Then* $\{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{c, d\}\}$ *is the set of all supra* α*-closed sets in X and* $S\text{-}\alpha\text{-}LC^{**}(X,\mu) = P(X) - \{\{a, d\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. *If* $A = \{a, b, c\}$, *then* $cl^{\mu}_{\alpha}(A) - A = \{d\}$ *is supra* α*-closed and* $A \cup [(X - cl^{\mu}_{\alpha}(A)] = \{a, b, c\}$ *is supra* α*-open but* $A \notin S\text{-}\alpha\text{-}LC^{**}(X, \mu)$.

Remark 3.17. *Let* $A \in S\text{-}\alpha\text{-}LC(X,\mu)$ *and* $B \in S\text{-}\alpha\text{-}LC(X,\mu)$

(i). Even if A and B are supra α*–separated,* $A \cup B \notin S\text{-}\alpha\text{-}LC(X,\mu)$.

(ii). Even if A and B are supra α*–separated,* $A \cup B \notin S\text{-}\alpha\text{-}LC^{*}(X,\mu)$.

(iii). Even if A and B are supra α*–separated,* $A \cup B \notin S\text{-}\alpha\text{-}LC^{**}(X,\mu)$.

Example 3.18. *Let* $X = \{a, b, c, d\}$ *with supra topological space* $\mu = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$. *Let* $A = \{a\} \in S\text{-}\alpha\text{-}LC(X,\mu)$ *(respectively,* $S\text{-}\alpha\text{-}LC^{*}(X,\mu)$ *and* $S\text{-}\alpha\text{-}LC^{**}(X,\mu)$*) and* $B = \{d\} \in S\text{-}\alpha\text{-}LC(X,\mu)$ *(respectively,* $S\text{-}\alpha\text{-}LC^{*}(X,\mu)$ *and* $S\text{-}\alpha\text{-}LC^{**}(X,\mu)$*). Here A and B are supra* α*-separated, because* $A \cap cl^{\mu}_{\alpha}(B) = B \cap cl^{\mu}_{\alpha}(A) = \phi$. *Then* $A \cup B = \{a, d\} \notin S\text{-}\alpha\text{-}LC(X,\mu)$ *(respectively,* $S\text{-}\alpha\text{-}LC^{*}(X,\mu)$ *and* $S\text{-}\alpha\text{-}LC^{**}(X,\mu)$*).*

Definition 3.19. *Let* (X,μ) *be a supra topological space. A subset A of* (X,μ) *is called supra dense, if* $cl^{\mu}(A) = X$.

Definition 3.20. *A supra topological space* (X,μ) *is called supra submaximal, if every supra dense subset is supra open in X.*

Definition 3.21. *Let* (X,μ) *be a supra topological space. A subset A of* (X,μ) *is called supra* α*-dense, if* $cl^{\mu}_{\alpha}(A) = X$.

Definition 3.22. *A supra topological space* (X,μ) *is called supra* α*-submaximal, if every supra* α*-dense subset is supra* α*-open in X.*

Example 3.23. *Consider the supra topological space* (X,μ) *with* $X = \{a, b, c, d\}$ *and* $\mu = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. *Here X and* $\{a, b, c\}$ *are the supra* α*-dense sets and also supra* α*-open sets in X. Therefore X is supra* α*-submaximal.*

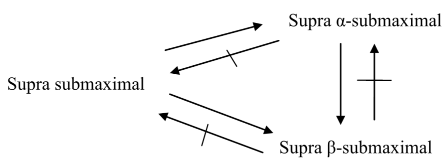
Remark 3.24.

(1). Every supra submaximal space is supra α*-submaximal.*

(2). Every supra submaximal space is supra β*-submaximal.*

(3). Every supra α*-submaximal space is supra* β*-submaximal.*

Remark 3.25. *The converses of the above statements are not true. The following diagram and examples illustrates this fact.*



Example 3.26. *Consider the supra topological space (X, μ) with X= $\{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. In this supra topological space, the subsets X and $\{a, b, c\}$ are supra dense (resp., supra α-dense and resp., supra β-dense). Thus the supra topological space (X, μ) is supra submaximal (resp., supra α-submaximal space and resp., supra β-submaximal).*

Example 3.27. *Consider the supra topological space (X, μ) with X= $\{a, b, c, d\}$ and $\mu = \{\phi, X, \{a\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. In this supra topological space, the supra α-open sets are ϕ, X, $\{a\}$, $\{a, b\}$, $\{b, c\}$, $\{a, b, c\}$ and $\{a, b, d\}$. The supra β-open sets are ϕ, X, $\{a\}$, $\{b\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{a, b, c\}$, $\{a, b, d\}$ and $\{a, c, d\}$. Since all supra β-dense sets are supra β-open sets, (X, μ) is supra β-submaximal space. Here the subset $\{a, c\}$ is supra α-dense and not supra α-open set. Thus the supra topological space (X, μ) is not supra α-submaximal space. Also the subset $\{a, b, d\}$ is supra dense and not supra open set. Therefore the supra topological space (X, μ) is not supra submaximal space.*

Theorem 3.28. *A supra topological space (X, μ) is supra α-submaximal if and only if P(X) = S-α-LC(X) holds.*

Proof. **Necessity:** Let A $\in$ P(X) and G = A $\cup$ [X- $cl^{\mu}_{\alpha}(A)$]. Then $cl^{\mu}_{\alpha}(G)$ = X and so G is supra α-dense and hence supra α-open by assumption. By Theorem 3.10, A$\in$ S-α-LC(X). Hence P(X) = S-α-LC(X).

Sufficiency: Let every subset of X be supra α-locally closed. Let A be supra α-dense in X. Then $cl^{\mu}_{\alpha}(A)$= X. Now A = A$\cup$[X- $cl^{\mu}_{\alpha}(A)$]. By Theorem: 3.10, A is supra α-open. Hence X is supra α-submaximal. ☐

Theorem 3.29. *Let (X, μ) and (Y, λ) be the supra topological spaces.*

(1) If M $\in$ S-α-LC(X, μ) and N $\in$ S-α-LC(Y, λ), then M $\times$ N$\in$S-α-LC(X $\times$ Y, μ $\times\lambda$).

(2) If M $\in$ S-α-LC(X, μ) and N $\in$ S-α-LC*(Y, λ), then M $\times$ N$\in$S-α-LC*(X $\times$ Y, μ $\times\lambda$).*

*(3) If M $\in$ S-α-LC**(X, μ) and N $\in$ S-α-LC**(Y, λ), then M $\times$ N $\in$ S-α-LC**(X $\times$Y, μ $\times\lambda$).*

Proof. Let M $\in$ S-α-LC(X, μ) and N $\in$ S-α-LC(Y, λ). Then there exist a supra α-open sets P and P' of (X, μ) and (Y, λ) and supra semi-closed sets Q and Q' of (X, μ) and (Y, λ) respectively such that M = P $\cap$ Q and N = P' $\cap$ Q'. Then M $\times$ N = (P $\times$ P') $\cap$ (Q $\times$Q') holds. Hence M $\times$ N$\in$S-α-LC(X $\times$ Y, μ $\times\lambda$).

The proofs of (2) and (3) are similar to that of (1). ☐

Theorem 3.30. *If A is supra α-locally closed set in (X, μ), Then A is supra β-locally closed set in (X, μ).*

Proof. Since every supra α-open set is supra β-open, S-α-LC(X, μ) $\subseteq$ S-β-LC(X, μ), for any supra topological space (X, μ). ☐

Remark 3.31. *A supra β-locally closed set need not be a supra α-locally closed set. The following example supports this fact.*

Example 3.32. *Consider the supra topological space in Example 3.8, the subset $\{a, b\}$ is a supra β-locally closed set and not a supra α-locally closed set.*

4. Supra α-Locally Continuous Functions

In this section we define a new type of functions called Supra α-locally continuous functions (S-α-L-continuous functions), supra α-locally irresolute functions (S-α-L-irresolute functions) and study some of their properties.

Definition 4.1. *Let (X,τ) and (Y,σ) be two topological spaces and $\tau \subseteq \mu$. A function $f : (X,\tau) \rightarrow (Y, \sigma)$ is called S-α-L-continuous (resp., S-α-L*-continuous, resp., S-α-L**-continuous), if $f^{-1}(A) \in$ S-α-LC(X,μ), (resp., $f^{-1}(A) \in$ S-α-LC*(X,μ), resp., $f^{-1}(A) \in$ S-α-LC**(X,μ)) for each $A \in \sigma$.*

Definition 4.2. *Let (X,τ) and (Y,σ) be two topological spaces and μandλ be the supra topologies associated with τ and σ respectively. A function $f : (X,\tau) \rightarrow (Y, \sigma)$ is said to be S-α-L-irresolute (resp., S-α-L*- irresolute, resp., S-α-L**-irresolute) if $f^{-1}(A) \in$ S-α-LC (X,μ), (resp., $f^{-1}(A) \in$ S-α-LC* (X,μ), resp., $f^{-1}(A) \in$ S-α-LC**(X,μ)) for each $A \in$ S-α-LC(Y, λ) (resp., $A \in$ S-α-LC*(Y, λ), resp., $A \in$ S-α-LC**(Y, λ)).*

Theorem 4.3. *Let (X,τ) and (Y,σ) be two topological spaces and μ be a supra topology associated with τ. Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a function. If f is S-α-L*-continuous or S-α-L**-continuous, then it is S-α-L-continuous.*

Theorem 4.4. *Let (X,τ) and (Y,σ) be two topological spaces and μandλ be the supra topologies associated with τ and σ respectively. Let f: $(X,\mu) \rightarrow (Y,\sigma)$ be a function. If f is S-α-L-irresolute (respectively S-α-L* – irresolute, respectively S-α-L**-irresolute), then it is S-α-L-continuous. (respectively S-α-L*-continuous, respectively S-α-L**-continuous).*

Remark 4.5. *Converse of Theorem 4.3 need not be true as seen from the following example.*

Example 4.6. *Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a, c, d\}\}$, $\sigma = \{\{\phi, Y, \{b, c, d\}\}$ and $\mu = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$. Define f: $(X,\mu) \rightarrow (Y,\sigma)$ is identity function. Here f is not S-α-L**-continuous, but it is S-α-L-continuous and S-α-L*-continuous.*

Remark 4.7. *The following example provides a function which is S-α-L**- continuous function but not S-α-L**- irresolute function.*

Example 4.8. *Let $X = Y = \{a, b, c, d\}$ with $\tau = \{\phi, X, \{a, b\}, \{a, b, d\}\}$, $\sigma = \{\{\phi, X, \{a\}, \{a, b, c\}\}$, $\mu = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$ and $\lambda = \{\phi, Y, \{a\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}\}$. Define f: $(X,\mu) \rightarrow (Y,\sigma)$ by f(a) = b, f(b) = a, f(c) = d and f(d) = c.Here f is S-α-L**-continuous and it is not S-α-L**-irresolute.*

Theorem 4.9. *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be supra α-LC-continuous and A be supra α-open in X. Then the restriction $f\,|A : A \rightarrow Y$ is S-α-L-continuous.*

Proof. Let U be supra open in Y. Then $f^{-1}(U)$ in supra α-LC in X. So $f^{-1}(U)$ = G∩H where G is supra α-open and H is supra α-closed in X. Now $(f \quad A\,)^{-1}(U)$= (G∩H)∩A = G∩(H∩A) (resp. (G∩A)∩H) where H∩A is supra α-closed (resp. G∩A is supra α-open) in X. Therefore $(f \quad A\,)^{-1}(U)$ is supra α-LC in X. Hence f | A is supra α-L-continuous. □

Theorem 4.10. *A supra topological space (X,μ) is supra α-submaximal if and only if every function having (X,μ) as domain is supra α-L-continuous.*

Proof. **Necessity:** Let (X,μ) be supra α-submaximal. Then α-LC(X) = P(X) by Theorem: 3.28. Let f: $(X,\mu) \rightarrow$ (Y, λ) be a function and A $\in\sigma$. Then$f^{-1}(A)\in$S-α-LC(X) and so f is S-α-L-continuous.

Sufficiency: Let every function having (X,μ) as domain be supra α-L-continuous. Let Y= {0, 1} and $\sigma = \{\phi$, Y, {0}}. Let A $\subset(X,\mu)$ and f: $(X,\mu) \rightarrow$ (Y, λ) be defined by f(x) = 0 if x ∈A and f(x) = 1 if x ∉A. Since f is supra α-L-continuous, A∈S-α-LC(X,μ). Hence P(X) = S-α-LC(X). Therefore X is supra α-submaximal by Theorem: 3.28. □

Theorem 4.11. *If g: $X \rightarrow Y$ is S-α-L-continuous and h: $Y \rightarrow Z$ is supra continuous, then h∘g: $X \rightarrow Z$ is S-α-L-continuous.*

Proof. Let g: X → Y is S- α-L-continuous and h: Y → Z is supra continuous. By the Definitions, g^{-1}(V) ∈ S-α-LC (X), V ∈Y and h^{-1}(W) ∈Y , W∈ Z. Let W ∈ Z. Then $(h\circ g)^{-1}$(W) = $(g^{-1}h^{-1})$(W) = $g^{-1}(h^{-1}$(W)) = g^{-1}(V), for V ∈ Y. From this, $(h\circ g)^{-1}$(W) = g^{-1}(V) ∈ S-α-LC (X), W ∈ Z. Therefore $h\circ g$ is S-α-L- continuous. □

Remark 4.12. *If g: X → Y is S-α-L-irresolute and h: Y → Z is S-α-L-continuous, then h∘g: X → Z is S-α-L-continuous.*

Proof. Let g: X → Y is S-α-L-irresolute and h: Y → Z is S-α-L-continuous. By the Definitions, g^{-1}(V) ∈ S-α-LC (X), for V ∈S-α-LC (Y) and h^{-1}(W) ∈S-α-LC (Y), for W∈ Z. Let W ∈ Z. Then $(h \circ g)^{-1}$(W) = $(g^{-1}h^{-1})$(W) = $g^{-1}(h^{-1}$(W)) = g^{-1}(V), for V ∈ S-α-LC(Y). This implies, $(h \circ g)^{-1}$(W) = g^{-1}(V) ∈ S-α-LC (X), W ∈ Z. Hence $h \circ g$ is S-α-L- continuous. □

Theorem 4.13. *If g: X → Y and h: Y → Z are S-α-L-irresolute, then h∘g: X → Z is also S-α-L-irresolute.*

Proof. By the hypothesis and the Definitions, we have g^{-1}(V) ∈ S-α-LC (X), for V ∈S-α-LC (Y) and h^{-1}(W) ∈S-α-LC (Y), for W∈ S-α-LC(Z). Let W ∈S-α-LC(Z). Then $(h \circ g)^{-1}$(W) = $(g^{-1}h^{-1})$(W) = $g^{-1}(h^{-1}$(W)) = g^{-1}(V), for V ∈ S-α-LC(Y). Therefore, $(h \circ g)^{-1}$(W) = g^{-1}(V) ∈ S-α-LC(X), W ∈ S-α-LC(Z). Thus $h \circ g$ is S-α-L-irresolute. □

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