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Divisor Cordial Labeling for Vertex Switching and Duplication of Special Graphs

Research Article

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Abstract: A divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \ldots, |V|\}$ such that an edge $e = uv$ is assigned label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and label 0 otherwise, then $|e_f(0) - e_f(1)| \leq 1$. A graph which admits divisor cordial labeling is called divisor cordial graph. In this paper we prove that vertex switching of gear graph, shell graph, flower graph are divisor cordial. We also prove some result on divisor cordial labeling for the graphs resulted from the duplication of graph elements.

MSC: 05C78

Keywords: Divisor cordial graph, Vertex switching, Duplication of graph.

1. Introduction

By a graph, we mean finite, simple, undirected graph. For terms not defined here, we refer to Gross and Yellen [3]. For standard terminology and notations related to number theory we refer to Burton [1] and The most recent findings on various graph labeling techniques can be found in Gallian [2]. In [6], Varatharajan et al. introduced the concept of divisor cordial labeling of graph. The divisor cordial labeling of various types of graphs are presented in [4, 7]. The brief summary of definitions which are necessary for the present investigation are provided below.

1.1. Definitions

Definition 1.1 ([6]). *Let $G = (V(G), E(G))$ be a simple graph, $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ be a bijection and induced function $f^* : E(G) \to \{0, 1\}$ is defined by*

$$f^*(e = uv) = \begin{cases} 1; & \text{if } f(u)|f(v) \quad \text{or} \quad f(v)|f(u) \\ 0; & \text{otherwise} \end{cases}$$

The function f is called divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits divisor cordial labeling is called divisor cordial graph.

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Definition 1.2 ([3]). *The gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of C_n.*

Definition 1.3 ([3]). *The shell S_n is obtained by taking $n-3$ concurrent chords in cycle C_n. The vertex at which all the chords are concurrent is called the apex vertex.*

Definition 1.4 ([3]). *The flower fl_n is obtained from a helm H_n by joining each pendant vertex to the apex vertex of the helm. Here we consider rim vertices as internal vertices.*

Definition 1.5 ([3]). *Star $K_{1,n}$ is the graph with one vertex of degree n called apex and n vertices of degree one (pendant vertices).*

Definition 1.6 ([4]). *The vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which is not adjacent to v in G.*

Definition 1.7 ([5]). *Duplication of vertex v of graph G produces a new graph G' by adding a new vertex v' such that $N(v') = N(v)$. In other words a vertex v' is said to be duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G'.*

Definition 1.8 ([5]). *Duplication of vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) = \{v_k, v''_k)$ and $N(v''_k) = \{v_k, v'_k\}$.*

Definition 1.9 ([5]). *Duplication of an edge $e = uv$ by a new vertex w in a graph G produces a new graph G' such that $N(w) = \{u, v\}$.*

Definition 1.10 ([5]). *Duplication of an edge $e = uv$ of a graph G produces a new graph G' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.*

Vaidya and Prajapati [5] discussed prime labeling for the graphs obtained by duplication of graph elements. In the present paper we investigate further results in this context on divisor cordial graphs.

Throughout this paper $|V(G)|$ and $|E(G)|$ denote the cardinality of vertex set and edge set respectively.

2. Main Results

Theorem 2.1. *The graph obtained by vertex switching of any vertex (except apex vertex) of gear graph G_n is divisor cordial.*

Proof. Let u_0 be the apex vertex and $u_1, u_2, \ldots u_{2n}$ be other vertices of gear graph G_n, where $deg(u_i) = 2$ when i is even and $deg(u_i) = 3$ when i is odd. Now the graph obtained by vertex switching of rim vertices u_i and u_j of degree 2 are isomorphic to each other for all i and j. Similarly the graph obtained by vertex switching of rim vertices u_i and u_j of degree 3 are isomorphic to each other for all i and j. Hence we require to discuss two cases.

(i) Vertex switching of an arbitrary vertex say u_1 of G_n of degree 3.

(ii) Vertex switching of an arbitrary vertex say u_2 of G_n of degree 2.

Let $(G_n)u_i$ denote the vertex switching of G_n with respect to the vertex $u_i, i = 1, 2$.

Case 1: Vertex switching of G_n with respect to vertex u_1 of degree 3.

Here $|V(G)| = 2n+1$ and $|E(G)| = \frac{5n-6}{2}$. We define labeling function $f : V((G_n)u_i) \to \{1, 2, \ldots, 2n+1\}$ as follows. Our aim is to generate $\frac{5n-6}{2}$ edges with label 1 and $\frac{5n-6}{2}$ edges with label 0.

Let $f(u_0) = p$, which generates $n-1$ edges with label 0, where p is largest prime. $f(u_1) = 1$ which generates $2n-3$ edges with label 1. Now it remains to generate $k = \frac{5n-6}{2} - (2n-3)$ edges with label 1. For the vertices $u_2, u_3, \ldots, u_{2n}$ assign the vertex label as per following ordered pattern upto it generate k edges with label 1.

$2, 2 \times 2^1, 2 \times 2^2 \ldots, 2 \times 2^{k_1}$,

$3, 3 \times 2^1, 3 \times 2^2 \ldots, 3 \times 2^{k_2}$,

$5, 5 \times 2^1, 5 \times 2^2 \ldots, 5 \times 2^{k_3}$,

$\ldots, \ldots, \ldots, \ldots,$

where $(2m-1)^{2^{k_m}} \leq 2n+1$ and $m \geq 1, k \geq 0$. Observe that $(2m-1)2^{\alpha} \mid (2m-1)2^{\alpha+1}$ and $(2m-1)2^{k_i}$ does not divide $2m+1$. Then for remaining vertices of G, assign the vertex labels such that the consecutive vertices do not generate edge label 1. In view of above labeling pattern we have,

$$\text{when } n \text{ is even:} \begin{cases} e_f(1) = \frac{5n-6}{2} \\ e_f(0) = \frac{5n-6}{2} \end{cases}$$

$$\text{when } n \text{ is odd:} \begin{cases} e_f(1) = \frac{5n-5}{2} \\ e_f(0) = \frac{5n-7}{2} \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$, in this case.

Case 2: Vertex switching of u_2 of degree 2.

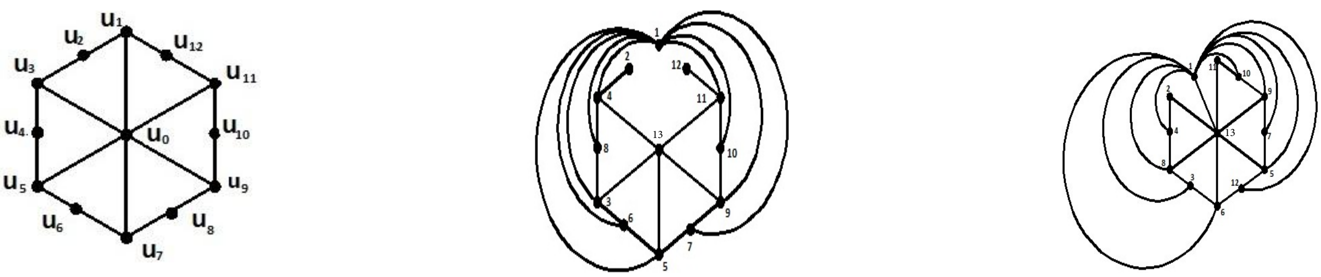
Here $|V(G)| = 2n+1$ and $|E(G)| = \frac{5n-4}{2}$. We follow the same labeling pattern as in Case 1 Then we have,

$$\textit{when } n \textit{ is even:} \begin{cases} e_f(1) = \frac{5n-4}{2} \\ e_f(0) = \frac{5n-4}{2} \end{cases}$$

$$\textit{when } n \textit{ is odd:} \begin{cases} e_f(1) = \frac{5n-3}{2} \\ e_f(0) = \frac{5n-5}{2} \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$. Thus the graph obtained by vertex switching of any vertex of G_n is divisor cordial. $\square$

Example 2.2. *Divisor cordial labeling for the graph obtained by vertex switching of gear graph G_6 with respect to vertex of degree 3 and degree 2 is shown in Fig. 1(b) and Fig.1(c) respectively.*



(a) G_6 (b) (c)

Figure 1:

Theorem 2.3. *The graph obtained vertex switching of shell graph S_n (except apex vertex) is divisor cordial.*

Proof. Let u_0 be the apex vertex and $u_1, u_2, \ldots, u_{n-1}$ be the other vertices of shell S_n, where $deg(u_1) = 2$, $deg(u_{n-1}) = 2$ and $deg(u_i) = 3, i = 2, 3, \ldots, n-2$. The graphs obtained by vertex switching of vertices of same degree are isomorphic to each other. Hence we require to discuss two cases

(i) vertex switching of an arbitrary vertex say u_2 of S_n of degree 3.

(ii) vertex switching of an arbitrary vertex say u_1 of S_n of degree 2.

Let $(S_n)_{u_i}$ denote the vertex switching of S_n with respect to the vertex u_i of S_n, $i = 1, 2$.

Case 1: Vertex switching of u_2 of S_n of degree 3.

Here $|V(G)| = n$ and $|E(G)| = 3n - 10$. We define labeling function as

For $n = 6$

$f(u_i) = i; i = 0, 1,$

$f(u_i) = i - 1; i = 3, 4$,

$f(u_3) = p$, where p is largest prime number.

$f(u_n) = n$.

For $n \neq 6, n \in \mathbb{N}$, label the vertices $u_0, u_1, \ldots u_{n-1}$ as per the following pattern.

$1, 1 \times 2^1, 1 \times 2^2 \ldots, 1 \times 2^{k_1},$

$3, 3 \times 2^1, 3 \times 2^2 \ldots, 3 \times 2^{k_2},$

$5, 5 \times 2^1, 5 \times 2^2 \ldots, 5 \times 2^{k_3},$

$\ldots \ldots \ldots \ldots$

In view of above labeling pattern we have,

$$\textit{when } n \textit{ is even:} \begin{cases} e_f(1) = \frac{3n-10}{2} \\ e_f(0) = \frac{3n-10}{2} \end{cases}$$

$$\textit{when } n \textit{ is odd:} \begin{cases} e_f(1) = \frac{3n-9}{2} \\ e_f(0) = \frac{3n-11}{2} \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$, in this case.

Case 2: Vertex switching of u_1 of S_n of degree 2.

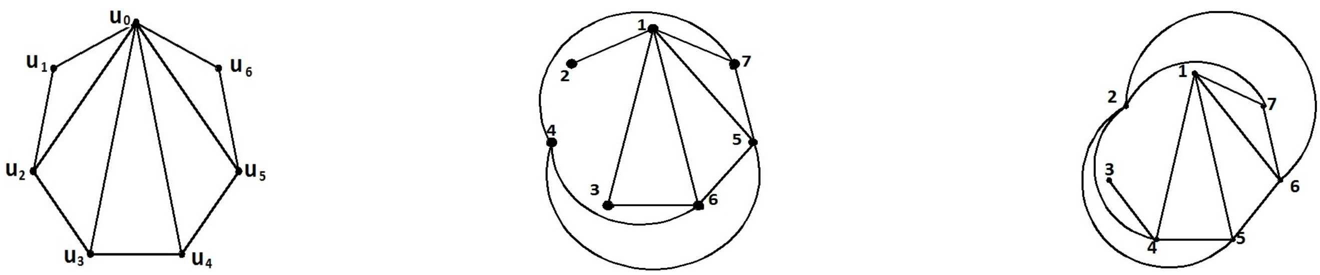
Here $|V(G)| = n$ and $|E(G)| = 3n - 8$. We define labeling function as $f(u_i) = i + 1; 0 \leq i \leq n - 1$. In view of above labeling pattern we have,

$$\textit{when } n \textit{ is even:} \begin{cases} e_f(1) = \frac{3n-8}{2} \\ e_f(0) = \frac{3n-8}{2} \end{cases}$$

$$\textit{when } n \textit{ is odd:} \begin{cases} e_f(1) = \frac{3n-7}{2} \\ e_f(0) = \frac{3n-9}{2} \end{cases}$$

Thus $|e_f(0) - e_f(1)| \leq 1$, in this case. Thus the graph obtained by vertex switching of any vertex of S_n is divisor cordial. □

Example 2.4. *Divisor cordial labeling of the graph obtained by vertex switching of shell graph S_7 with respect to vertex of degree 3 and degree 2 is shown in Fig. 2(b) and Fig. 2(c) respectively*



(a) S_7 (b) (c)

Figure 2:

Theorem 2.5. *The graph obtained vertex switching of flower graph fl_n (except apex vertex) is divisor cordial.*

Proof. Let u_0 be the apex vertex of helm $H_n, n \geq 3$. Let $u_1, u_2, \ldots, u_n$ be rim vertices of wheel W_n in H_n. Let $e_1, e_2, \ldots, e_n$ be the spoke edges where $e_i = u_i v_i, i = 1, 2, \ldots, n$, $deg(u_i) = 4$ and $deg(v_i) = 2$. Then flower graph fl_n is obtained by joining the vertices $v_1, v_2, \ldots, v_n$ to apex vertex u_0 of H_n.

Now the graph obtained by vertex switching of internal vertices u_i and u_j are isomorphic to each other for all i and j. Similarly the graph obtained by vertex switching of external vertices u_i and u_j are isomorphic to each other for all i and j. Hence it is required to discuss two cases

(i) vertex switching of an internal vertex say u_1 of fl_n of degree 4,

(ii) vertex switching of external vertex say v_1 of fl_n of degree 2.

Let $(fl_n)_{u_1}$ and $(fl_n)_{v_1}$ denote the vertex switching of fl_n with respect to the vertex u_1 and v_1 respectively.

Case 1: Vertex switching of an internal vertex say u_1 of fl_n of degree 4.

Here $|V(G)| = 2n + 1$and $|E(G)| = 6n - 8$. We define labeling function as

$f(u_0) = 1,$

$f(u_i) = 2i; 1 \leq i \leq n,$

$f(v_i) = 2i + 1; 1 \leq i \leq n.$

In view of above labeling pattern we have, $e_f(1) = 3n - 4$ and $e_f(0) = 3n - 4$. Thus $|e_f(0) - e_f(1)| \leq 1$, in this case.

Case 2: Vertex switching of an external vertex say v_1 of fl_n of degree 2.

Here $|V(G)| = 2n + 1$and $|E(G)| = 6n - 4$.

We define labeling function as

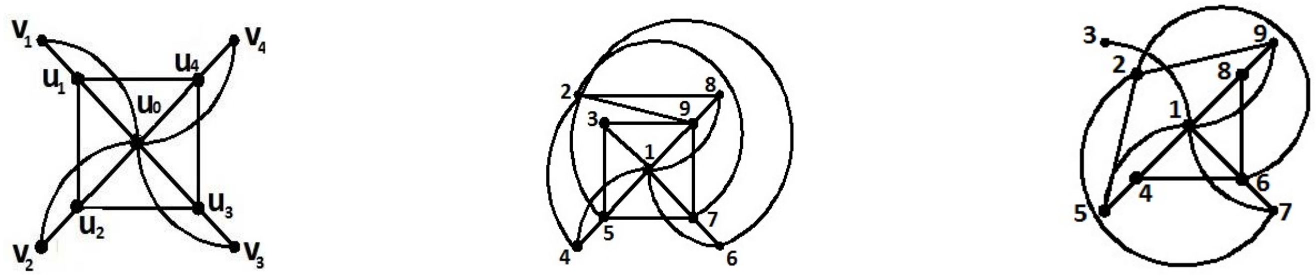
$f(u_0) = 1,$

$f(u_i) = 2i + 1; 1 \leq i \leq n,$

$f(v_i) = 2i; 1 \leq i \leq n.$

In view of above labeling pattern we have, $e_f(1) = 3n - 2$ and $e_f(0) = 3n - 2$. Thus $|e_f(0) - e_f(1)| \leq 1$, in this case. Hence the graph obtained by vertex switching of any vertex of fl_n (except apex vertex) is divisor cordial graph. □

Example 2.6. *Divisor cordial labeling of the graph obtained by vertex switching of flower graph fl_4 with respect to vertex of degree 4 and degree 2 is shown in Fig. 3(b) and Fig. 3(c) respectively*



(a) fl_4 (b) (c)

Figure 3:

Theorem 2.7. *The graph obtained by duplication of a vertex in $K_{1,n}$ is divisor cordial.*

Proof. Let v_0 be the apex vertex of star graph $K_{1,n}$ and $v_1, v_2, \ldots, v_n$ are pendant vertices of $K_{1,n}$, Let G be the graph obtained by duplication of any vertex in $K_{1,n}$. We consider the following cases.

Case 1: Duplication of apex vertex.

Duplication of apex vertex of $K_{1,n}$ is the complete bipartite graph $K_{2,n}$ which is divisor cordial. [6]

Case 2: Duplication of pendant vertex.

Duplication of any pendant vertex of $K_{1,n}$ is $K_{1,n+1}$ which is divisor cordial. [6] □

Theorem 2.8. *The graph obtained by duplication of a vertex by an edge in $K_{1,n}$ is divisor cordial.*

Proof. Let v_0 be the apex vertex of star graph $K_{1,n}$ and $v_1, v_2, \ldots, v_n$ are pendant vertices of $K_{1,n}$, Let G be the graph obtained by duplication of a vertex v_j by an edge $v'_j v''_j$ in $K_{1,n}$. We consider the following cases.

Case 1: Duplication of apex vertex v_0 by an edge $v'_0 v''_0$.

We define $f : V(G) \to \{1, 2, 3, \ldots n+2, n+3\}$ as

$f(v_0) = f(v_j) = 2,$

$f(v_1) = 1,\ f(v_i) = i + 1; 2 \leq i \leq n.$

$f(v'_0) = n + 2, f(v''_0) = n + 3.$

Then f is an injection and it admits divisor labeling for G.

Case 2: Duplication of pendant vertex v_j by an edge $v'_j v''_j$.

Without loss of generality we assume that $v_j = v_n$. Then in G we have a cycle of length three having vertices v_n, v'_n and v''_n

We define $f : V(G) \to \{1, 2, 3, \ldots n+2, n+3\}$ as

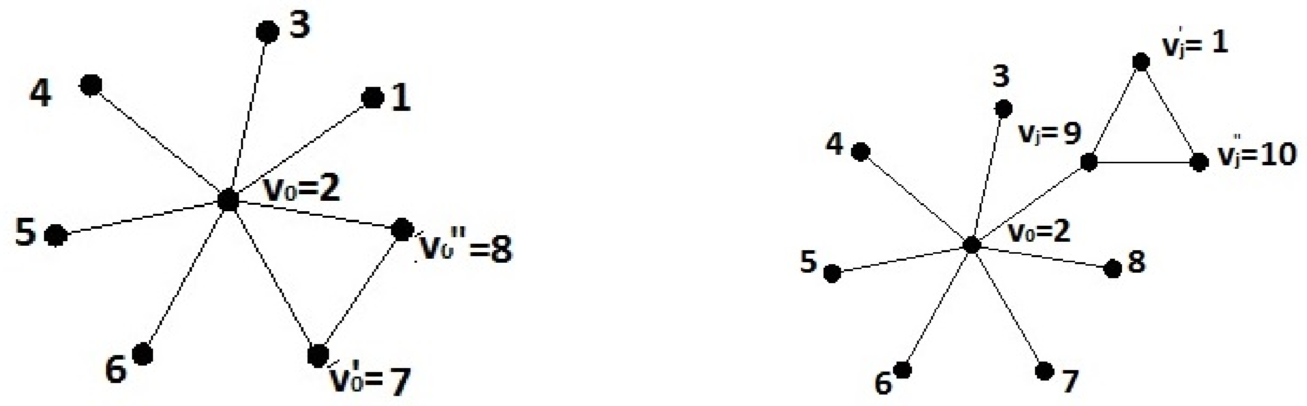
$f(v_0) = 2,$

$f(v'_n) = 1, f(v''_n) = n + 3,$

$f(v_i) = i + 2; 1 \leq i \leq n.$

This labeling admits a divisor labeling for G. □

Example 2.9. *Divisor cordial labeling of the graph obtained by duplication of apex vertex an edge and pendant vertex by an edge in $K_{1,5}$ is shown in Fig.4(a) and Fig.4(b) respectively.*



(a) (b)

Figure 4:

Theorem 2.10. *The graph obtained by duplication of an edge by a vertex in $K_{1,n}$ is divisor cordial.*

Proof. Let v_0 be the apex vertex and $v_1, v_2, \ldots, v_n$ be the consecutive pendant vertices of $K_{1,n}$. Let G be the graph obtained by duplication of the edge v_0v_n by a vertex v_n'. We define $f : V(G) \to \{1, 2, \ldots, n+2\}$ as

$f(v_0) = 2, f(v_1) = 1,$

$f(v_i) = i + 1, 2 \leq i \leq n.$

$f(v_n') = n + 2.$

Then obviously f is an injection and hence G admits divisor cordial labeling. i.e. G is divisor cordial graph. □

Example 2.11. *Divisor cordial labeling of the graph obtained by duplication of an edge v_0v_n by a vertex v_n' in star graph $K_{1,5}$ is shown in Fig. 5.*

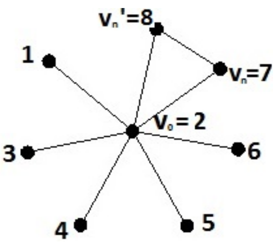


Figure 5:

Theorem 2.12. *The graph obtained by duplication of any edge in $K_{1,n}$ is divisor cordial.*

Proof. Let v_0 be the apex and $v_1, v_2, \ldots, v_n$ be the consecutive pendant vertices of $K_{1,n}$. Let G be the graph obtained by duplication of edge $e = v_0v_n$ by a new edge $e' = v_0'v_n'$ in $K_{1,n}$. $|V(G)| = n + 4$ and $|E(G)| = 2n + 2$. We define $f : V(G) \to \{1, 2, \ldots, n+4\}$ as $f(v_0) = 1, f(v_0') = p$, where p is largest prime $\leq n + 4$. Assign the remaining labels to the remaining vertices in any order. Then f is an injection and hence G admits divisor cordial labeling. i.e. G is a divisor cordial graph. □

Example 2.13. *Divisor cordial labeling of the graph obtained by duplication of edge v_0v_n by edge $v_0'v_n'$ in star graph $K_{1,5}$ is shown in Fig. 6.*

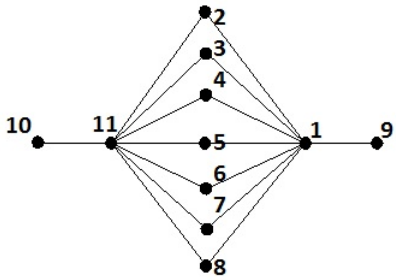


Figure 6:

3. Concluding Remarks

It is already proved that flower graph, gear graph, shell and star graphs are divisor cordial. We have discussed divisor cordiality of these graphs in context of vertex switching of graphs with respect to any arbitrary vertex. We have also investigated some result on divisor cordial labeling for duplication of graph elements.

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