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Cost Analysis for a Telecommunication System with Standby Transmitter

Research Article

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Abstract: This paper deals with the Cost analysis for telecommunication system with standby transmitter. The failure and repair rates of the subsystems follow the exponential distribution. One Standby control unit has been taken to improve system performance. Supplementary variables techniques have been used for mathematical formulation of the model. The steady state availability expression has been derived using normalizing conditions. Laplace transform is being utilized to solve the mathematical equations. Some particular cases and asymptotic behavior of the system have also been derived to improve the practical utility of the model. Cost function and availability of the system have been computed. The findings of the present paper will be highly useful to the telecommunication system to enhance system performance.

MSC: 90B25.

Keywords: Reliability function, Supplementary variables, Perfect Switching, Standby redundancy, Availability and cost function.

1. Introduction

A communications network is a collection of transmitters, receivers, and communications channels that send messages to one another. In this research, the authors have studied about cost estimation [5] of telecommunication system. Telecommunication, system configuration has been shown in Fig 1 and 2, respectively. The whole system has been divided into four subsystems namely A, C, D and E.

The system under consideration consists of four subsystems; viz; A, C, D, E:

Transmitter(A): The Transmitter performed the function of modulating the audio signal and radiates the same in the form of electromagnetic wave. The Transmitter performs signal processing operation and thereby it couples the input message signal to the communication channel. Transmitter functions pertaining to the signal processing are:

- Amplifications.
- Filtering.
- Modulation.

Thus resultant modulated wave could be transmitted into air through antenna or transmitting aerial, which transmits these signals uniformly in all direction in the form of electromagnetic waves.

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Communication channel (C): Communicating data from one location to another requires some form of pathway or medium. These pathway ys, called communication channels. It refers either to a physical transmission medium such as a wire or to a logical connection over a multiplexed medium such as a radio channel. A channel is used to convey an information signal, for example a digital bit stream, from one or several senders (or transmitters) to one or several receivers. A channel has a certain capacity for transmitting information, often measured by its bandwidth in Hz or its data rate in bits per second.

Receiver (D): In communications, a receiver is an electronic device that receives radio waves and converts the information carried by them to a usable form. It is used with an antenna. The antenna intercepts radio waves (electromagnetic waves) and converts them to tiny alternating currents which are applied to the receiver, and the receiver extracts the desired information. The receiver uses electronic filters to separate the desired radio frequency signal from all the other signals picked up by the antenna, an electronic amplifier to increase the power of the signal for further processing, and finally recovers the desired information through demodulation. The information produced by the receiver may be in the form of sound (an audio signal), images (a video signal) or data (a digital signal).

Destination (E): In this model, the authors have taken one stand by redundant transmitter. So, the subsystem A has two standby redundant units A1 and A2. On failure of main unit A1, we can online standby unit A2 through a switching device B. The capability of a system is affected by all the considered units in the system. The whole system gets fail if any of its subsystems stop working. All failures follow exponential time distribution whereas all repairs follow general time distribution. A set of difference-differential equations has been obtained that governing the behaviour of considered system. Laplace transform and supplementary variable technique have been used to solve and formulate mathematical model. All failures follow exponential time distribution whereas all repairs follow general time distribution. Laplace transforms of various state probabilities have been obtained. A numerical example together with its graphical illustration has appended in last to highlight important results of this study. Block diagram, considered system's diagram and transition state diagram have been shown respectively in Figure 1, Figure 2 and Figure 3.

2. Assumptions

Assumptions associated with this model are as follows:

- Initially, the whole system is new and operable.
- All failures follow exponential time distribution and are S-independent.
- All Repairs follow general time distribution and are perfect.
- Switching device used to online standby unit of subsystem A is imperfect.
- The subsystem A can be repaired after complete failure
- Only one change can take place in one transition.
- The failure rates of both units of subsystem A are equal.

3. Notations

The following notations have been used throughout in this model:

$P_0(t)$	The probability that at time t, the system is in operable state (good state).
f_i	Failure rate of i^{th} subsystem of the system, $i = A,\ C,\ D,\ E$, for subsystem A, C, D and E, respectively.
$1-\alpha$	Failure rate of Switching device B.
$r_i(j)\Delta$	The first order probability that the i^{th} subsystem will be repaired in the time interval $(j,\ j+\Delta)$, conditioned that it was not repaired up to the time j, where $i = A,\ C,\ D,\ E$ and $j = m,\ x,\ y,\ z$ respectively.
$P_A(t)$	The probability that system is operable through A_2 unit while A_1 unit is already failed.
$P_B(t)$	The probability that system is failed due to failure of switching device B.
$P_{A1i}(j,\ t)\Delta$	The probability that at time t, the i^{th} subsystem is failed after the failure of A_1 unit of the subsystem A. Elapsed repair time lies in the interval $(j,\ j+\Delta)$.
$R(t)$	Reliability function.
μ	Repair rate of switching device B.
$\overline{P}(s)$	Laplace transform of function $P(t)$.
$S_{i\setminus}(j)$	$r_i(j).\exp\left\{-\int r_i(j)dj\right\}$. For all i and j.
$P_i(j,\ t)\Delta$	The probability that the system is failed at time t due to failure of i^{th} subsystem and elapsed repair time lies in the interval.
$P_i(t)$	The probability that system is failed due to failure of subsystem i, where i= C, D and E.

4. Formulation of Mathematical Model

Using elementary probability considerations and limiting procedure, we obtain the following set ofdifference-differential equations governing the behavior of considered system, continuous in time and discrete in space:

$$\left[\frac{d}{dt} + \alpha f_A + f_C + f_D + f_E\right] P_0(t) = \mu P_B(t) + \int_0^\infty P_A(m,t) r_A(m) dm + \int_0^\infty P_C(x,t) r_c(x) dx + \int_0^\infty P_D(y,t)\mu_D(y)dy + \int_0^\infty P_E(z,t)\mu_E(z)dz \tag{1}$$

Similarly, the difference-differential equations for other states are:

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + r_C(x)\right] P_C(x,t) = 0 \tag{2}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + r_D(y)\right] P_D(y,t) = 0 \tag{3}$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + r_E(z)\right] P_E(z,t) = 0 \tag{4}$$

$$\left[\frac{d}{dt} + f_A + f_C + f_D + f_E + (1-\alpha)\right] P_{A1}(t) = \alpha f_A P_0\left(t\right) + \int_0^\infty P_{A1C}\left(x,t\right) r_C\left(x\right) dx + \int_0^\infty P_{A1D}\left(y,t\right) r_D\left(y\right) dy + \int_0^\infty P_{A1E}\left(z,t\right) r_E\left(z\right) dz \tag{5}$$

$$\left[\frac{d}{dt} + \mu\right] P_B\left(t\right) = (1-\alpha) P_{A1}(t) \tag{6}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+r_C\left(x\right)\right]P_{A1C}\left(x,t\right)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+r_D\left(y\right)\right]P_{A1D}\left(y,t\right)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+r_E\left(z\right)\right]P_{A1E}\left(z,t\right)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+r_A\left(m\right)\right]P_{A}\left(m,t\right)=0 \tag{10}$$

Boundary conditions:

$$P_C\left(0,t\right)=f_C.P_0\left(t\right) \tag{11}$$

$$P_D\left(0,t\right)=f_D.P_0\left(t\right) \tag{12}$$

$$P_E\left(0,t\right)=f_E.P_0\left(t\right) \tag{13}$$

$$P_{A1C}\left(0,t\right)=f_C.P_{A1}\left(t\right) \tag{14}$$

$$P_{A1D}\left(0,t\right)=f_D.P_{A1}\left(t\right) \tag{15}$$

$$P_{A1E}\left(0,t\right)=f_E.P_{A1}\left(t\right) \tag{16}$$

$$P_{A}\left(0,t\right)=f_A.P_{A1}\left(t\right) \tag{17}$$

Initial conditions: $P_0\left(0\right)=1$, and all other state probabilities are zero at

$$t=0 \tag{18}$$

5. Solution of the Model

In order to solve the model, we shall obtain all transition state probabilities by solving equations equations (1) till (17) subjected to initial conditions (18), we have:

$$\overline{P}_0\left(s\right)=\frac{1}{U(s)} \tag{19}$$

$$\overline{P}_C\left(s\right)=\frac{f_C D_C(s)}{U(s)} \tag{20}$$

$$\overline{P}_D\left(s\right)=\frac{f_D D_D(s)}{U(s)} \tag{21}$$

$$\overline{P}_E\left(s\right)=\frac{f_E D_E(s)}{U(s)} \tag{22}$$

$$\overline{P}_{A1}(s)=\frac{W}{U(s)} \tag{23}$$

$$\overline{P}_{B}(s)=\frac{(1-\alpha)W}{(S+\mu)U(s)} \tag{24}$$

$$\overline{P}_{A1C}(s)=\frac{f_C W D_C(s)}{U(s)} \tag{25}$$

$$\overline{P}_{A1}(s)=\frac{W}{U(s)} \tag{26}$$

$$\overline{P}_{A1}(s)=\frac{W}{U(s)} \tag{27}$$

$$\overline{P}_{A}(s)=\frac{f_A W D_A}{U(s)} \tag{28}$$

where

$$W=\frac{\alpha f_A}{(s+f_A+f_C+f_D+f_E)\left[(1-\alpha)-f_C\overline{S}_C(s)-f_D\overline{S}_D(s)-f_E\overline{S}_E(s))\right]} \tag{29}$$

$$D_i\left(s\right)=\frac{1-\overline{S}_i\left(s\right)}{s},\quad \forall i=C,D,E \tag{30}$$

and

$$U(s)=s+\alpha f_A+f_C+f_D+f_E-\frac{(1-\alpha)W\mu}{s+\mu}-f_AW\overline{S}_A(s)-f_CW\overline{S}_C(s)-f_DW\overline{S}_D(s)-f_EW\overline{S}_E(s) \tag{31}$$

Sum of equations (19) through (27)= $\frac{1}{s}$

6. Asymtotic Behaviour Analysis

By using final value theorem in Laplace transform; viz $\lim\limits_{t\to\infty} P\left(t\right)=\lim\limits_{s\to 0} s\overline{P}\left(s\right)=P$ (say), provided the limit on LHS exists, we can obtain the time independent state probabilities from equations (19) through (27) as follows:

$$P_0=\lim_{t\to\infty}P_0(t)=\lim_{s\to 0}s\overline{P}_0(s)=\frac{1}{U'(0)} \tag{32}$$

Similarly $P_i=\frac{f_iM_i}{U'(0)}$

$$i=C,D,E\ \text{ respectively} \tag{33}$$

$$P_A=\frac{T}{F'(0)} \tag{34}$$

$$P_B=\frac{(1-\alpha)T}{\mu F'(0)} \tag{35}$$

$$P_{A1i}=\frac{TM_if_i}{F'(0)},\ \text{ and} \tag{36}$$

$$P_{A1}=\frac{TM_Af_A}{F'(0)},\ \text{ and} \tag{37}$$

$$M=-\overline{S_i}(0)=\text{Mean time to repair i}^{\text{th}}\text{ subsystem and} \tag{38}$$

$$T=\frac{\alpha f_A}{f_A+(1-\alpha)} \tag{39}$$

6.1. Availability of the Considered System

L.T. of availability of considered system is given by also, $P_{up}(t)=\overline{P_0}(s)+\overline{P_{A1}}(s)$ or

$$\overline{P}_{up}(s)=\frac{1}{(s+\alpha f_A+f_C+f_D+f_E)}\left[1+\frac{\alpha f_A}{s+f_A+f_C+f_D+f_E+(1-\alpha)}\right]$$

Taking inverse Laplace transform, we obtain

$$P_{up}(t)=\left(1+\frac{\alpha f_A}{(1+\alpha)(1+f_A)}\right)e^{-(\alpha f_A+f_C+f_D+f_E)t}-\left(\frac{\alpha f_A}{(1+\alpha)(1+f_A)}\right)e^{-(f_A+f_C+f_D+f_E+1-\alpha)t} \tag{40}$$

6.2. Cost Analysis

Cost function for the considered system is given by

$$G(t)=C_1\int_0^t P_{up}(t)dt-C_2t \tag{41}$$

where, C_1 = revenue per unit up time, C_2 = repair cost per unit time and

$$\int_0^t P_{up}(t)dt=(1+H)\frac{1-e^{-(\alpha f_A+f_C+f_D+f_E)t}}{\alpha f_A+f_C+f_D+f_E}-H\frac{1-e^{-(\alpha f_A+f_C+f_D+f_E+1-\alpha)t}}{f_A+f_C+f_D+f_E+(1-\alpha)} \tag{42}$$

Where,

$$H=\left(\frac{\alpha f_A}{(1-\alpha)(1+f_A)}\right)$$

7. Numerical Illustration

For a numerical illustration, we consider the values: $\alpha = 0.7$, $f_A = 0.002$, $f_C = 0.004$, $f_D = 0.001$, $f_E = 0.006$, $C_1 = Rs.9.00$, $C_2 = Rs.4.00$ and $t = 0, 1, 2, \ldots, 16$. By using these values, we compute the Table 1 and 2, respectively. Corresponding graphs have been shown in Figure 3 and 4, respectively.

8. Results and Discussion

Table 1 computes availability of considered system for various values of time t. Its graph has been shown in Figure 3. A study of Table 1 and Figure 3 reveals that availability of considered system decreases rapidly initially but thereafter it decrease approximately in constant manner. $P_{up}(t)$ approaches to zero for a very large value of time t. Table 2 gives the cost function G(t) for various values of time t. Its graph has been shown in Figure 4. Examination of Table 2 and Figure 4 yields that cost function increases constantly up to $t = 16$ and appears its maximum value.

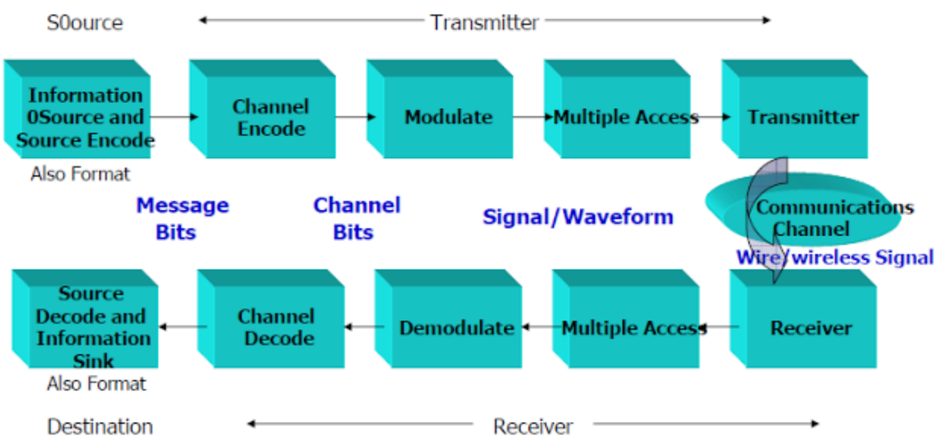

Figure 1. Represents the diagram of Communication system

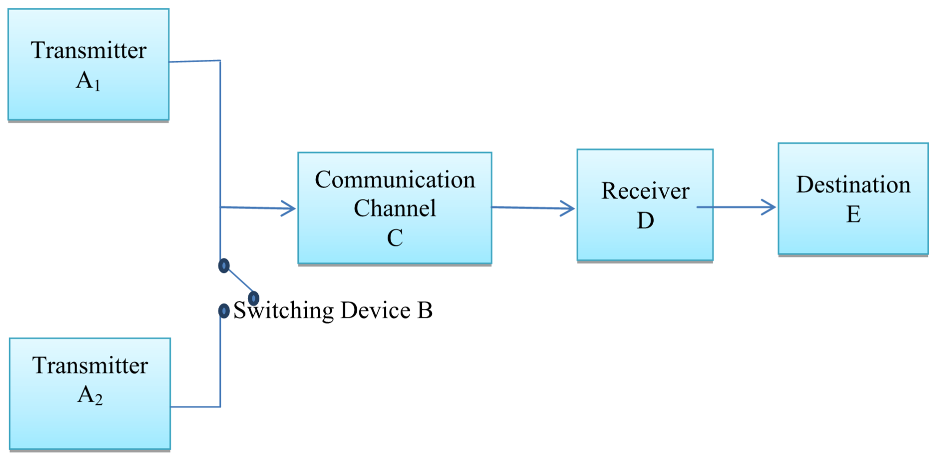

Figure 2. Represents the system configuration of Communication system

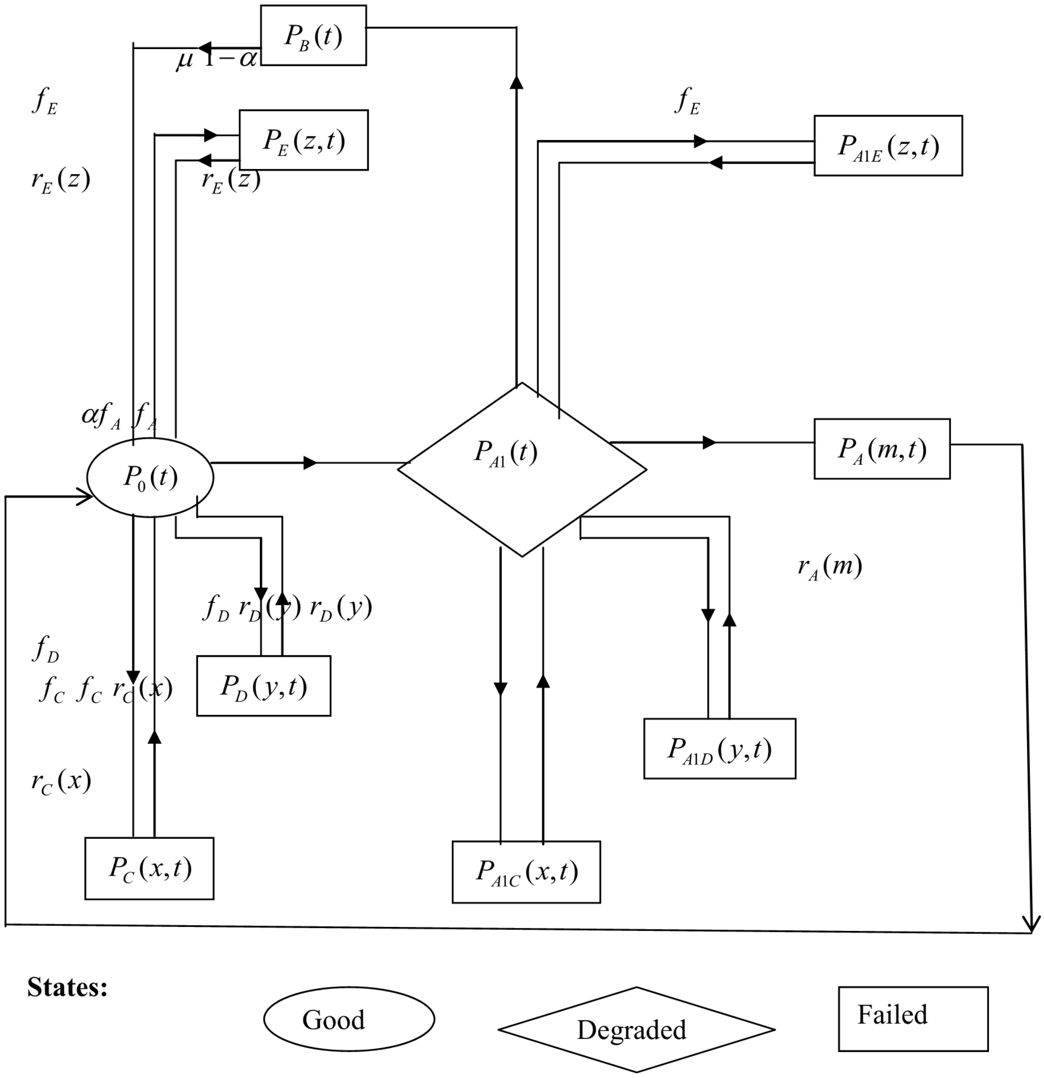

Figure 3. State-transition diagram

t	$P_{up}(t)$
0	1
1	0.987952
2	0.975979
3	0.964099
4	0.952325
5	0.940668
6	0.929134
7	0.917726
8	0.906447
9	0.895299
10	0.884282
11	0.873397
12	0.862642
13	0.852018
14	0.841523
15	0.831156
16	0.820916

Table 1.

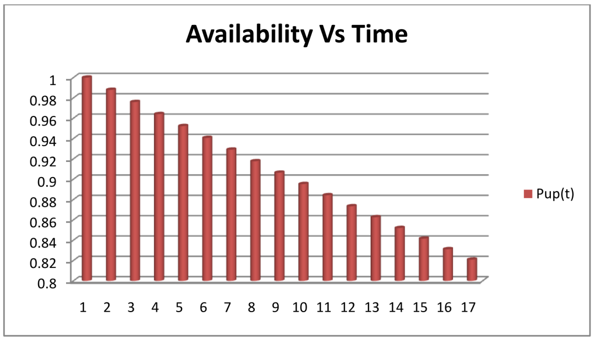

Figure 4.

t	G(t)
0	0
1	4.94935
2	9.797611
3	14.54356
4	19.18665
5	23.72684
6	28.16441
7	32.49994
8	36.73415
9	40.86792
10	44.90221
11	48.83805
12	52.67649
13	56.41863
14	60.06558
15	63.61845
16	67.07836

Table 2. **Cost Function Vs Time**

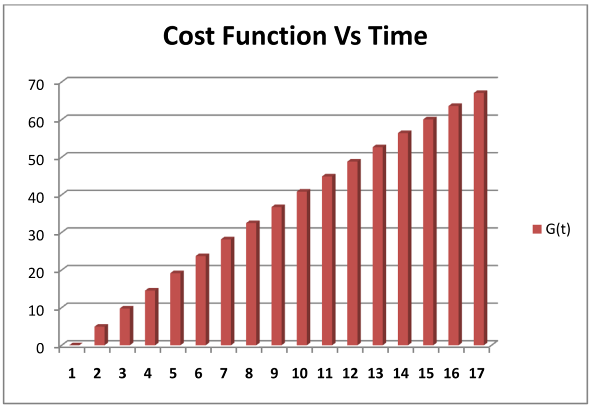

Figure 5.

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