



# Radial Variation of Axial and Radial Velocity of Blood in Stenosed Artery in the Presence of Body Accelerations

Research Article

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**Abstract:** The paper studies the flow of blood through a stenosed artery in the presence of body accelerations. The mathematical model is constructed by considering blood to be a Newtonian fluid. The artery is assumed to have axisymmetric shape. The shape of stenosis is time dependent. The present paper calculates both radial and axial velocities. The governing blood flow equations are solved using finite difference approximation numerically and radial variation of axial and radial velocities is discussed. These velocities are plotted for different times.

**Keywords:** Axial velocity, radial velocity, Body acceleration.

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## 1. Introduction

One of the most serious problem related with biological fluid flow in human body is narrowing of arteries due to deposition of fatty material inside it. Blood flow in the human circulatory system depends upon the pumping action of the heart, which creates a pressure gradient throughout the system. But in many situations of day to day life such as driving a vehicle, flying in an aircraft etc. human body may be subjected to vibratory or acceleratory motion. Prolonged acceleration may cause various physical disorders like headache, increase in pulse rate etc. Hence it is important to study the pulsatile blood flow under the action of body acceleration.

Mandal P. K et al [1] studied the effect of body acceleration on unsteady pulsatile flow of non-newtonian fluid through stenosed artery. Chakravarty S. et al [2] presented non-linear mathematical model in constricted artery. They observed the blood flow in the presence of body accelerations. The paper presented the unsteady behaviour of blood treating blood as non-Newtonian fluid. The laminar pulsatile flow of blood under the influence of externally imposed body accelerations was studied by J.C.Mishra et.al [3]. They have developed the model by treating blood as a non -Newtonian fluid using biviscosity model. The governing equations are solved using finite difference technique. The mathematical model to study the characteristic of blood flowing through an arterial segment in the presence of a couple of stenoses with surface irregularities was developed by N.Mustapha et.al [4]. They have solved governing equations numerically by MAC (Marker and Cell) method in cylindrical polar coordinate system in staggered grids. Chakravarty. S, Mandal P. K [5] studied blood flow in tapered arteries in the presence of stenosis. The paper analysed the effects of tapering, arterial wall motion and the pressure

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gradient on the blood flow characteristic. This present paper attempts to study radial variation of axial as well as radial velocity of blood under the diseased conditions. Finite difference scheme involving central differences is implemented to solve governing equations. Effect of body acceleration on axial velocity is studied and it is compared with blood flow velocity in the absence of body acceleration.

## 2. Governing Equations

We consider the stenotic blood flow experiencing body accelerations to be unsteady, axisymmetric and laminar. The basic equations of motion governing such flow may be written as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) + B(t) \quad (3)$$

Where  $r, z$  are radial and axial directions.  $u$  and  $w$  are velocity components along radial and axial directions respectively.  $\rho$  and  $\mu$  are density and viscosity of blood,  $p$  is the pressure. The pressure gradient  $-\frac{\partial p}{\partial z}$  is due to the pumping action of the heart which is given by  $-\frac{\partial p}{\partial z} = A_0 + A_1 \cos \omega t$ , Burton, A. C [6]. Where  $A_0$  is taken as the constant amplitude and  $A_1$  is the amplitude of the pulsatile component which gives rise to systolic and diastolic pressure. Here  $w = 2\pi f_p r$ , where  $f_p$  is pulse frequency.  $B(t)$  is the periodic body acceleration in axial direction which have been made use of Sankar D.S [7], given by  $B(t) = r_0 \cos(\omega_p t + \phi)$ ,  $t \geq 0$ . Where  $r_0$  is its amplitude,  $\omega_p = 2\pi f_p$ ,  $f_p$  is its frequency and  $\phi$  is its phase difference.

## 3. The Geometry of Stenosis

Young D. F [8] described the time dependent geometry of the stenosis as

$$R(z, t) = \begin{cases} a_1(t) \left\{ a - \frac{\tau_m}{2} [1 + \cos \{\pi(z - l_1)/l_0\}] \right\}, & d < z < d + 2l_0 \\ a a_1(t), & \text{Otherwise.} \end{cases}$$

Where  $a_1(t) = 1 + K_R \cos(\omega t - \phi)$  in which  $\omega$  is angular frequency,  $\phi$  is the phase difference and  $K_R$  is a constant.  $l_1$  is the centre of the stenosis,  $l_0$  is the half length of the stenosis region,  $\tau_m$  is the maximum height of the stenosis and  $a$  is the radius of artery in non stenotic region.

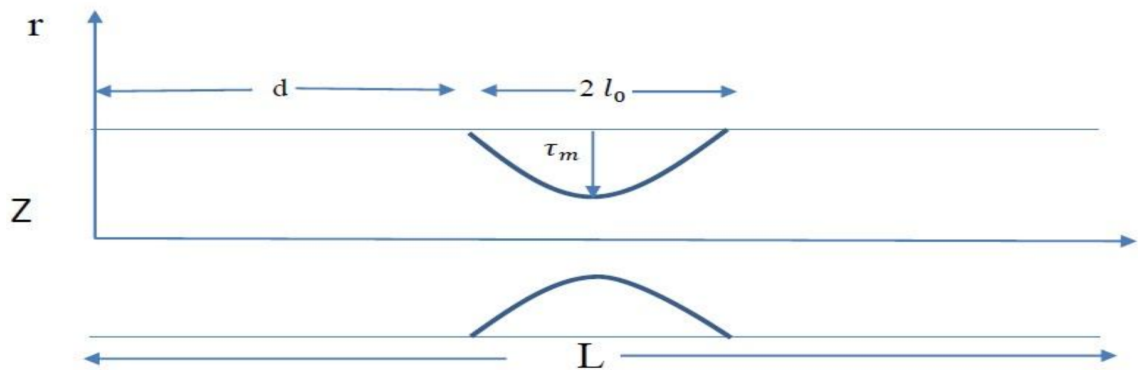


Figure 1: The geometry of the stenosis in an artery

## 4. Boundary Conditions

The velocities at the inlet and outlet of an arterial segment of finite length are taken as Sarifuddin et al [9]

$$u(r, z, t) = 0 \text{ and } w(r, z, t) = \frac{5}{3} \left( 1 - \left( \frac{r}{R(z, t)} \right)^3 \right) \text{ at } z = 0 \quad (4)$$

$$\frac{\partial w(r, z, t)}{\partial z} = 0 = \frac{\partial u(r, z, t)}{\partial z} \text{ at } z = L \quad (5)$$

It is assumed that initially radial and axial velocity both are zero. That is when system is at rest there is no flow through artery. That is

$$u(r, z, 0) = 0, \quad w(r, z, 0) = 0 \quad (6)$$

Axially, there is no radial flow, therefore the radial velocity is zero and the axial velocity gradient of the blood may be assumed to be equal to zero. This may be stated as

$$\frac{\partial w}{\partial r} = 0, \quad u(r, z, t) = 0 \text{ on } r = 0 \quad (7)$$

On the artery wall the axial velocity is zero due to no slip condition and radial velocity is rate of change in shape of the stenosis which is written as

$$w(r, z, t) = 0, \quad u(r, z, t) = \frac{\partial R}{\partial t} \text{ on } r = R(z, t) \quad (8)$$

## 5. Numerical Method and Implementation

Equations (1), (3) along with boundary conditions (4)-(8) take the form after introducing radial co-ordinate transformation,

$$x = \frac{r}{R(z, t)}$$

$$\frac{1}{R} \frac{\partial u}{\partial x} + \frac{u}{xR} + \frac{\partial w}{\partial z} - \frac{x}{R} \frac{\partial R}{\partial z} \frac{\partial w}{\partial x} = 0 \quad (9)$$

$$\frac{\partial w}{\partial t} = \frac{1}{R} \left[ x \left( w \frac{\partial R}{\partial z} + \frac{\partial R}{\partial t} \right) - u \right] \frac{\partial w}{\partial x} - w \frac{\partial w}{\partial z} + \frac{\mu}{\rho R^2} \left( \frac{\partial^2 w}{\partial x^2} + \frac{1}{x} \frac{\partial w}{\partial x} \right) - \frac{1}{\rho} \frac{\partial p}{\partial z} + B(t) \quad (10)$$

$$u(x, z, t) = 0 \text{ and } w(x, z, t) = \frac{5}{3} (1 - x^3) \text{ at } z = 0 \quad (11)$$

$$\frac{\partial w(x, z, t)}{\partial z} = 0 = \frac{\partial u(x, z, t)}{\partial z} \text{ at } z = L \quad (12)$$

$$u(x, z, 0) = 0, \quad w(x, z, 0) = 0 \quad (13)$$

$$\frac{\partial w}{\partial x} = 0, \quad u(x, z, t) = 0 \text{ on } x = 0 \quad (14)$$

$$w(x, z, t) = 0, \quad u(x, z, t) = \frac{\partial R}{\partial t} \text{ on } x = 1 \quad (15)$$

Multiplying equation (9) by  $xR$  and integrating w.r.t.  $x$  within limits 0 to  $x$ .

$$\int_0^x x \frac{\partial u}{\partial x} dx + \int_0^x u dx + \int_0^x xR \frac{\partial w}{\partial z} dx - \int_0^x x^2 \frac{\partial R}{\partial z} \frac{\partial w}{\partial x} dx = 0 \quad (16)$$

Which gives us

$$u(x, z, t) = xR \frac{\partial w}{\partial z} - \frac{R}{x} \int_0^x x \frac{\partial w}{\partial z} dx - \frac{2}{x} \frac{\partial R}{\partial z} \int_0^x xw dx \quad (17)$$

Now, using boundary condition (11), we will get

$$\int_0^1 x \frac{\partial w}{\partial z} dx - \int_0^1 x \left[ \frac{2}{R} \frac{\partial R}{\partial z} w + \frac{1}{R} \frac{\partial R}{\partial t} f(x) \right] dx = 0 \quad (18)$$

Here, choice of  $f(x)$  is arbitrary. Let  $f(x)$  be of the form  $f(x) = \frac{10}{3}(1-x^3)$  satisfying  $\int_0^1 xf(x) dx = 1$ . Considering equality between the integrals to integrands, we will get

$$\frac{\partial w}{\partial z} = -\frac{2}{R} \frac{\partial R}{\partial z} w + \frac{10}{3R} \frac{\partial R}{\partial t} (x^3 - 1)$$

Using this in equation (17),

$$u(x, z, t) = xw \frac{\partial R}{\partial z} + 0.3333 \frac{\partial R}{\partial t} (5x - 2x^4) \quad (19)$$

Solving equation (10) using finite difference approximations in which central differences have been used.

$$\begin{aligned} \frac{\partial w}{\partial x} &= \frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x} \\ \frac{\partial w}{\partial z} &= \frac{w_{i+1,j}^k - w_{i-1,j}^k}{2\Delta z} \\ \frac{\partial w}{\partial t} &= \frac{w_{i,j}^{k+1} - w_{i,j}^k}{\Delta t} \\ \frac{\partial^2 w}{\partial x^2} &= \frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{(\Delta x)^2} \end{aligned}$$

Where  $x_j = (j-1)\Delta x$ ,  $z_i = (i-1)\Delta z$  and  $t_k = (k-1)\Delta t$ .  $\Delta x$ ,  $\Delta z$  are increments in radial and axial directions respectively.

$$\begin{aligned} w_{i,j}^{k+1} &= w_{i,j}^k + \Delta t \left\{ \frac{-1}{\rho} \left( \frac{\partial p}{\partial z} \right)^{k+1} - w_{i,j}^k \left( \frac{w_{i+1,j}^k - w_{i-1,j}^k}{2\Delta z} \right) + \left( \frac{x_j}{R_i^k} w_{i,j}^k \left( \frac{\partial R}{\partial z} \right)_i^k + \frac{x_j}{R_i^k} \left( \frac{\partial R}{\partial t} \right)_i^k - \frac{u_{i,j}^k}{R_i^k} \right) \left( \frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x} \right) \right. \\ &\quad \left. + \frac{\mu}{\rho(R_i^k)^2} \left[ \frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{(\Delta x)^2} + \frac{1}{x_j} \left( \frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x} \right) \right] + B(k+1) \right\} \quad (20) \end{aligned}$$

We solve equation (20) for value of  $w$  by using boundary conditions (11)-(15). After obtaining axial velocity, radial velocity can be obtained using equation (19). For numerical calculations following data is used. This data have been made use of Mandal et al [1]

$d = 6 \text{ mm}$ ,  $l_0 = 7.5 \text{ mm}$ ,  $L = 30 \text{ mm}$ ,  $a = 0.8 \text{ mm}$ ,  $\rho = 1.06 \times 10^3 \text{ kg/m}^3$ ,  $\mu = 0.035 \text{ P}$ ,  $\tau_m = 0.2a$ ,  $A_0 = 100 \text{ Kg m}^{-2} \text{ s}^{-2}$ ,  $A_1 = 0.2A_0$ ,  $K_R = 0.05$ ,  $r_0 = 100 \text{ mm/s}^2$ .

## 6. Results

Using the parameters shown above, the shape function  $R(z, t)$  is plotted over the length of the artery  $z$  for different values of time taken into consideration. Figure 2 shows distribution of time dependent stenosis over the length of the artery for different time periods. The stenosis is sinusoidal which has single peak where height of stenosis is maximum. Also the curves follow the shape of the stenosis over the length of artery and remain constant before and after the stenosis region. The figure 3 shows radial variation of axial velocity in the presence of body accelerations for different times. As time increases the axial velocity increases. In figure 4 curves are plotted by removing body acceleration term. In the absence of body acceleration the axial velocity is lower as compare to its value in the presence of body accelerations. Figure 5 shows radial variation of radial velocity for different times at  $z = 20 \text{ mm}$  from the inlet of artery. It is observed that all the curves of radial velocity decrease from zero on the axis as one moves away from the axis and finally increase towards the wall to attend some finite value on the wall which is due to the presence of wall motion. Also, it is clear from the graph that as time increases, the radial velocity also increases in the same pattern.

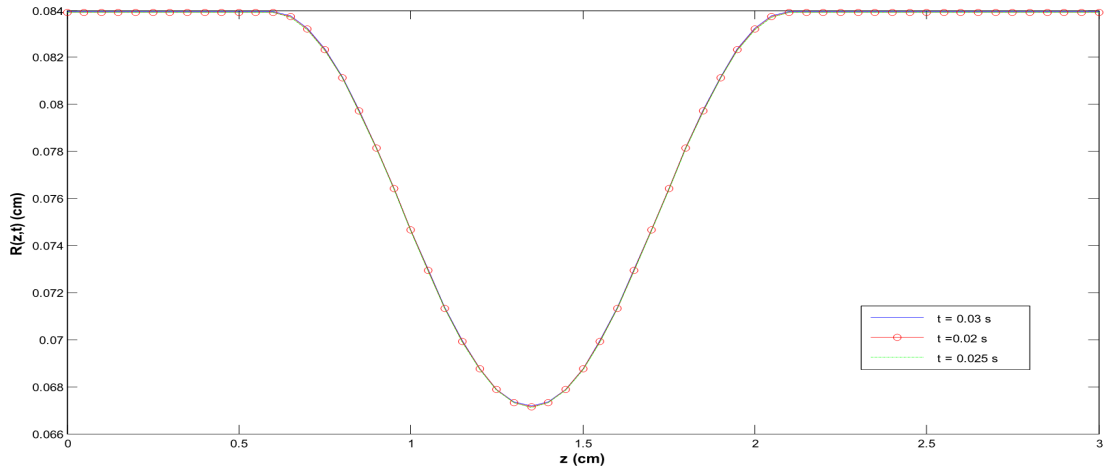


Figure 2: Shape function for varying time

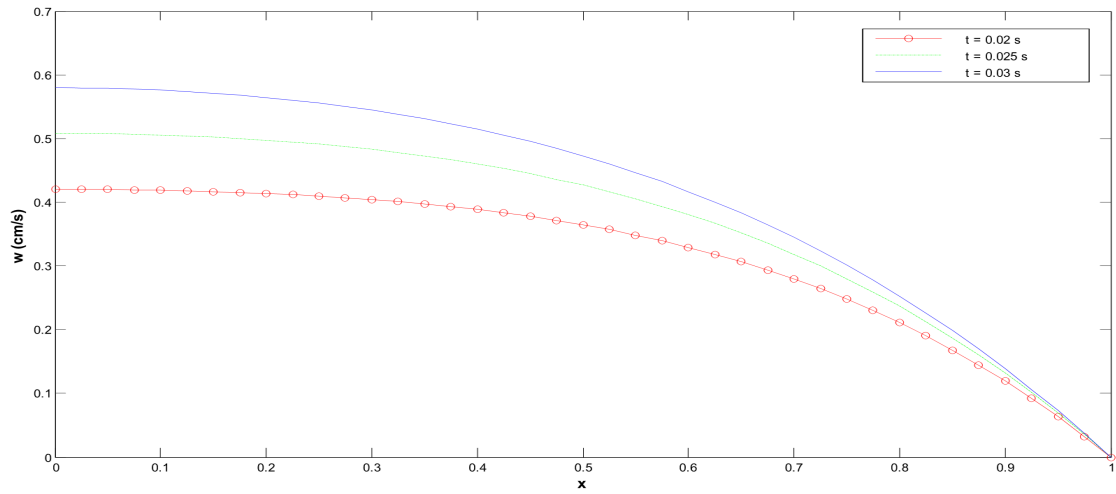


Figure 3: Radial variation of axial velocity for different times with body acceleration and  $\theta = 0^0$

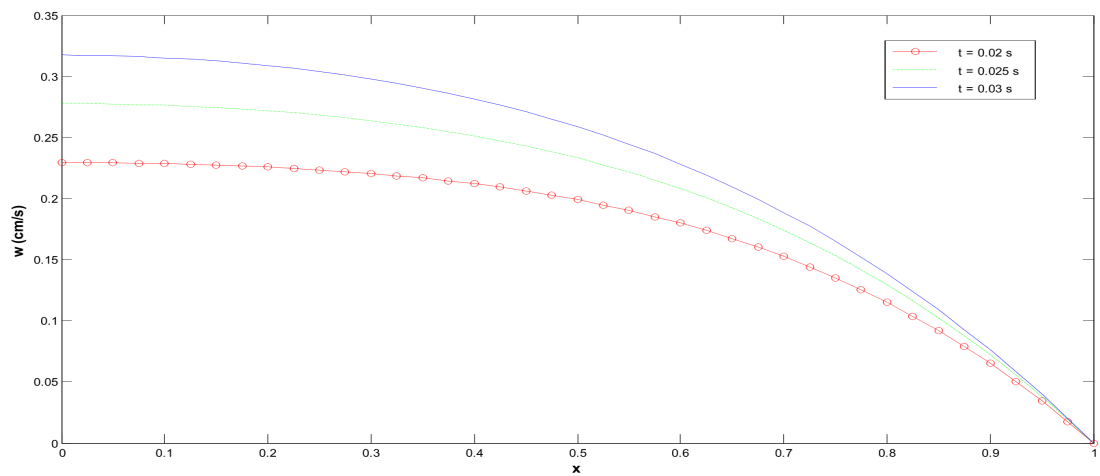


Figure 4: Radial variation of axial velocity for different times without body acceleration

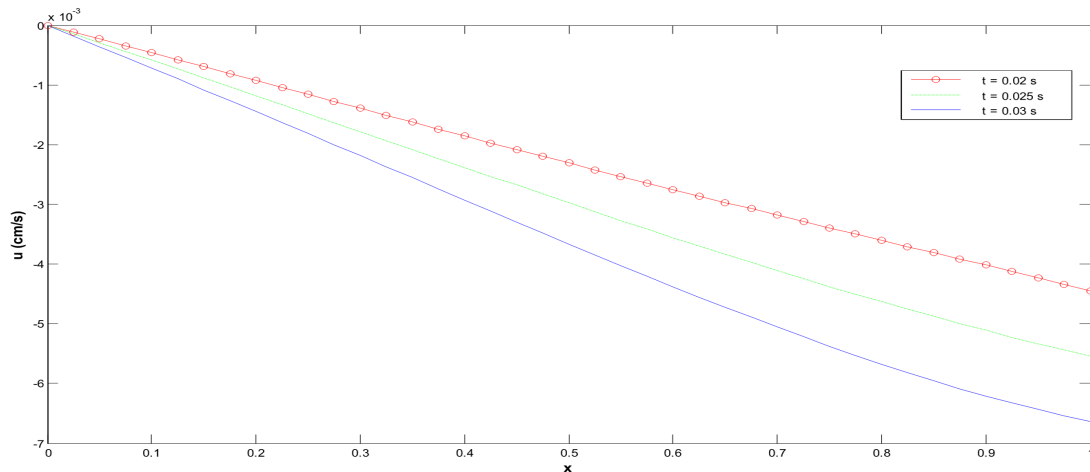


Figure 5: Radial variation of radial velocity for different times ( $z = 20\text{mm}$ )

## 7. Conclusion

The axial velocity of the blood increases as time increases. It decreases from its maximum to zero. On the axis of symmetry it is maximum and becomes zero at the wall of the artery due to no slip condition. There is impact of body acceleration on blood velocities. Whenever human body is subjected to body accelerations axial velocity increases in comparison with the body without body accelerations. Also the blood flow follows the shape of the stenosis.

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