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On Semi α-Regular Pre-Semi Closed Sets in Bitopological Spaces

Research Article

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Abstract: In this paper, we introduce a new class of closed sets and open sets in bitopological spaces, namely, $\tau_1\tau_2$-s αrps-closed sets and $\tau_1\tau_2$-s αrps-open sets and characterize the properties of this sets.

Keywords: $\tau_1\tau_2$-s αrps-closed, $\tau_1\tau_2$-s αrps-open and τ_1-αrps-open.

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1. Introduction

A triple $(X,\ \tau_1,\ \tau_2)$, where X is a non empty set and τ_1 and τ_2 are topologies on X is called a bitopological space. In 1963, Kelly initiated the study of bitopological spaces. In 1985, Fukutake introduce the concept of g-closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. The aim of this paper is to extend the same concept in bitopological spaces.

2. Preliminaries

Let $(X,\ \tau_1,\ \tau_2)$ or simply X denotes a bitopological space. For any subset $A \subseteq X$, τ_2-intA and τ_2-clA denote the interior and closure of a set A with respect to the topology τ_2 respectively. X \ A denotes the complement of A in X. We shall now require the following known definitions.

Definition 2.1. *A subset A of a space X is called*

(1). pre-open [9] if $A \subseteq int\ clA$ and pre-closed if $cl\ intA \subseteq A$.

(2). semi-open [8] if $A \subseteq cl\ intA$ and semi-closed if $int\ clA \subseteq A$.

(3). semi-pre-open [1] if $A \subseteq cl\ int\ clA$ and semi-pre-closed if $int\ cl\ intA \subseteq A$.

(4). α-open [10] if $A \subseteq int\ cl\ intA$ andα-closed if $cl\ int\ clA \subseteq A$.

(5). regular open [14] if $A = int\ clA$ and regular closed if $cl\ intA = A$.

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(6). b-open [2] if $A \subseteq cl\ intA \bigcup int\ clA$ and b-closed if $cl\ intA \bigcap int\ clA \subseteq A$.

The τ_2-semi-closure (resp. τ_2-pre-closure, resp. τ_2-semi-pre-closure, resp. τ_2-α-closure, resp. τ_2-b-closure) of a subset A of X is the intersection of all semi-closed (resp. pre-closed, resp. semi-pre-closed, resp. α -closed, resp. b-closed) sets containing A with respect to the topology τ_2 and is denoted by τ_2-sclA (resp. τ_2-pclA, resp. τ_2-spclA, resp. τ_2-αclA, resp. τ_2-bclA).

Definition 2.2. *A subset A of a space X is called*

(1). $\tau_1\tau_2$-generalized closed [5] (briefly $\tau_1\tau_2$-g-closed) if τ_2-clA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.

(2). $\tau_1\tau_2$-regular generalized closed [7] (briefly $\tau_1\tau_2$-rg-closed) if τ_2-clA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-regular open.

(3). $\tau_1\tau_2$-α-generalized closed [11] (briefly $\tau_1\tau_2$-αg-closed) if τ_2-$\alpha clA \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.

(4). $\tau_1\tau_2$-generalized semi-closed [6] (briefly $\tau_1\tau_2$-gs-closed) if τ_2-sclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-open.

(5). $\tau_1\tau_2$-generalized αb-closed [19] (briefly $\tau_1\tau_2$-gαb-closed) if τ_2-bclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-α-open.

(6). $\tau_1\tau_2$-regular generalized b-closed [4] (briefly $\tau_1\tau_2$-rgb-closed) if τ_2-bclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-regular open.

(7). $\tau_1\tau_2$-$g^{\#}$-semi-closed [18] (briefly $\tau_1\tau_2$-$g^{\#}s$-closed) if τ_2-sclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1- αg-open.

(8). $\tau_1\tau_2$-strongly generalized closed [12] (briefly $\tau_1\tau_2$-g*-closed) if τ_2-clA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-g-open.

(9). $\tau_1\tau_2$-g*-preclosed [17] (briefly $\tau_1\tau_2$-g*p-closed) if τ_2-pclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-g-open.

Lemma 2.3 ([16]). *For a subset A of X, sint(sclA $\setminus$ A) = Ø.*

3. On Semi α-regular Pre-Semi Closed Sets In Bitopological Spaces

In this section, we introduce the concept of $\tau_1\tau_2$-$s\alpha rps$-closed sets in bitopological spaces and discuss some of the related properties.

Definition 3.1. *A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-semi α-regular pre-semi closed (briefly $\tau_1\tau_2$-sαrps-closed) if τ_2-sclA $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-αrps-open in X.*

Remark 3.2. *If $\tau_1 = \tau_2$ in the above definition, a $\tau_1\tau_2$-sαrps-closed is a sαrps-closed in the sense of point set topology. The set of all $\tau_1\tau_2$-sαrps-closed sets in the bitopological space (X,τ_1,τ_2) is denoted by $S\alpha RPSC(X,\tau_1,\tau_2)$.*

Remark 3.3. *In general, the set of all $\tau_1\tau_2$-sαrps-closed sets need not be equal to the set of all $\tau_2\tau_1$-sαrps-closed sets as seen in the following example.*

Example 3.4. *Let X = {a,b,c} with the topologies τ_1 = {ϕ,{a},{b},{a,b},X} and τ_2 = {ϕ,{a},{b,c},X}. ThenSαRPSC(X,τ_1,τ_2) = {ϕ,{a},{c},{a,c},{b,c},X} and SαRPSC(X,τ_2,τ_1) = P(X). Therefore SαRPSC(X,τ_1,τ_2) $\neq$SαRPSC(X,τ_2,τ_1).*

Remark 3.5. *Difference between two $\tau_1\tau_2$-sαrps-closed sets need not be a $\tau_1\tau_2$-sαrps-closed set as seen in the Example 3.6.*

Example 3.6. *From Example 3.4, A = {b,c} and B = {c} are $\tau_1\tau_2$-sαrps-closed sets. But A $\setminus$ B = {b} is not $\tau_1\tau_2$-sαrps-closed.*

Proposition 3.7. *Let (X,τ_1,τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.*

(1). Every τ_2-semi-closed set is $\tau_1\tau_2$-sαrps-closed.

(2). Every τ_2-regular closed set is $\tau_1\tau_2$-sαrps-closed.

(3). Every τ_2-α-closed set is $\tau_1\tau_2$-sαrps-closed.

(4). Every $\tau_1\tau_2$-$g^{\#}$- semi-closed set is $\tau_1\tau_2$-sαrps-closed.

Proof.

(1). Let A be a τ_2-semi-closed subset of X. Let A $\subseteq$ U and U is τ_1-αrps-open. Since A is τ_2-semi-closed, τ_2-sclA = A. Therefore τ_2-sclA $\subseteq$ U. Hence A is $\tau_1\tau_2$-sαrps-closed.

(2). Let A be a τ_2-regular closed subset of X. Since every τ_2-regular closed set is τ_2-semi-closed and by (1), we have A is $\tau_1\tau_2$-sαrps-closed.

(3). Let A be a τ_2-α-closed subset of X. Since every τ_2-α -closed set is τ_2-semi-closed and by (1), we have A is $\tau_1\tau_2$-sαrps-closed.

(4). Let A be a $\tau_1\tau_2$-$g^{\#}$-semi-closed subset of a space X. Let A $\subseteq$ U and U is τ_1-αrps-open. Since every τ_1-αrps-open set is τ_1-αg-open and since A is $\tau_1\tau_2$-$g^{\#}$-semi-closed, τ_2-sclA$\subseteq$ U. Hence A is $\tau_1\tau_2$-sαrps-closed.

$\square$

The reverse implications are not true as shown in the Examples 3.8 and 3.9.

Example 3.8. *Let $X = \{a,b,c\}$ with the topologies $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. Then $\{c\}$ is $\tau_1\tau_2$-sαrps-closed, but not τ_2-semi-closed, not τ_2-regular closed and not τ_2-α -closed.*

Example 3.9. *Let $X = \{a,b,c\}$ with the topologies $\tau_1 = \{\phi,\{b,c\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. Then $\{c\}$ is $\tau_1\tau_2$-sαrps-closed, but not $\tau_1\tau_2$-$g^{\#}$-semi-closed.*

Proposition 3.10. *Let (X,τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.*

(1). Every $\tau_1\tau_2$-sαrps-closed set is $\tau_1\tau_2$-gs-closed.

(2). Every $\tau_1\tau_2$-sαrps-closed set is $\tau_1\tau_2$-gαb-closed.

(3). Every $\tau_1\tau_2$-sαrps-closed set is $\tau_1\tau_2$-rgb-closed.

Proof.

(1). Let A be a $\tau_1\tau_2$-sαrps-closed subset of a space X. Let A $\subseteq$ U and U is τ_1-open. Since every τ_1-open set is τ_1-αrps-open and since A is $\tau_1\tau_2$-sαrps-closed, τ_2-sclA $\subseteq$ U. Hence A is $\tau_1\tau_2$-gs-closed.

(2). Let A be a $\tau_1\tau_2$-sαrps-closed subset of a space X. Let A $\subseteq$ U and U is τ_1-α-open. Since every τ_1-α-open set is τ_1-αrps-open and since A is $\tau_1\tau_2$-sαrps-closed, τ_2-sclA $\subseteq$ U. But τ_2-bclA $\subseteq$ τ_2-sclA. Hence A is $\tau_1\tau_2$-gαb-closed.

(3). Let A be a $\tau_1\tau_2$-sαrps-closed subset of a space X. Let A $\subseteq$ U and U is τ_1-regular open. Since every τ_1-regular open set is τ_1-αrps-open and since A is $\tau_1\tau_2$-sαrps-closed, τ_2-sclA $\subseteq$ U. But τ_2-bclA $\subseteq$ τ_2-sclA. Hence A is $\tau_1\tau_2$-rgb-closed.

$\square$

The reverse implications are not true as shown in the Examples 3.11 and 3.12.

Example 3.11. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},$ $\{a,b\},\{b,c\},\{a,b,c\},X\}$. *Then* $\{a,b,c\}$ *is* $\tau_1\tau_2$-*gs-closed and* $\tau_1\tau_2$-*rgb-closed, but not* $\tau_1\tau_2$-*sαrps-closed.*

Example 3.12. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a,b\},\{a,b,c\},X\}$. *Then* $\{a\}$ *is* $\tau_1\tau_2$-*gαb-closed, but not* $\tau_1\tau_2$-*sαrps-closed. The concept* $\tau_1\tau_2$-*sαrps-closed is independent from the concepts* $\tau_1\tau_2$-*αg-closed,* $\tau_1\tau_2$-*g*-closed,* $\tau_1\tau_2$-*g*p-closed,* $\tau_1\tau_2$-*rg-closed and* $\tau_1\tau_2$-*g-closed as seen in the following examples.*

Example 3.13. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$. *Then*

(1). $\{a\}$ is $\tau_1\tau_2$ -sαrps-closed but not $\tau_1\tau_2$-αg-closed and $\{a,b,c\}$ is $\tau_1\tau_2$-αg-closed but not $\tau_1\tau_2$-sαrps-closed.

(2). $\{a\}$ is $\tau_1\tau_2$-sαrps-closed but not $\tau_1\tau_2$-g-closed and $\{a,b,c\}$ is $\tau_1\tau_2$-g-closed but not $\tau_1\tau_2$-sαrps-closed.

Example 3.14. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{b,c\},\{a,b,c\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},X\}$. *Then* $\{a\}$ *is* $\tau_1\tau_2$-*sαrps-closed but not* $\tau_1\tau_2$-*rg-closed and* $\{a,b\}$ *is* $\tau_1\tau_2$-*rg-closed but not* $\tau_1\tau_2$-*sαrps-closed.*

Example 3.15. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},X\}$. *Then* $\{a\}$ *is* $\tau_1\tau_2$-*sαrps-closed but not* $\tau_1\tau_2$-*g*-closed and not* $\tau_1\tau_2$-*g*p-closed.*

Example 3.16. *Let* $X = \{a,b,c\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,c\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. *Then* $\{b\}$ *is* $\tau_1\tau_2$-*g*-closed and* $\tau_1\tau_2$-*g*p-closed, but not* $\tau_1\tau_2$-*sαrps-closed.*

Theorem 3.17. *If A and B are* $\tau_1\tau_2$-*sαrps-closed sets, then* $A \bigcap B$ *is also* $\tau_1\tau_2$-*sαrps-closed.*

Proof. Let $A \bigcap B \subseteq U$ and U be τ_1-αrps-open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\tau_1\tau_2$-sαrps-closed sets, τ_2-sclA $\subseteq$ U and τ_2-sclB $\subseteq$ U. Therefore $(\tau_2$-sclA$) \bigcap (\tau_2$-sclB$) \subseteq$ U. But τ_2-scl$(A \bigcap B) \subseteq ($ τ_2-sclA$) \bigcap (\tau_2$-sclB$)$ [3]. Therefore τ_2-scl$(A \bigcap B) \subseteq$ U. Hence $A \bigcap B$ is $\tau_1\tau_2$-sαrps- closed. $\square$

Corollary 3.18.

(1). If A is $\tau_1\tau_2$-sαrps-closed and F is τ_2-semi-closed, then $A \bigcap F$ is $\tau_1\tau_2$-sαrps-closed.

(2). If A is $\tau_1\tau_2$-sαrps-closed and F is τ_2-regular closed, then $A \bigcap F$ is $\tau_1\tau_2$-sαrps-closed.

Proof.

(1). Since F is τ_2-semi-closed, by Proposition 3.7(i), F is $\tau_1\tau_2$-sαrps-closed. Since A is $\tau_1\tau_2$-sαrps-closed, by Theorem 3.17, $A \bigcap F$ is $\tau_1\tau_2$-sαrps-closed.

(2). Since F is τ_2-regular-closed, by Proposition 3.7 (2), F is $\tau_1\tau_2$-sαrps-closed. Since A is $\tau_1\tau_2$ -sαrps-closed, by Theorem 3.17, $A \bigcap F$ is $\tau_1\tau_2$-sαrps-closed.

$\square$

Remark 3.19. *The union of two* $\tau_1\tau_2$-*sαrps-closed sets need not be* $\tau_1\tau_2$-*sαrps-closed as seen in the following example.*

Example 3.20. *From Example 3.13, $A = \{a\}$ and $B = \{b,c\}$ are $\tau_1\tau_2$-sαrps-closed sets. But $A \bigcup B = \{a,b,c\}$ is not $\tau_1\tau_2$-sαrps-closed.*

Theorem 3.21. *If a set A is* $\tau_1\tau_2$-*sαrps-closed then, τ_2-sclA $\setminus$ A does not contain a non empty τ_1-αrps-closed set.*

Proof.   Suppose that A is $\tau_1\tau_2$-$s\alpha rps$-closed in X. Let F be a τ_1-αrps-closed subset of τ_2-sclA $\setminus$ A. Then F $\subseteq \tau_2$-sclA $\bigcap$ (X $\setminus$ A) $\subseteq$ X $\setminus$ A and so A $\subseteq$ X $\setminus$ F. Since A is $\tau_1\tau_2$-$s\alpha rps$- closed and since X $\setminus$ F is τ_1-αrps-open, τ_2-sclA $\subseteq$ X $\setminus$ F that implies F $\subseteq$ X $\setminus \tau_2$-sclA. As we have already F $\subseteq \tau_2$-sclA. It follows that F $\subseteq \tau_2$-sclA $\bigcap$ (X $\setminus \tau_2$-sclA) = $\emptyset$. Thus F = $\emptyset$. Therefore τ_2-sclA $\setminus$ A does not contain a non empty τ_1-αrps-closed set. $\square$

Theorem 3.22. *Let A be $\tau_1\tau_2$-$s\alpha rps$-closed. Then A is τ_2-semi-closed if and only if τ_2-sclA $\setminus$ A is τ_1-αrps-closed.*

Proof.   If A is τ_2-semi-closed, then τ_2-sclA = A and so τ_2-sclA $\setminus$ A = $\emptyset$ which is τ_1-αrps-closed.

Conversely suppose that τ_2-sclA $\setminus$ A is τ_1-αrps-closed. Since A is $\tau_1\tau_2$-$s\alpha rps$-closed, by Theorem 3.21, τ_2-sclA $\setminus$ A = $\emptyset$. That is τ_2-sclA = A and hence A is τ_2-semi-closed. $\square$

Theorem 3.23. *If A is $\tau_1\tau_2$-$s\alpha rps$-closed and τ_1-αrps-open, then A is τ_2-semi-closed.*

Proof.   Since A is $\tau_1\tau_2$-$s\alpha rps$-closed and τ_1-αrps-open, τ_2-sclA $\subseteq$ A. Therefore A = τ_2-sclA. Hence A is τ_2-semi-closed. $\square$

Theorem 3.24. *If A is $\tau_1\tau_2$-$s\alpha rps$-closed and if A $\subseteq$ B $\subseteq \tau_2$-sclA, then*

(1). B is $\tau_1\tau_2$-$s\alpha rps$-closed.

(2). τ_2-sclB $\setminus$ B contains no non empty τ_1-αrps-closed set.

Proof.   A $\subseteq$ B $\subseteq \tau_2$-sclA $\Rightarrow \tau_2$-sclA = τ_2-sclB.

(1). Let B $\subseteq$ U and U be τ_1-αrps-open. Then A $\subseteq$ U. Since A is $\tau_1\tau_2$-$s\alpha rps$-closed, τ_2 -sclA $\subseteq$ U. That implies τ_2-sclB $\subseteq$ U. This proves (1).

(2). Since B is $\tau_1\tau_2$-$s\alpha rps$-closed, (2) follows from Theorem 3.21.

$\square$

Theorem 3.25. *For every point x of a space X, X $\setminus \{x\}$ is $\tau_1\tau_2$-$s\alpha rps$-closed or τ_1-αrps-open.*

Proof.   Suppose X $\setminus \{x\}$ is not τ_1-αrps-open. Then X is the only τ_1-αrps-open set containing X $\setminus \{x\}$.This implies τ_2-scl(X $\setminus \{x\}$) $\subseteq$ X. Then by using Definition 3.1, X $\setminus \{x\}$ is $\tau_1\tau_2$-$s\alpha rps$- closed. $\square$

4.   On Semi α-regular Pre-Semi Open Sets in Bitopological Spaces

In this section, we introduce the concept of $\tau_1\tau_2$-$s\alpha rps$-open sets in bitopological spaces and discuss some of the related properties.

Definition 4.1. *A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-semi α-regular pre-semi open (briefly $\tau_1\tau_2$-$s\alpha rps$-open) if its complement is $\tau_1\tau_2$-$s\alpha rps$-closed.*

Remark 4.2. *If τ_1 =τ_2 in the above definition, $\tau_1\tau_2$-$s\alpha rps$-open is $s\alpha rps$-open in the sense of point set topology. The set of all $\tau_1\tau_2$-$s\alpha rps$-open sets in the bitopological space (X,τ_1, τ_2) is denoted by $S\alpha RPSO(X,\tau_1,\tau_2)$.*

Remark 4.3. *In general, the set of all $\tau_1\tau_2$-$s\alpha rps$-open sets need not be equal to the set of all $\tau_2\tau_1$-$s\alpha rps$-open sets as seen in the following example.*

Example 4.4. *Let X = $\{a,b,c\}$ with the topologies τ_1 = $\{\phi,\{a\},\{b\},\{a,b\},X\}$ and τ_2 = $\{\phi,\{a\},\{b,c\},X\}$. Then$S\alpha RPSO(X,\tau_1,\tau_2)$ = $\{\phi,\{b,c\},\{a,b\},\{b\},\{a\},X\}$ and $S\alpha RPSO(X,\tau_2,\tau_1)$ = $P(X)$. Therefore $S\alpha RPSO(X,\tau_1,\tau_2)$ $\neq S\alpha RPSO(X,\tau_2,\tau_1)$.*

Remark 4.5. *Difference between two $\tau_1\tau_2$-sαrps-open sets need not be a $\tau_1\tau_2$-sαrps-open set as seen in the Example 4.6.*

Example 4.6. *From Example 4.4, $A = \{b,c\}$ and $B = \{b\}$ are $\tau_1\tau_2$-sαrps-open sets. But $A \setminus B = \{c\}$ is not $\tau_1\tau_2$-sαrps-open.*

Proposition 4.7. *Let (X,τ_1,τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.*

(1). Every τ_2-semi-open set is $\tau_1\tau_2$-sαrps-open.

(2). Every τ_2-regular open set is $\tau_1\tau_2$-sαrps-open.

(3). Every τ_2-α-open set is $\tau_1\tau_2$-sαrps-open.

(4). Every $\tau_1\tau_2$-$g^{\#}$- semi-open set is $\tau_1\tau_2$-sαrps-open.

Proof.

(1). Let A be a τ_2-semi-open subset of a space X. Then $X \setminus A$ is τ_2-semi-closed. Since every τ_2-semi-closed set is $\tau_1\tau_2$-sαrps-closed, $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Therefore A is $\tau_1\tau_2$-sαrps-open in X.

(2). Let A be a τ_2-regular open subset of a space X. Then $X \setminus A$ is τ_2-regular closed. Since every τ_2-regular closed set is $\tau_1\tau_2$-sαrps-closed, $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Therefore A is $\tau_1\tau_2$-sαrps-open in X.

(3). Let A be a τ_2-α-open subset of a space X. Then $X \setminus A$ is τ_2-α-closed. Since every τ_2-α-closed set is $\tau_1\tau_2$-sαrps-closed, $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Therefore A is $\tau_1\tau_2$-sαrps-open in X.

(4). Let A be a $\tau_1\tau_2$-$g^{\#}$-semi-open subset of a space X. Then $X \setminus A$ is $\tau_1\tau_2$-$g^{\#}$- semi-closed. Since every $\tau_1\tau_2$-$g^{\#}$-semi-closed set is $\tau_1\tau_2$-sαrps-closed, $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Therefore A is $\tau_1\tau_2$-sαrps-open in X.

$\square$

The reverse implications are not true as shown in the Examples 4.8 and 4.9.

Example 4.8. *Let $X = \{a,b,c\}$ with the topologies $\tau_1 = \{\phi,\{a\},\{b\},\{a,b\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. Then $\{a,b\}$ is $\tau_1\tau_2$-sαrps-open, but not τ_2-semi-open, not τ_2-regular open and not τ_2-α-open.*

Example 4.9. *Let $X = \{a,b,c\}$ with the topologies $\tau_1 = \{\phi,\{b,c\},X\}$ and $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. Then $\{a,b\}$ is $\tau_1\tau_2$-sαrps-open, but not $\tau_1\tau_2$-$g^{\#}$-semi-open.*

Proposition 4.10. *Let (X,τ_1,τ_2) be a bitopological space and $A \subseteq X$. Then the following are true.*

(1). Every $\tau_1\tau_2$-sαrps-open set is $\tau_1\tau_2$-gs-open.

(2). Every $\tau_1\tau_2$-sαrps-open set is $\tau_1\tau_2$-gαb-open.

(3). Every $\tau_1\tau_2$-sαrps-open set is $\tau_1\tau_2$-rgb-open.

Proof.

(1). Let A be a $\tau_1\tau_2$-sαrps-open subset of a space X. Then $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Since every $\tau_1\tau_2$-sαrps-closed set is $\tau_1\tau_2$-gs-closed, $X \setminus A$ is $\tau_1\tau_2$-gs-closed. Therefore A is $\tau_1\tau_2$-gs-open in X.

(2). Let A be a $\tau_1\tau_2$-sαrps-open subset of a space X. Then $X \setminus A$ is $\tau_1\tau_2$-sαrps-closed. Since every $\tau_1\tau_2$-sαrps-closed set is $\tau_1\tau_2$-gαb-closed, $X \setminus A$ is $\tau_1\tau_2$-gαb-closed. Therefore A is $\tau_1\tau_2$-gαb-open in X.

(3). Let A be a $\tau_1\tau_2$-$s\alpha rps$-open subset of a space X. Then X \ A is $\tau_1\tau_2$-$s\alpha rps$-closed. Since every $\tau_1\tau_2$-$s\alpha rps$-closed set is

$\tau_1\tau_2$-rgb-closed, X \ A is $\tau_1\tau_2$-rgb-closed. Therefore A is $\tau_1\tau_2$-rgb-open in X.

$\square$

The reverse implications are not true as shown in the Examples 4.11 and 4.12.

Example 4.11. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},$ $\{a,b\},\{b,c\},\{a,b,c\},X\}$.*Then* $\{d\}$ *is* $\tau_1\tau_2$-*gs-open and* $\tau_1\tau_2$-*rgb-open, but not* $\tau_1\tau_2$-*sαrps-open.*

Example 4.12. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a,b\},\{a,b,c\},X\}$. *Then* $\{b,c,d\}$ *is* $\tau_1\tau_2$-*gαb-open, but not* $\tau_1\tau_2$-*sαrps-open. The concept* $\tau_1\tau_2$-*sαrps-open is independent from the concepts* $\tau_1\tau_2$-*αg-open,* $\tau_1\tau_2$-*g*-open,* $\tau_1\tau_2$-*g*p -open,* $\tau_1\tau_2$-*rg -open and* $\tau_1\tau_2$-*g-open as seen in the following examples.*

Example 4.13. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},X\}$. *Then*

(1). $\{b,c,d\}$ *is* $\tau_1\tau_2$-*sαrps-open but not* $\tau_1\tau_2$-*αg-open and* $\{d\}$ *is* $\tau_1\tau_2$-*αg-open but not* $\tau_1\tau_2$-*sαrps-open.*

(2). $\{b,c,d\}$ *is* $\tau_1\tau_2$-*sαrps-open but not* $\tau_1\tau_2$-*g-open and* $\{d\}$ *is* $\tau_1\tau_2$-*g-open but not* $\tau_1\tau_2$-*sαrps-open.*

Example 4.14. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{b,c\},\{a,b,c\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},X\}$. *Then* $\{b,c,d\}$ *is* $\tau_1\tau_2$-*sαrps-open but not* $\tau_1\tau_2$-*rg-open and* $\{c,d\}$ *is* $\tau_1\tau_2$-*rg-open but not* $\tau_1\tau_2$-*sαrps-open.*

Example 4.15. *Let* $X = \{a,b,c,d\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,b\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},X\}$. *Then* $\{b,c,d\}$ *is* $\tau_1\tau_2$-*sαrps-open but not* $\tau_1\tau_2$-*g*-open and not* $\tau_1\tau_2$-*g*p-open.*

Example 4.16. *Let* $X = \{a,b,c\}$ *with the topologies* $\tau_1 = \{\phi,\{a\},\{a,c\},X\}$ *and* $\tau_2 = \{\phi,\{a\},\{b,c\},X\}$. *Then* $\{a,c\}$ *is* $\tau_1\tau_2$-*g*-open and* $\tau_1\tau_2$-*g*p-open, but not* $\tau_1\tau_2$-*sαrps-open.*

Theorem 4.17. *If A and B are* $\tau_1\tau_2$-*sαrps-open sets of X, then* $A \bigcup B$ *is also a* $\tau_1\tau_2$-*sαrps-open set in X.*

Proof. Let A and B be $\tau_1\tau_2$-$s\alpha rps$-open sets of X. Then X \ A and X \ B are $\tau_1\tau_2$-$s\alpha rps$-closed sets of X. By Theorem 3.17, $(X \setminus A) \bigcap (X \setminus B)$ is $\tau_1\tau_2$-$s\alpha rps$-closed in X. But $(X \setminus A) \bigcap (X \setminus B) = X \setminus (A \bigcup B)$. Therefore $A \bigcup B$ is $\tau_1\tau_2$-$s\alpha rps$-open in X. $\square$

Remark 4.18. *The intersection of two* $\tau_1\tau_2$-*sαrps-open sets need not be* $\tau_1\tau_2$-*sαrps-open as shown in the following example.*

Example 4.19. *From Example 4.13,* $A = \{a,d\}$ *and* $B = \{b,d\}$ *are* $\tau_1\tau_2$-*sαrps-open sets. But their intersection* $A \bigcap B = \{d\}$ *is not a* $\tau_1\tau_2$-*sαrps-open set.*

Theorem 4.20. *Every singleton point set in a bitopological space X is either* $\tau_1\tau_2$-*sαrps-open or* τ_1-*αrps-closed.*

Proof. Let X be a bitopological space. Let $x \in X$. Then by Theorem 3.25, $X\setminus\{x\}$ is $\tau_1\tau_2$-$s\alpha rps$-closed or τ_1-αrps-open. Therefore $\{x\}$is $\tau_1\tau_2$-$s\alpha rps$-open or τ_1-αrps-closed. $\square$

Theorem 4.21. *A set* $A \subseteq X$ *is* $\tau_1\tau_2$-*sαrps-open if and only if* $F \subseteq \tau_2$-*sintA whenever* $F \subseteq A$ *and F is* τ_1-*αrps-closed.*

Proof. Let A $\subseteq$ X be $\tau_1\tau_2$-*sαrps*-open. Let F be τ_1-*αrps*-closed and F $\subseteq$ A. Then X \ A $\subseteq$ X \ F where X \ F is τ_1-*αrps*-open. Since X \ A is $\tau_1\tau_2$-*sαrps*-closed, τ_2-*scl* (X \ A) $\subseteq$ X \ F and hence X \ τ_2-*sint*A $\subseteq$ X \ F that implies F $\subseteq$ τ_2-*sint*A.

Conversely assume that F $\subseteq$ τ_2-*sint*A whenever F $\subseteq$ A and F is τ_1-*αrps*-closed. Let X \ A $\subseteq$ U and U be τ_1-*αrps*-open. Then X \ U $\subseteq$ A. Since X \ U is τ_1-*αrps*-closed, by assumption X \ U $\subseteq$ τ_2-*sint*A. That implies X \ τ_2-*sint*A $\subseteq$ U. That is τ_2-*scl* (X \ A) $\subseteq$ U. Therefore X \ A is $\tau_1\tau_2$-*sαrps*-closed. Hence A is $\tau_1\tau_2$-*sαrps*-open. $\qquad\square$

Theorem 4.22. *If τ_2-sintA $\subseteq$ B $\subseteq$ A and A is $\tau_1\tau_2$-sαrps-open, then B is $\tau_1\tau_2$-sαrps-open.*

Proof. Let A be $\tau_1\tau_2$-*sαrps*-open and τ_2-*sint*A $\subseteq$ B $\subseteq$ A. Then X \ A $\subseteq$ X \ B $\subseteq$ X \ τ_2-*sint*A that implies X \ A $\subseteq$ X \ B $\subseteq$ τ_2-*scl* (X \ A). Since X \ A is $\tau_1\tau_2$-*sαrps*-closed, by Theorem 3.24 (i), X \ B is $\tau_1\tau_2$-*sαrps*-closed. This proves that B is $\tau_1\tau_2$-*sαrps*-open. $\qquad\square$

Theorem 4.23. *If $A \subseteq X$ is $\tau_1\tau_2$-sαrps-closed, then τ_2-sclA \ A is $\tau_1\tau_2$-sαrps-open.*

Proof. Let A $\subseteq$ X be $\tau_1\tau_2$-*sαrps*-closed and let F be a τ_1-*αrps*-closed set such that F $\subseteq$ τ_2-*scl*A \ A. Then by Theorem 3.21, F = $\varnothing$ that implies F $\subseteq$ τ_2-*sint* (τ_2-*scl*A \ A). By using Theorem 4.21, τ_2-*scl*A \ A is $\tau_1\tau_2$-*sαrps*-open. $\qquad\square$

Theorem 4.24. *Let $A \subseteq B \subseteq X$ and let τ_2-sclA \ A be $\tau_1\tau_2$-sαrps-open. Then τ_2-sclA \ B is also $\tau_1\tau_2$-sαrps-open.*

Proof. Suppose that τ_2-*scl*A \ A is $\tau_1\tau_2$-*sαrps*-open. Let F be a τ_1-*αrps*-closed subset of X with F $\subseteq$ τ_2-*scl*A \ B. Then F $\subseteq$ τ_2-*scl*A \ A. Since τ_2-*scl*A \ A is $\tau_1\tau_2$-*sαrps*-open, by Theorem 4.21, F $\subseteq$ τ_2-*sint* (τ_2-*scl*A \ A). By Lemma 2.3, F = $\varnothing$. Therefore F $\subseteq$ τ_2-*sint* (τ_2-*scl*A \ B). Hence τ_2-*scl*A \ B is $\tau_1\tau_2$-*sαrps*-open. $\qquad\square$

Theorem 4.25. *If a set A is $\tau_1\tau_2$-sαrps-open in X and if U is τ_1-αrps-open in X with τ_2-sintA $\bigcup$ (X \ A) $\subseteq$ U, then U = X.*

Proof. Let U be τ_1-*αrps*-open in X with τ_2-*sint*A $\bigcup$ (X \ A) $\subseteq$ U. Now X \ U $\subseteq$ τ_2-*scl* (X \ A) $\bigcap$ A = τ_2-*scl* (X \ A) \ (X \ A). Suppose that A is $\tau_1\tau_2$-*sαrps*-open. Since X \ U is τ_1-*αrps*-closed and since X \ A is $\tau_1\tau_2$-*sαrps*-closed, by Theorem 3.21, X \ U = $\varnothing$ and hence U = X. $\qquad\square$

Theorem 4.26. *Let X be a topological space and A, B $\subseteq$ X. If B is $\tau_1\tau_2$-sαrps-open and τ_2-sintB $\subseteq$ A, then A $\bigcap$ B is $\tau_1\tau_2$-sαrps-open.*

Proof. Since τ_2-*sint*B $\subseteq$ A, τ_2-*sint*B $\subseteq$ A $\bigcap$ B $\subseteq$ B. Since B is $\tau_1\tau_2$-*sαrps*-open, by Theorem 4.22, A $\bigcap$ B is $\tau_1\tau_2$-*sαrps*-open. $\qquad\square$

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