International Journal of Mathematics And its Applications
Volume 4, Issue 3–B (2016), 8–16.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Reliability Importance of Components in Health Care System

Research Article

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Abstract: In Today's Scenario, it's very rare to find an healthy person and the number of patients are increasing rapidly, reflecting recent life style changes. One of the Most Common disease in the World is Diabetes Mellitus. In this paper, we discuss the Diabetes Mellitus disease and the study of Reliability importance helps to take preventive measures from its disease. The knowledge of System Reliability helps the System in Health Care to be more perfect and improve its status for the betterment of the Society. Here, we deal with Series-Parallel-Series (SPS) Configuration Systems and using Fault Tree Analysis Approach we determine the Reliability of Cure for the treatment of Diabetes in a Diabetic Health Centre.

Keywords: System Reliability, Reliability Importance, Series-Parallel-Series Configuration System, Fault Tree Analysis Approach.

1. Introduction

Reliability is the probability that a device or an item perform its function adequately over time interval (0, t). The device under consideration may be an entire system [2]. In practice the system is broken down to subsystems and elements whose individual Reliability factors can be estimated or determined depending on the manner in which these sub-systems and elements are connected to constitute the given system. The combinatorial rule of probability is applied to obtain the System Reliability [7].

System Reliability method helps us to analyse the Reliability of Systems in Healthcare, from the Reliability factors of subsystems and elements [3]. Generally Reliability Importance is a function of operation time of failure and repair characteristics (of all components in the system) and of the System Structure. So far, all Reliability Importance indices are calculated through combinatorial approaches such as Reliability Block Diagram or Fault Tree Analysis (FTA) or Markov Modeling [4].

Diabetes is a Complex group disease with a variety of causes. People with Diabetes have high blood glucose, also called high blood sugar or hyperglycemia. Diabetes is a disorder of metabolism the way the body uses digested food for energy. The digestive tract breaks down Carbohydrates, Sugars and Starches found in many Foods into Glucose, a form of sugar that enters the bloodstream. With the help of hormone insulin, cells throughout the body absorb glucose and use it

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for energy. Diabetes develops when the body does not make enough insulin or is not able to use insulin effectively or both [5].

In this Paper we discuss the Type 1 and Type 2 Diabetes Mellitus Disease and the study of Reliability of cure for the treatment of Diabetes in a Diabetic Health Centre. The paper is organized as follows. In this Section 1, we give the Introduction. In Section 2, we give the basic definitions and concepts. In Section 3, we discuss the Series-Parallel-Series Configuration. Model of the System using Fault-Tree-Analysis approach and derive the formula for Reliability of the System and Reliability Importance Indices of Components in the System are Calculated. In Section 4, we consider the Medical Example of Type 1 and Type 2 Diabetes and we related this with Series-Parallel-Series Configuration System. We discuss with Numerical illustration. In Section 5, we draw the Conclusion.

2. Basic Definitions

2.1. Reliability

Reliability is defined as the probability of a device(or an item) performing its purpose for the period intended under the given operating conditions. It can also be defined as the probability of non-failure. If F(t) is the failure probability, then $1 - F(t)$ gives the non-failure probability. Thus, the Reliability of the device (or an item) for time $T = t$ (i.e., the device functions satisfactorily for time $T \geq t$ is

$$
\begin{aligned}
R(t) &= 1 - F(t) \\
&= 1 - \int_{-\infty}^{t} f(x)\,dx \;\; text(or) \\
R(t) &= \int_{t}^{\infty} f(x)\,dx
\end{aligned}
$$

where $R(t)$ is the Reliability at time t.

2.2. Series System

In Series System, all Components in the System should be operating to maintain the required operation of the System. Thus, the failure of any one Component of the System will cause failure of the Whole System. The System Reliability of the Series System is given by

$$R_S(t) = R_1.R_2 \ldots R_m = \prod_{i=1}^{m} R_i(t)$$

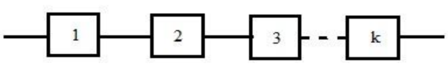


Figure 1. **series configuration**

where $R_i(t)$ is the Reliability of component i at time t, $R_S(t)$ is the series system Reliability at time t.

2.3. Parallel System

In a Parallel System, the System operates if one or more Components operate, and the System fails if all Components fail. The Parallel n-Components are represented by the following diagram.

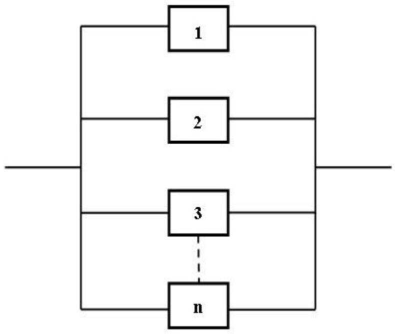


Figure 2. **Parallel Configuration**

The System Reliability of the Parallel System is given by

$$R_P\left(t\right) = 1 - \prod_{i=1}^{n} \left(1 - \left(R_i\left(t\right)\right)\right)$$

where, $R_i\left(t\right)$ is the Reliability of Component i at time t. $R_P\left(t\right)$ is the Parallel System Reliability at time t.

2.4. General Series Parallel System

The System consist of stage 1, stage 2, . . . , stage k, connected in series. Each stage contains a number of redundant elements, stage i consists of n_i redundant elements connected in parallel. The Reliability of the system is the product of the Reliabilities of each stage. Stage i with n_i elements will have the Reliability

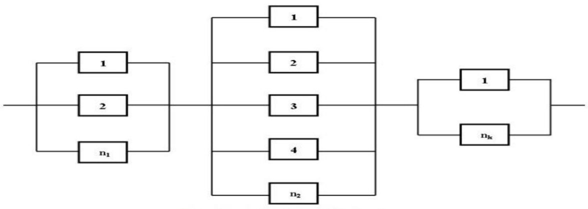


Figure 3. **General Series Parallel Configuration**

$$R_i = 1 - \left[1 - P\left(X_{i1}\right)\right]\left[1 - P\left(X_{i2}\right)\right] \ldots \left[1 - P\left(X_{in_i}\right)\right] = 1 - \prod_{j=1}^{n_i} \left[1 - P\left(X_{ij}\right)\right]$$

Therefore the system Reliability

$$R\left(S\right) = \prod_{i=1}^{k} \left[1 - \prod_{j=1}^{n_i} \left(1 - P\left(X_{ij}\right)\right)\right]$$

2.5. Fault Tree Analysis (FTA) Approach

Fault Tree Analysis Approach is a top-down approach of a system analysis that is used to determine the possible occurrence of undesirable events or failures. Over the years, the method has gained favor over other reliability analysis approaches because of its versatility in degree of detail of the complex systems. There are many symbols used to construct fault trees. The basic four symbols are given below. The Figure 4 denotes a fault event that occurs from the logical combination of fault events through the input of logic gates such as OR and AND

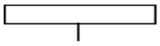


Figure 4. **Fault Event**

The Figure 5 denotes a basic fault event or the failure of an elementary system component. The events occurrence probability, failure and repair rates are usually obtained from empirical data or other source.

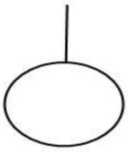


Figure 5. **Basic Fault Event**

The Figure 6 denotes the OR gate. It denotes the output fault event if one or more of input fault event

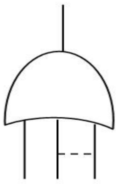


Figure 6. **Output Fault Event**

The Figure 7 denotes the AND gate which shows that an output fault tree event occurs if all the input fault events occur

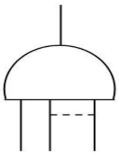


Figure 7. **Output fault tree event**

2.6. Reliability Importance

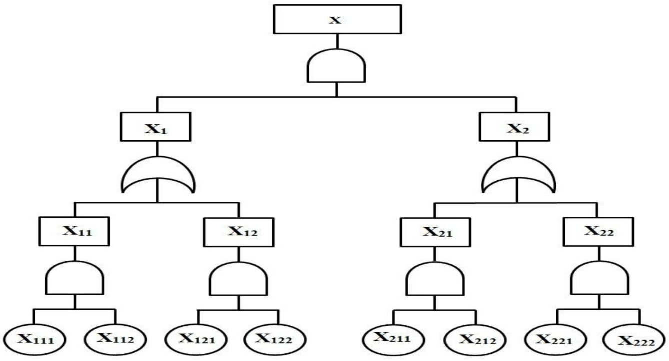
One of the most widely used Reliability Importance indices is Birnbaum's Component Importance [7]. Analytically this is defined by

$$I_K^B(t) = \frac{\partial R_S(t)}{\partial R_k(t)} = \frac{\partial F(t)}{\partial F_k(t)}$$

Where $I_K^B(t)$ is Reliability Importance of the Kth component. $R_S(t)$ and F(t) are the system Reliability and Unreliability at time t, respectively. For identical units $\lambda_i = \lambda$, $R_S(t) = 1 - \left(1 - e^{-\lambda t}\right)^n$.

3. Series Parallel Series Configuration Model (SPS)

Consider a system which has X_i (i=1, 2) Connected in series. Each subsystem has two subunits connected in parallel X_{ij} (i, j=1,2). Each subsystem has two subunits connected in series X_{ijk} (i, j, k=1,2). All the units involved in this system are independent and the base events probabilities $P(X_{ijk})$ are determined. Fault Tree Analysis diagram for a Series Parallel Series (SPS) Configuration system model is given below.



The Reliability for Series-Parallel-Series Configuration is given by,

$$R\left(S\right)=\prod_{i=1}^{2}\left[1-\prod_{j=1}^{2}\left[1-\prod_{k=1}^{2}P_{ijk}\right]\right] \tag{1}$$

Substituting $P(X_{ijk})=r_{ijk}$ and expanding the formulae

$$\begin{aligned}
&=1-[1-r_{111}r_{112}-r_{121}r_{122}+r_{111}r_{112}r_{121}r_{122}]\\
&=1-[1-r_{211}r_{212}-r_{221}r_{222}+r_{211}r_{212}r_{221}r_{222}]\\
&=1-\{1-[r_{111}r_{112}\left(1-r_{121}r_{122}\right)+r_{121}r_{122}(1-r_{111}r_{112})+r_{111}r_{112}r_{121}r_{122}]\}\\
&=1-\{1-[r_{211}r_{212}\left(1-r_{221}r_{222}\right)+r_{221}r_{222}(1-r_{211}r_{212})+r_{211}r_{212}r_{221}r_{222}]\}
\end{aligned} \tag{2}$$

The above formula can be written as,

$$R\left(S\right)=1-\left\{\prod_{i=1}^{2}\left\{1-\sum_{j=1}^{2}\left(\prod_{k=1}^{2}r_{ijk}\left[1-\prod_{k=1}^{2}r_{ij+1k}\right]\right)\right\}+\prod_{j=\text{‘}1}^{2}\left[\prod_{k=1}^{2}r_{ijk}\right]\right\} \tag{3}$$

Here, j+1=1, when j=2. In general,

$$R\left(S\right)=1-\left\{\prod_{i=1}^{n}\left\{1-\sum_{j=1}^{n}\left(\prod_{k=1}^{n}r_{ijk}\left[1-\prod_{k=1}^{n}r_{ij+1k}\right]\right)\right\}+\prod_{j=\text{‘}1}^{n}\left[\prod_{k=1}^{n}r_{ijk}\right]\right\} \tag{4}$$

The Reliability Importance of Independent Components in the Systems are derived

$$\begin{aligned}
\frac{\partial R}{\partial r_{11r}}=&\left[r_{11k}\left(1-\prod_{k=1}^{2}r_{12k}\right)-\prod_{\substack{k=1\\k\neq 1}}^{2}r_{11k}\prod_{k=1}^{2}r_{12k}+\prod_{\substack{k=1\\k\neq 1}}^{2}r_{11k}\prod_{k=1}^{2}r_{12k}\right]\\
&\left\{1-\left\{1-\left\{\sum_{j=1}^{2}\left[\prod_{k=1}^{2}r_{2jk}\left(1-\prod_{k=1}^{2}r_{2j+1k}\right)\right]\right\}+\prod_{j=1}^{2}\left(\prod_{k=1}^{2}r_{2jk}\right)\right\}\right\}
\end{aligned} \tag{5}$$

$$\begin{aligned}
\frac{\partial R}{\partial r_{12r}}=&\left[r_{12k}\left(1-\prod_{k=1}^{2}r_{11k}\right)-\prod_{k=1}^{2}r_{11k}\prod_{\substack{k=1\\k\neq 1}}^{2}r_{12k}+\prod_{k=1}^{2}r_{11k}\prod_{\substack{k=1\\k\neq 1}}^{2}r_{12k}\right]\\
&\left\{1-\left\{1-\left\{\sum_{j=1}^{2}\left[\prod_{k=1}^{2}r_{2jk}\left(1-\prod_{k=1}^{2}r_{2j+1k}\right)\right]\right\}+\prod_{j=1}^{2}\left(\prod_{k=1}^{2}r_{2jk}\right)\right\}\right\}
\end{aligned} \tag{6}$$

$$\frac{\partial R}{\partial r_{21r}} = \left[r_{21k}\left(1-\prod_{k=1}^{2} r_{22k}\right) - \prod_{k=1}^{2} r_{22k}\prod_{\substack{k=1\\k\neq 1}}^{2} r_{21k} + \prod_{k=1}^{2} r_{22k}\prod_{\substack{k=1\\k\neq 1}}^{2} r_{21k}\right]$$
$$\left\{1-\left\{1-\left\{\sum_{j=1}^{2}\left[\prod_{k=1}^{2} r_{1jk}\left(1-\prod_{k=1}^{2} r_{1j+1k}\right)\right]\right\}+\prod_{j=1}^{2}\left(\prod_{k=1}^{2} r_{1jk}\right)\right\}\right\} \quad (7)$$

$$\frac{\partial R}{\partial r_{22r}} = \left[r_{22k}\left(1-\prod_{k=1}^{2} r_{21k}\right) - \prod_{k=1}^{2} r_{21k}\prod_{\substack{k=1\\k\neq 1}}^{2} r_{22k} + \prod_{k=1}^{2} r_{21k}\prod_{\substack{k=1\\k\neq 1}}^{2} r_{22k}\right]$$
$$\left\{1-\left\{1-\left\{\sum_{j=1}^{2}\left[\prod_{k=1}^{2} r_{1jk}\left(1-\prod_{k=1}^{2} r_{1j+1k}\right)\right]\right\}+\prod_{j=1}^{2}\left(\prod_{k=1}^{2} r_{1jk}\right)\right\}\right\} \quad (8)$$

In general,

$$\frac{\partial R}{\partial r_{pqr}} = \left[\prod_{\substack{k=1\\k\neq 1}}^{n} r_{ijk}\left(1-\prod_{k=1}^{n} r_{ij+1k}\right) - \prod_{\substack{k=1\\i=p\\j=p}}^{n} r_{ij+1k}\prod_{\substack{k=1\\k\neq r\\i=p\\j=p}}^{n} r_{ijk} + \prod_{\substack{k=1\\j\neq q\\i=p}}^{n} r_{ijk}\prod_{\substack{k=1\\k\neq r\\j=q\\i=p}}^{n} r_{ijk}\right]$$
$$1-\left\{\prod_{\substack{i=1\\i\neq p}}^{n}\left\{1-\sum_{j=1}^{n}\left(\prod_{k=1}^{n} r_{ijk}\left[1-\prod_{k=1}^{n} r_{ij+1k}\right]\right)\right\} + \prod_{j=`1}^{n}\left[\prod_{k=1}^{n} r_{ijk}\right]\right\} \quad (9)$$

4. Medical Problem

The two main Types of diabetes are Type 1 and Type 2 diabetes.

Type 1 diabetes is usually caused by autoimmune destruction of the pancreatic beta cells and produce insulin that unlocks the cells of the body, allowing glucose to enter into blood and fuel them. Therefore, people with Type 1 diabetes have to inject everyday for the rest of their lives and this is one of the most challenging metabolic disorders because of the demands it imposes on day to day life (Ramesh Goyal 2010).

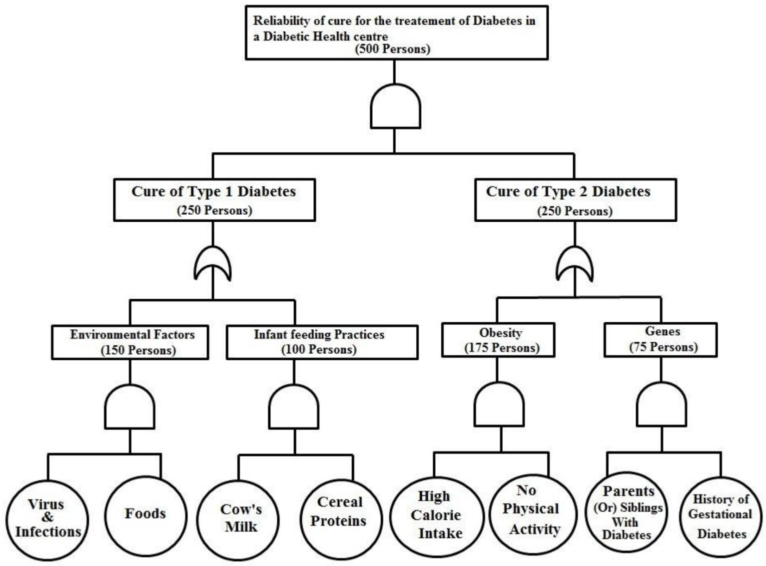
Globally there are close to 5, 00,000 children under the age of 15 with Type 1 diabetes. Every day 200 children develop Type 1 diabetes (International Diabetes Federation (IDF), 2008). Recent recommendations for age, sex and body size of the general population. In case of being overweight or obese, weight control strategies should be applied [8].

Type 2 diabetes is caused by a combination of genetic factors related to impaired insulin secretion and insulin resistance and environmental factors such as obesity, overeating, lack of exercise, and stress as well as aging. It is typically a multiple genes and environmental factors to varying extents [6].

4.1. Numerical Example

Consider the example for the Series Parallel Series (SPS) Configuration System related to Medical Problem which deals with the Reliability of the Cure of treatment of Diabetes in an Health Centre in particular year. The Diabetic patients arriving are of two different types (i.e.). of Type 1 and Type 2. Each of the above 2 types are classified into two subfactors, where each of the subfactors are caused by other factors.The above problem can be modeled as a Series-Parallel-Series Configuration System provided the base subfactors are independent.

Let as assume the probability of the base events for a small sample as,



$P(r_{111})$−Probability of Cure of patients with Virus and Infections = 49/80 = 0.6125.

$P(r_{112})$−Probability of Cure of patients with taking Foods = 55/70 = 0.7857.

$P(r_{121})$−Probability of Cure of patients with taking Cow's Milk = 20/25 = 0.8.

$P(r_{122})$−Probability of Cure of patients with taking Cereal Proteins = 52/75 =0.6933.

$P(r_{211})$−Probability of Cure of patients with taking High Calorie Intake = 60/75 = 0.8.

$P(r_{212})$−Probability of Cure of patients with No physical Activity = 70/100 = 0.7.

$P(r_{221})$−Probability of Cure of patients with affected by Parents or Siblings with Diabetes = 27/40 = 0.67.

$P(r_{222})$−Probability of Cure of patients with affected by with History of Gestational Diabetes = 22/35 = 0.6285.

Using equation (3) the reliability of Cure for the treatment of Diabetes in a Diabetic Health Centre, is R(S) = 0.5729

$$\frac{\partial R}{\partial r_{111}} = 0.2607$$

$$\frac{\partial R}{\partial r_{112}} = 0.2032$$

$$\frac{\partial R}{\partial r_{121}} = 0.2680$$

$$\frac{\partial R}{\partial r_{122}} = 0.3092$$

$$\frac{\partial R}{\partial r_{211}} = 0.3116$$

$$\frac{\partial R}{\partial r_{212}} = 0.3561$$

$$\frac{\partial R}{\partial r_{221}} = 0.2126$$

$$\frac{\partial R}{\partial r_{222}} = 0.2266$$

From the graph, it is clear that the Reliability Importance of Cure of patients affected due to Infant Feeding by Cow's milk and Obesity by taking High Calorie Intake are high. Whereas Reliability Importance of Cure of patients who are affected by Diabetes with various other factors seems to be very low, that Components Reliability can be improved to increase the Cure of Diabetes Patients.

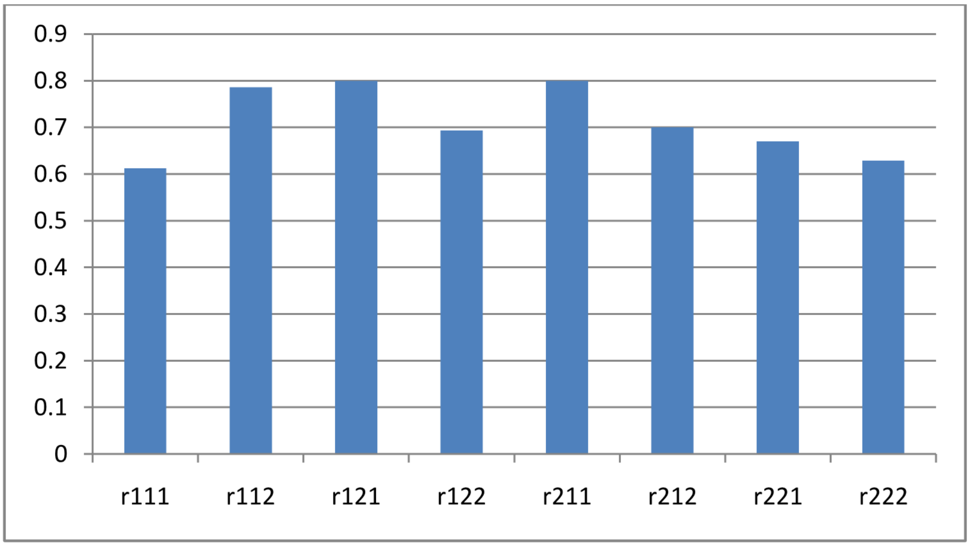


Figure 8. **Reliability of cure for the treatments of Diabetes in a Diabetic Health Centre**

5. Conclusion

In this Paper, we have derived the System Reliability and Reliability Importance of the Components in the System for SPS Configuration. We analyzed this with Numerical example from the medical field. We have found that by Calculating the Reliability Importance of the Components in the System, Certain Component's whose Reliability Importance is very low can be improved for the effective functioning of the System.

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