International Journal of Mathematics And its Applications
Volume 4, Issue 3–B (2016), 87–96.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Fixed Point Theorems in Generalized Intuitionistic Fuzzy Metric Spaces

Research Article

R.Muthuraj[1] and R.Pandiselvi[2]*

1 PG and Research Department of Mathematics, H.H.The Rajah's College, Pudukkottai, Tamilnadu, India.

2 Department of Mathematics, The Madura college, Madurai, Tamilnadu, India.

Abstract: The aim of this paper is to introduce the concept of generalized intuitionistic fuzzy metric spaces with property (E) and prove common fixed point theorem in Ten weakly Compatible mappings in generalized intuitionistic fuzzy metric spaces with property (E).

MSC: 54H25, 47H10.

Keywords: Weakly Compatible mappings, Generalized intuitionistic fuzzy metric spaces.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [13] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michlek [6] and George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norms. As a generalization of fuzzy sets, Atanassov [2] introduced and studied the concept of intutionstic fuzzy sets. Park [8] using the idea of intutionistic fuzzy sets defined the notion of intutionistic fuzzy metric spaces with the help of continuous t-norms and continuous t conorm as a generalization of fuzzy metric spaces. Geroge and Veeramani [5] had showed that every metric induces an intutionistic fuzzy metric every fuzzy metric space is an intutionistic fuzzy metric space and found a necessary and sufficient condition for an intutionistic fuzzy metric space to be complete. Choudhary [4] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramosil and Michlek [6] introduced the notion of Cauchy sequences in an intutionistic fuzzy metric space and proved the well known fixed point theorem of Banach [3], Turkoglu et al [12] gave the generalization of jungek's common fixed point theorem to intutionistic fuzzy metric spaces.

In 2006, Sedghi and Shobe [10] defined $\mathcal{M}$-fuzzy metric spaces and proved a common fixed point theorem for four weakly Compatible mappings in this spaces. In 2009, Mehra and Gugnani [7] defined the notion of an intutionistic $\mathcal{M}$-fuzzy metric spaces due to Sedghi and Shobe [10] and proved a common fixed point theorem for six mappings for property (E) in this newly defined space. Our result is an intutionistic fuzzy Version of the results of Mehra and Gugnani [7] results in $\mathcal{M}$-fuzzy metric space.

We introduce the concept of an generalized intutionistic fuzzy metric spaces as follows.

* *E-mail: rpselvi@gmail.com*

2. Preliminaries

Definition 2.1. *A binary operation* $* : [0,1] \times [0,1]$ *is a continuous t-norm if it satisfies the following condition.*

(1). $*$ *is associative and commutative*

(2). $*$ *is continuous*

(3). $a * 1 = a$ *for all* $a \in [0,1]$

(4). $a * b < c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a, b, c, d \in [0,1]$.

Two typical example of a continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$

Definition 2.2. *A binary operation* $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-conorm if it satisfies the following conditions:*

(1). $\diamondsuit$ *is associative and commutative,*

(2). $\diamondsuit$ *is continuous,*

(3). $a \diamondsuit 0 = a$ *for all* $a \in [0,1]$

(4). $a \diamondsuit b \leq c \diamondsuit d$ *whenever* $a \leq c$ *and* $b \leq d$*, for each* $a, b, c, d \in [0,1]$.

Two typical examples of a continuous t-conorm are $a \diamondsuit b = \min(1, a+b)$ and $a \diamondsuit b = \max(a, b)$.

Definition 2.3. *A 5-tuple* $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ *is called an generalized intutionistic fuzzy metric space if* X *is an arbitrary (non-empty) set,* $*$ *is a continuous t-norm,* $\diamondsuit$ *a continuous t-conorm and* $\mathcal{M}$*,* $\mathcal{N}$ *are fuzzy sets on* $X^3 \times (0, \infty)$*, satisfying the following conditions: for each* $x, y, z, a \in X$ *and* $t, s > 0$.

(a). $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) \leq 1$.

(b). $\mathcal{M}(x, y, z, t) > 0$.

(c). $\mathcal{M}(x, y, z, t) = 1$ *if and only if* $x = y = z$*,*

(d). $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$*, where* p *is a permutation function,*

(e). $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t+s)$

(f). $\mathcal{M}(x, y, z, \cdot) : (0, \infty) \to [0,1]$ *is continuous*

(g). $\mathcal{N}(x, y, z, t) > 0$*,*

(h). $\mathcal{N}(x, y, z, t) = 0$*, if and only if* $x = y = z$*,*

(i). $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$ *where* p *is a permutation function,*

(j). $\mathcal{N}(x, y, z, a, t) \mathcal{N}(a, z, z, s) \geq \mathcal{N}(x, y, z, t+s)$*,*

(k). $\mathcal{N}(x, y, z, \cdot) : (0, \infty) \to [0,1]$ *is continuous*

Then $(\mathcal{M}, \mathcal{N})$ *is called an generalized intutionistic fuzzy metric on* X.

Example 2.4. *Let $X = R$ and $\mathcal{M}(x,y,z,t) = \frac{t}{t+|x-y|+|y-z|+|z-x|}$, $\mathcal{N}(x,y,z,t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}$ for every x, y, z and $t > 0$, let A and B be defined as $Ax = 2x+1$, $Bx = x+2$, consider the sequence $x_n = \frac{1}{n}+1$, $n = 1,2,\ldots$. Thus we have $\lim_{n\to\infty}\mathcal{M}(Ax_n,3,3,t) = \lim_{n\to\infty}\mathcal{M}(Bx_n,3,3,t) = 1$ and $\lim_{n\to\infty}\mathcal{N}(Ax_n,3,3,t) = \lim_{n\to\infty}\mathcal{N}(Bx_n,3,3,t) = 0$, for every $t > 0$. Then A and B satisfying the property (E).*

Lemma 2.5. *Let $(X,\mathcal{M},\mathcal{N},*,\diamondsuit)$ be an generalized intuitionistic fuzzy metric space. Then $\mathcal{M}(x,y,z,t)$ and $\mathcal{N}(x,y,z,t)$ are non-decreasing with respect to t, for all x, y, z in X.*

Proof. By Definition 2.3, for each $x,y,z,a \in X$ and $t,s > 0$ we have $\mathcal{M}(x,y,a,t) * \mathcal{M}(a,z,z,s) \le \mathcal{M}(x,y,z,t+s)$. If we set $a = z$, we get $\mathcal{M}(z,y,z,t) * \mathcal{M}(z,z,z,s) \le \mathcal{M}(x,y,z,t+s)$, that is $\mathcal{M}(x,y,z,t+s) \ge \mathcal{M}(x,y,z,t)$. Similarly, $\mathcal{N}(x,y,a,t)\diamondsuit\mathcal{N}(a,z,z,s) \ge \mathcal{N}(x,y,z,t+s)$, for each $x,y,z,a \in X$ and $t,s > 0$, by definition of $(X,\mathcal{N},\diamondsuit)$. If we set $a = z$, we get $\mathcal{N}(x,y,z,t)\diamondsuit\mathcal{N}(z,z,z,s) \ge \mathcal{N}(x,y,z,t+s)$ that is $\mathcal{N}(x,y,z,t+s) \le \mathcal{N}(x,y,z,t)$. Hence in IGFMS $(X,\mathcal{M},\mathcal{N},*,\diamondsuit)$, $\mathcal{M}(x,y,z,t)$ and $\mathcal{N}(x,y,z,t)$ are non-decreasing with respect to t, for all x, y, z in X. ☐

3. Main Results

Theorem 3.1. *Let A, B, S, T, I, J, L, K, P and Q be self mappings satisfying the following conditions:*

[3.1.1] $P(X) \subseteq ABI(X) \cup L(X)$ and $Q(X) \subseteq STJ(X) \cup K(X)$ and $ABI(X)$ or $STJ(X)$ and $K(X)$ are Complete IGFM subspace of X.

[3.1.2] $AB = BA$, $AI = IA$, $AL = LA$, $BI = IB$, $BL = LB$, $IL = LI$, $QL = LQ$, $QI = IQ$, $QB = BQ$, $ST = TS$, $SJ = JS$, $SK = KS$, $TJ = JT$, $TK = KT$, $JK = KJ$, $PK = KP$, $PJ = JP$, $PT = TP$.

[3.1.3] The pairs (P, STJ), (P, K) and (Q, ABI), (Q, L) are weakly compatible and (P, STJ) or (Q, ABI) and (Q, L) satisfies the property (E).

[3.1.4] If there exists a number $r > 1$ such that

$$\begin{aligned}\mathcal{M}(Px,Qy,Qz,t) \ge \varphi\{&\mathcal{M}(STJx,ABIy,Kx,rt),\mathcal{M}(Kx,ABIy,ABIz,rt),\mathcal{M}(STJx,ABIy,Lz,rt),\mathcal{M}(STJx,Qy,Lz,rt),\\ &\mathcal{M}(STJx,Ly,Qz,rt),\mathcal{M}(ABIz,Qz,Lz,rt),\mathcal{M}(Lz,Qy,Qz,rt),\mathcal{M}(Kx,Qy,Lz,rt),\\ &\mathcal{M}(Qy,ABIy,Lz,rt),\mathcal{M}(STJx,ABIy,ABIz,rt),\mathcal{M}(STJx,Qy,ABIz,rt),\mathcal{M}(STJx,ABIy,Qz,rt),\\ &\mathcal{M}(ABIz,Qy,Ly,rt),\mathcal{M}(Ly,Qy,Qz,rt),\mathcal{M}(Qy,ABIz,Ly,rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Px,Qy,Qz,t) \le \varphi'\{&\mathcal{N}(STJx,ABIy,Kx,rt),\mathcal{N}(Kx,ABIy,ABIz,rt),\mathcal{N}(STJx,ABIy,Lz,rt),\mathcal{N}(STJx,Qy,Lz,rt),\\ &\mathcal{N}(STJx,Ly,Qz,rt),\mathcal{N}(ABIz,Qz,Lz,rt),\mathcal{N}(Lz,Qy,Qz,rt),\mathcal{N}(Kx,Qy,Lz,rt),\\ &\mathcal{N}(Qy,ABIy,Lz,rt),\mathcal{N}(STJx,ABIy,ABIz,rt),\mathcal{N}(STJx,Qy,ABIz,rt),\mathcal{N}(STJx,ABIy,Qz,rt),\\ &\mathcal{N}(ABIz,Qy,Ly,rt),\mathcal{N}(Ly,Qy,Qz,rt),\mathcal{N}(Qy,ABIz,Ly,rt)\}\end{aligned}$$

for all $x,y,z \in X$ and $t > 0$. Then A, B, S, T, I, J, L, K, P and Q have a unique common fixed point in X.

Proof. Suppose that pairs (Q, ABI) and (Q, L) satisfy the property (E), then there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty}\mathcal{M}(Qx_n,u,u,t) = \lim_{n\to\infty}\mathcal{M}(ABIx_n,u,u,t) = 1$ and $\lim_{n\to\infty}\mathcal{N}(Qx_n,u,u,t) = \lim_{n\to\infty}\mathcal{N}(ABIx_n,u,u,t) = 0$ and $\lim_{n\to\infty}\mathcal{M}(Qx_n,u,u,t) = \lim_{n\to\infty}\mathcal{M}(Lx_n,u,u,t) = 1$ and $\lim_{n\to\infty}\mathcal{N}(Qx_n,u,u,t) = \lim_{n\to\infty}\mathcal{N}(Lx_n,u,u,t) = 0$ for some $u \in X$ and for every $t > 0$.

As $Q(x) \subset STJ(x) \cup K(x)$, There exists a sequence $\{y_n\}$ such that $Qx_n = STJy_n = Ky_n = u$. Hence $\lim_{n\to\infty} \mathcal{M}(STJy_n, u, u, t) = 1$, $\lim_{n\to\infty} \mathcal{N}(STJy_n, u, u, t) = 0$ and $\lim_{n\to\infty} \mathcal{M}(Ky_n, u, u, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(Ky_n, k, k, t) = 0$. We prove that $\lim_{n\to\infty} \mathcal{M}(Py_n, u, u, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(Py_n, u, u, t) = 0$.

Step 1: Putting $x = y_n$, $y = x_n$ and $z = x_{n+1}$ in [3.1.4], we obtain

$$\begin{aligned}\mathcal{M}(Py_n, Qx_n, Qx_{n+1}, t) \geq \varphi\{&\mathcal{M}(STJy_n, ABIx_n, Ky_n, rt), \mathcal{M}(Ky_n, ABIx_n, ABIx_{n+1}, rt),\\ &\mathcal{M}(STJy_n, ABIx_n, Lx_{n+1}, rt), \mathcal{M}(STJy_n, Qx_n, Lx_{n+1}, rt), \mathcal{M}(STJy_n, Lx_n, Qx_{n+1}, rt),\\ &\mathcal{M}(ABIx_{n+1}, Qx_{n+1}, Lx_{n+1}, rt), \mathcal{M}(Lx_{n+1}, Qx_n, Qx_{n+1}, rt), \mathcal{M}(Ky_n, Qx_n, Lx_{n+1}, rt),\\ &\mathcal{M}(Qx_n, ABIx_n, Lx_{n+1}, rt), \mathcal{M}(STJy_n, ABIx_n, ABIx_{n+1}, rt), \mathcal{M}(STJy_n, Qx_n, ABIx_{n+1}, rt),\\ &\mathcal{M}(STJy_n, ABIx_n, Qx_{n+1}, rt), \mathcal{M}(ABIx_{n+1}, Qx_n, Lx_n, rt), \mathcal{M}(Lx_n, Qx_n, Qx_{n+1}, rt),\\ &\mathcal{M}(Qx_n, ABIx_{n+1}, Lx_n, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Py_n, Qx_n, Qx_{n+1}, t) \leq \varphi'\{&\mathcal{N}(STJy_n, ABIx_n, Ky_n, rt), \mathcal{N}(Ky_n, ABIx_n, ABIx_{n+1}, rt),\\ &\mathcal{N}(STJy_n, ABIx_n, Lx_{n+1}, rt), \mathcal{N}(STJy_n, Qx_n, Lx_{n+1}, rt), \mathcal{N}(STJy_n, Lx_n, Qx_{n+1}, rt),\\ &\mathcal{N}(ABIx_{n+1}, Qx_{n+1}, Lx_{n+1}, rt), \mathcal{N}(Lx_{n+1}, Qx_n, Qx_{n+1}, rt), \mathcal{N}(Ky_n, Qx_n, Lx_{n+1}, rt),\\ &\mathcal{N}(Qx_n, ABIx_n, Lx_{n+1}, rt), \mathcal{N}(STJy_n, ABIx_n, ABIx_{n+1}, rt)\mathcal{N}(STJy_n, Qx_n, ABIx_{n+1}, rt),\\ &\mathcal{N}(STJy_n, ABIx_n, Qx_{n+1}, rt), \mathcal{N}(ABIx_{n+1}, Qx_n, Lx_n, rt), \mathcal{N}(Lx_n, Qx_n, Qx_{n+1}, rt),\\ &\mathcal{N}(Qx_n, ABIx_{n+1}, Lx_n, rt)\}\end{aligned}$$

Letting $n \to \infty$ in the above inequality, we get,

$$\lim_{n\to\infty} \mathcal{M}(Py_n, Qx_n, Qx_{n+1}, t) \geq \varphi\{\mathcal{M}(u, u, u, rt), \mathcal{M}(u, u, u, rt), \mathcal{M}(u, u, u, rt), \ldots, \mathcal{M}(u, u, u, rt)\} = 1 \quad \text{and}$$

$$\lim_{n\to\infty} \mathcal{N}(Py_n, Qx_n, Qx_{n+1}, t) \leq \varphi'\{\mathcal{N}(u, u, u, rt), \mathcal{N}(u, u, u, rt), \mathcal{N}(u, u, u, rt), \ldots, \mathcal{N}(u, u, u, rt)\} = 0.$$

Therefore $\lim_{n\to\infty} \mathcal{M}(Py_n, u, u, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(Py_n, u, u, t) = 0$. Hence $\lim_{n\to\infty} Py_n = \lim_{n\to\infty} STJy_n = \lim_{n\to\infty} Ky_n = \lim_{n\to\infty} Qx_n = \lim_{n\to\infty} ABIx_n = \lim_{n\to\infty} Lx_n = u$. Assume STJ(X) and K(X) are complete generalized intuitionistic fuzzy metric spaces, then there exists $x \in X$, such that $STJx = u$ and $Kx = u$.

Step 2: If $Px \neq u$, Putting $y = x_n$, and $z = x_{n+1}$ in [3.1.4] then we have

$$\begin{aligned}\mathcal{M}(Px, Qx_n, Qx_{n+1}, t) \geq \varphi\{&\mathcal{M}(STJx, ABIx_n, Kx, rt), \mathcal{M}(Kx, ABIx_n, ABIx_{n+1}, rt), \mathcal{M}(STJx, ABIx_n, Lx_{n+1}, rt),\\ &\mathcal{M}(STJx, Qx_n, Lx_{n+1}, rt), \mathcal{M}(STJx, Lx_n, Qx_{n+1}, rt), \mathcal{M}(ABIx_{n+1}, Qx_{n+1}, Lx_{n+1}, rt),\\ &\mathcal{M}(Lx_{n+1}, Qx_n, Qx_{n+1}, rt), \mathcal{M}(Kx, Qx_n, Lx_{n+1}, rt), \mathcal{M}(Qx_n, ABIx_n, Lx_{n+1}, rt),\\ &\mathcal{M}(STJx, ABIx_n, ABIx_{n+1}, rt), \mathcal{M}(STJx, Qx_n, ABIx_{n+1}, rt), \mathcal{M}(STJx, ABIx_n, Qx_{n+1}, rt),\\ &\mathcal{M}(ABIx_{n+1}, Qx_n, Lx_n, rt), \mathcal{M}(Lx_n, Qx_n, Qx_{n+1}, rt), \mathcal{M}(Qx_n, ABIx_{n+1}, Lx_n, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Px, Qx_n, Qx_{n+1}, t) \leq \varphi'\{&\mathcal{N}(STJx, ABIx_n, Kx, rt), \mathcal{N}(Kx, ABIx_n, ABIx_{n+1}, rt), \mathcal{N}(STJx, ABIx_n, Lx_{n+1}, rt),\\ &\mathcal{N}(STJx, Qx_n, Lx_{n+1}, rt), \mathcal{N}(STJx, Lx_n, Qx_{n+1}, rt), \mathcal{N}(ABIx_{n+1}, Qx_{n+1}, Lx_{n+1}, rt),\\ &\mathcal{N}(Lx_{n+1}, Qx_n, Qx_{n+1}, rt), \mathcal{N}(Kx, Qx_n, Lx_{n+1}, rt), \mathcal{N}(Qx_n, ABIx_n, Lx_{n+1}, rt),\\ &\mathcal{N}(STJx, ABIx_n, ABIx_{n+1}, rt), \mathcal{N}(STJx, Qx_n, ABIx_{n+1}, rt), \mathcal{N}(STJx, ABIx_n, Qx_{n+1}, rt),\\ &\mathcal{N}(ABIx_{n+1}, Qx_n, Lx_n, rt), \mathcal{N}(Lx_n, Qx_n, Qx_{n+1}, rt), \mathcal{N}(Qx_n, ABIx_{n+1}, Lx_n, rt)\}\end{aligned}$$

Letting $n \to \infty$, we get $\mathcal{M}(Px, u, u, t) = 1$. Hence $Px = u = STJx = Kx$ and $\mathcal{N}(Px, u, u, t) = 0$. If (P, STJ) and (P, U) are weakly compatible, we have $P(STJ)x = (STJ)Px$, so $P(Px) = P(STJ)x = (STJ)Px = (STJ)(STJ)x$. So, we have $Pu = STJu$ and $PKx = KPx$, $Pu = u$. Hence $Pu = STJu = u$.

Step 3: As $P(x) \subset ABI(x) \cup L(x)$, there exists $v \in X$ such that $Px = ABIv = Lv$, we prove that $ABIv = Qv$.

Case I : If $ABIV \neq QV$, Putting $y = v$ and $z = v$ in [3.1.4], then we have

$$\begin{aligned}
\mathcal{M}(Px, Qv, Qv, rt) \geq \varphi\{&\mathcal{M}(STJx, ABIv, Kx, rt), \mathcal{M}(Kx, ABIv, ABIv, rt), \mathcal{M}(STJx, ABIv, Lv, rt),\\
&\mathcal{M}(STJx, Qv, Lv, rt), \mathcal{M}(STJx, Lv, Qv, rt), \mathcal{M}(ABIv, Qv, Lv, rt),\\
&\mathcal{M}(Lv, Qv, Qv, rt), \mathcal{M}(Kx, Qv, Lv, rt), \mathcal{M}(Qv, ABIv, Lv, rt),\\
&\mathcal{M}(STJx, ABIv, ABIv, rt), \mathcal{M}(STJx, Qv, ABIv, rt), \mathcal{M}(STJx, ABIv, Qv, rt),\\
&\mathcal{M}(ABIv, Qv, Lv, rt), \mathcal{M}(Lv, Qv, Qv, rt), \mathcal{M}(Qv, ABIv, Lv, rt)\} \quad \text{and}\\
\mathcal{N}(Px, Qv, Qv, t) \leq \varphi'\{&\mathcal{N}(STJx, ABIv, Kx, rt), \mathcal{N}(Kx, ABIv, ABIv, rt), \mathcal{N}(STJx, ABIv, Lv, rt),\\
&\mathcal{N}(STJx, Qv, Lv, rt), \mathcal{N}(STJx, Lv, Qv, rt), \mathcal{N}(ABIv, Qv, Lv, rt),\\
&\mathcal{N}(Lv, Qv, Qv, rt), \mathcal{N}(Kx, Qv, Lv, rt), \mathcal{N}(Qv, ABIv, Lv, rt),\\
&\mathcal{N}(STJx, ABIv, ABIv, rt), \mathcal{N}(STJx, Qv, ABIv, rt), \mathcal{N}(STJx, ABIv, Qv, rt),\\
&\mathcal{N}(ABIv, Qv, Lv, rt),\ \mathcal{N}(Lv, Qv, Qv, rt), \mathcal{N}(Qv, ABIv, Lv, rt)\}
\end{aligned}$$

for all $x, y, z \in X$ and $t > 0$.

Case II: If $Qv \neq u$,

Then we have $\mathcal{M}(Px, Qv, Qv, t) > \mathcal{M}(Px, Qv, Qv, rt)$ and $\mathcal{N}(Px, Qv, Qv, t) < \mathcal{N}(Px, Qv, Qv, rt)$ which is contradiction. Then $ABIv = Qv = Px = Lv = u$.

Step 4: If (Q, ABI) and (Q, L) are weakly compatible mappings, then we get $Q(ABI)v = (ABI)Qv$ so, $(ABI)(ABI)v = (ABI)Qv = QQv$ so, $ABIu = QuandQLv = LQv$. We prove $Pu = u$ for $Qu = Lu$.

$$\begin{aligned}
\mathcal{M}(Pu, u, u, t) = \mathcal{M}(Pu, Qv, Qv, t) \geq \varphi\{&\mathcal{M}(STJu, ABIv, Ku, rt), \mathcal{M}(Ku, ABIv, ABIv, rt), \mathcal{M}(STJu, ABIv, Lv, rt),\\
&\mathcal{M}(STJu, Qv, Lv, rt), \mathcal{M}(STJu, Lv, Qv, rt), \mathcal{M}(ABIv, Qv, Lv, rt),\\
&\mathcal{M}(Lv, Qv, Qv, rt), \mathcal{M}(Ku, Qv, Lv, rt), \mathcal{M}(Qv, ABIv, Lv, rt),\\
&\mathcal{M}(STJu, ABIv, ABIv, rt), \mathcal{M}(STJu, Qv, ABIv, rt), \mathcal{M}(STJu, ABIv, Qv, rt),\\
&\mathcal{M}(ABIv, Qv, Lv, rt), \mathcal{M}(Lv, Qv, Qv, rt), \mathcal{M}(Qv, ABIv, Lv, rt)\} \quad \text{and}\\
\mathcal{N}(Pu, u, u, t) = \mathcal{N}(Pu, Qv, Qv, t) \leq \varphi'\{&\mathcal{N}(STJu, ABIv, Ku, rt), \mathcal{N}(Ku, ABIv, ABIv, rt), \mathcal{N}(STJu, ABIv, Lv, rt),\\
&\mathcal{N}(STJu, Qv, Lv, rt), \mathcal{N}(STJu, Lv, Qv, rt), \mathcal{N}(ABIv, Qv, Lv, rt),\\
&\mathcal{N}(Lv, Qv, Qv, rt), \mathcal{N}(Ku, Qv, Lv, rt), \mathcal{N}(Qv, ABIv, Lv, rt),\\
&\mathcal{N}(STJu, ABIv, ABIv, rt), \mathcal{N}(STJu, Qv, ABIv, rt), \mathcal{N}(STJu, ABIv, Qv, rt),\\
&\mathcal{N}(ABIv, Qv, Lv, rt), \mathcal{N}(Lv, Qv, Qv, rt), \mathcal{N}(Qv, ABIv, Lv, rt)\}
\end{aligned}$$

Step 5: If $Pu \neq u$, then we have $\mathcal{M}(Pu, u, u, t) > \mathcal{M}(Pu, u, u, rt)$ and $\mathcal{N}(Pu, u, u, t) < \mathcal{N}(Pu, u, u, rt)$ which is a contradiction. Thus

$$Pu = u = STJu = Ku \tag{1}$$

Step 6: Now we prove $Qu = u$. For

$$\begin{aligned}\mathcal{M}(u, Qu, Qu, t) = \mathcal{M}(Pu, Qu, Qu, t) \geq \varphi\{&\mathcal{M}(STJu, ABIu, Ku, rt), \mathcal{M}(Ku, ABIu, ABIu, rt), \mathcal{M}(STJu, ABIu, Lu, rt),\\ &\mathcal{M}(STJu, Qu, Lu, rt), \mathcal{M}(STJu, Lu, Qu, rt), \mathcal{M}(ABIu, Qu, Lu, rt),\\ &\mathcal{M}(Lu, Qu, Qu, rt), \mathcal{M}(Ku, Qu, Lu, rt), \mathcal{M}(Qu, ABIu, Lu, rt),\\ &\mathcal{M}(STJu, ABIu, ABIu, rt), \mathcal{M}(STJu, Qu, ABIu, rt), \mathcal{M}(STJu, ABIu, Qu, rt),\\ &\mathcal{M}(ABIu, Qu, Lu, rt), \mathcal{M}(Lu, Qu, Qu, rt), \mathcal{M}(Qu, ABIu, Lu, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(u, Qu, Qu, t) = \mathcal{N}(Pu, Qu, Qu, t) \geq \varphi'\{&\mathcal{N}(STJu, ABIu, Ku, rt), \mathcal{N}(Ku, ABIu, ABIu, rt), \mathcal{N}(STJu, ABIu, Lu, rt),\\ &\mathcal{N}(STJu, Qu, Lu, rt), \mathcal{N}(STJu, Lu, Qu, rt), \mathcal{N}(ABIu, Qu, Lu, rt),\\ &\mathcal{N}(Lu, Qu, Qu, rt), \mathcal{N}(Ku, Qu, Lu, rt), \mathcal{N}(Qu, ABIu, Lu, rt),\\ &\mathcal{N}(STJu, ABIu, ABIu, rt), \mathcal{N}(STJu, Qu, ABIu, rt), \mathcal{N}(STJu, ABIu, Qu, rt),\\ &\mathcal{N}(ABIu, Qu, Lu, rt), \mathcal{N}(Lu, Qu, Qu, rt), \mathcal{N}(Qu, ABIu, Lu, rt)\}\end{aligned}$$

Step7: If $Qu \neq u$, then we have $\mathcal{M}(u, Qu, Qu, t) > \mathcal{M}(u, Qu, Qu, rt)$ and $\mathcal{N}(u, Qu, Qu, t) < \mathcal{N}(u, Qu, Qu, rt)$ which is a contradiction. Then

$$Pu = Qu = STJu = ABIu = Lu = Ku = u \tag{2}$$

Step 8: Now, we show that $Ju = v$ by putting $x = Ju$, $y = x_{2n+1}$, and $z = x_{2n}$ in [3.1.4]. If $Ju \neq u$, then

$$\begin{aligned}\mathcal{M}(PJu, Qx_{2n+1}, Qx_{2n}, t) \geq \varphi\{&\mathcal{M}(STJJu, ABIx_{2n+1}, KJu, rt), \mathcal{M}(KJu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ &\mathcal{M}(STJJu, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{M}(STJJu, Qx_{2n+1}, Lx_n, rt),\\ &\mathcal{M}(STJJu, Lx_{2n+1}, Qx_{2n}, rt), \mathcal{M}(ABIx_{2n}, Qx_{2n}, Lx_{2n}, rt),\\ &\mathcal{M}(Lx_{2n}, Qx_{2n+1}, Qx_{2n}, rt), \mathcal{M}(KJu, Qx_{2n+1}, Lx_{2n}, rt),\\ &\mathcal{M}(Q_{2n+1}, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{M}(STJJu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ &\mathcal{M}(STJJu, Qx_{2n+1}, ABIx_{2n}, rt), \mathcal{M}(STJJu, ABIx_{2n+1}, Qx_{2n}, rt),\\ &\mathcal{M}(ABIx_{2n}, Qx_{2n+1}, Lx_{2n+1}, rt), \mathcal{M}(Lx_{2n+1}, Qx_{2n+1}, Qx_{2n}, rt),\\ &\mathcal{M}(Qx_{2n}, ABIx_{2n}, Lx_{2n+1}, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(PJu, Qx_{2n+1}, Qx_{2n+1}, t) \leq \varphi'\{&\mathcal{N}(STJJu, ABIx_{2n+1}, KJu, rt), \mathcal{N}(KJu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ &\mathcal{N}(STJJu, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{N}(STJJu, Qx_{2n+1}, Lx_n, rt),\\ &\mathcal{N}(STJJu, Lx_{2n+1}, Qx_{2n}, rt), \mathcal{N}(ABIx_{2n}, Qx_{2n}, Lx_{2n}, rt),\\ &\mathcal{N}(Lx_{2n}, Qx_{2n+1}, Qx_{2n}, rt), \mathcal{N}(KJu, Qx_{2n+1}, Lx_{2n}, rt),\\ &\mathcal{N}(Q_{2n+1}, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{N}(STJJu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ &\mathcal{N}(STJJu, Qx_{2n+1}, ABIx_{2n}, rt), \mathcal{N}(STJJu, ABIx_{2n+1}, Qx_{2n}, rt),\\ &\mathcal{N}(ABIx_{2n}, Qx_{2n+1}, Lx_{2n+1}, rt), \mathcal{N}(Lx_{2n+1}, Qx_{2n+1}, Qx_{2n}, rt),\\ &\mathcal{N}(Qx_{2n}, ABIx_{2n}, Lx_{2n+1}, rt)\}\end{aligned}$$

Since $TJ = JT$, $PJ = JP$ and $KJ = JK$, we have $P(Ju) = J(Pu) = Ju$, $TJ(Ju) = JT(Ju) = Ju$, $K(Ju) = J(Ku) = Ju$ letting $n \to \infty$, we have $\mathcal{M}(Ju, u, u, t) > \mathcal{M}(Ju, u, u, rt)$ and $\mathcal{N}(Ju, u, u, t) < \mathcal{N}(Ju, u, u, rt)$ which is a contradiction. Thus

$Ju = u$. Since $u = TJu$, we have $u = Tu$. Thus

$$u = Tu = Ju = Pu = Ku \tag{3}$$

Step 9: Now we show that $Su = u$. By putting $x = Su$, $y = x_{2n+1}$, and $z = x_{2n}$ in [3.1.4]. If $Su \neq u$, then

$$\begin{aligned}\mathcal{M}(PSu, Qx_{2n+1}, x_{2n}, t) \geq \varphi\, \{ & \mathcal{M}(STJSu, ABIx_{2n+1}, KSu, rt), \mathcal{M}(KSu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ & \mathcal{M}(STJSu, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{M}(STJSu, Qx_{2n+1}, Lx_{2n}, rt),\\ & \mathcal{M}(STJSu, Lx_{2n+1}, Qx_{2n}, rt), \mathcal{M}(ABIx_{2n}, Qx_{2n}, Lx_{2n}, rt),\\ & \mathcal{M}(Lx_{2n}, Qx_{2n+1}, Qx_{2n}, rt), \mathcal{M}(KSu, Qx_{2n+1}, Lx_{2n}, rt),\\ & \mathcal{M}(Qx_{2n+1}, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{M}(STJSu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ & \mathcal{M}(STJSu, Qx_{2n+1}, ABIx_{2n}, rt), \mathcal{M}(STJSu, ABIx_{2n+1}, Qx_{2n}, rt),\\ & \mathcal{M}(ABIx_{2n}, Qx_{2n+1}, Lx_{2n+1}, rt), \mathcal{M}(Lx_{2n+1}, Qx_{2n+1}, Qx_{2n}, rt),\\ & \mathcal{M}(Qx_{2n+1}, ABIx_{2n}, Lx_{2n+1}, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(PSu, Qx_{2n+1}, Qx_{2n}, t) \leq \varphi'\, \{ & \mathcal{N}(STJSu, ABIx_{2n+1}, KSu, rt), \mathcal{N}(KSu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ & \mathcal{N}(STJSu, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{N}(STJSu, Qx_{2n+1}, Lx_{2n}, rt),\\ & \mathcal{N}(STJSu, Lx_{2n+1}, Qx_{2n}, rt), \mathcal{N}(ABIx_{2n}, Qx_{2n}, Lx_{2n}, rt),\\ & \mathcal{N}(Lx_{2n}, Qx_{2n+1}, Qx_{2n}, rt), \mathcal{N}(KSu, Qx_{2n+1}, Lx_{2n}, rt),\\ & \mathcal{N}(Qx_{2n+1}, ABIx_{2n+1}, Lx_{2n}, rt), \mathcal{N}(STJSu, ABIx_{2n+1}, ABIx_{2n}, rt),\\ & \mathcal{N}(STJSu, Qx_{2n+1}, ABIx_{2n}, rt), \mathcal{N}(STJSu, ABIx_{2n+1}, Qx_{2n}, rt),\\ & \mathcal{N}(ABIx_{2n}, Qx_{2n+1}, Lx_{2n+1}, rt), \mathcal{N}(Lx_{2n+1}, Qx_{2n+1}, Qx_{2n}, rt),\\ & \mathcal{N}(Qx_{2n+1}, ABIx_{2n}, Lx_{2n+1}, rt)\}\end{aligned}$$

Since $ST = TS$, $SP = PS$ and $KS = SK$, we have $P(Su) = S(Pu) = Su$, $ST(Su) = TS(Su) = Su$, $K(Su) = S(Ku)$. Letting $n \to \infty$, we have $\mathcal{M}(Su, u, u, t) > \mathcal{M}(Su, u, u, rt)$ and $\mathcal{N}(Su, u, u, t) < \mathcal{N}(Su, u, u, rt)$, Thus $Su = u$. Since $u = STu$, we have $u = Tu$. Therefore $u = Tu = Su = Pu = Ku$.

Step 10: We show that $Iu = u$. By putting $x = u$, $y = Iu$ and $z = u$ in [3.1.4], If $Iu \neq u$, then

$$\begin{aligned}\mathcal{M}(Pu, Q(Iu), Qu, t) \geq \varphi\, \{ & \mathcal{M}(STJu, ABIIu, Ku, rt), \mathcal{M}(Ku, ABIIu, ABIu, rt), \mathcal{M}(STJu, ABIIu, Lu, rt),\\ & \mathcal{M}(STJu, QIu, Lu, rt), \mathcal{M}(STJu, LIu, Qu, rt), \mathcal{M}(ABIu, Qu, Lu, rt),\\ & \mathcal{M}(Lu, QIu, Qu, rt), \mathcal{M}(Ku, QIu, Lu, rt), \mathcal{M}(QIu, ABIIu, Lu, rt),\\ & \mathcal{M}(STJu, ABIIu, ABIu, rt), \mathcal{M}(STJu, QIu, ABIu, rt), \mathcal{M}(STJu, ABIIu, Qu, rt),\\ & \mathcal{M}(ABIu, QIu, LIu, rt), \mathcal{M}(LIu, QLu, Qu, rt), \mathcal{M}(QIu, ABIu, LIu, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Pu, QIu, Qu, t) \leq \varphi'\, \{ & \mathcal{N}(STJu, ABIIu, Ku, rt), \mathcal{N}(Ku, ABIIu, ABIu, rt), \mathcal{N}(STJu, ABIIu, Lu, rt),\\ & \mathcal{N}(STJu, QIu, Lu, rt), \mathcal{N}(STJu, LIu, Qu, rt), \mathcal{N}(ABIu, Qu, Lu, rt),\\ & \mathcal{N}(Lu, QIu, Qu, rt), \mathcal{N}(Ku, QIu, Lu, rt), \mathcal{N}(QIu, ABIIu, Lu, rt),\\ & \mathcal{N}(STJu, ABIIu, ABIu, rt), \mathcal{N}(STJu, QIu, ABIu, rt), \mathcal{N}(STJu, ABIIu, Qu, rt),\\ & \mathcal{N}(ABIu, QIu, LIu, rt), \mathcal{N}(LIu, QLu, Qu, rt), \mathcal{N}(QIu, ABIu, LIu, rt)\}\end{aligned}$$

Since $BI = IB$, $IQ = QI$ and $LI = IL$, We have $BI(Iu) = I(BIu) = Iu$, $QIu = IQu = Iu$ and $LIu = ILu = Iu$. Then $\mathcal{M}(u, Iu, u, t) > \mathcal{M}(u, Iu, u, rt)$ and $\mathcal{N}(u, Iu, u, t) < \mathcal{N}(u, Iu, u, rt)$, which is a contradiction. Therefore $Iu = u$, Since $u = BIu$, we have $u = Bu = Iu$.

Step 11: Finally we show that $Bu = u$. By putting $x = u$, $y = Bu$, $z = u$ in [3.1.4], If $Au \neq u$, then

$$\begin{aligned}\mathcal{M}(Pu, Q(Bu), Qu, t) \geq \varphi\{&\mathcal{M}(STJu, ABIBu, Ku, rt), \mathcal{M}(Ku, ABIBu, ABIu, rt), \mathcal{M}(STJu, ABIBu, Lu, rt),\\ &\mathcal{M}(STJu, QBu, Lu, rt), \mathcal{M}(STJu, LBu, Qu, rt), \mathcal{M}(ABIu, Qu, Lu, rt),\\ &\mathcal{M}(Lu, QBu, Qu, rt), \mathcal{M}(Ku, QBu, Lu, rt), \mathcal{M}(QBu, ABIBu, Lu, rt),\\ &\mathcal{M}(STJu, ABIBu, ABIu, rt), \mathcal{M}(STJu, QBu, ABIu, rt), \mathcal{M}(STJu, ABIBu, Qu, rt),\\ &\mathcal{M}(ABIu, QBu, LBu, rt), \mathcal{M}(LBu, Qu, Qu, rt), \mathcal{M}(QBu, ABIu, LBu, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(Pu, QBu, Qu, t) \leq \varphi'\{&\mathcal{N}(STJu, ABIBu, Ku, rt), \mathcal{N}(Ku, ABIBu, ABIu, rt), \mathcal{N}(STJu, ABIBu, Lu, rt),\\ &\mathcal{N}(STJu, QBu, Lu, rt), \mathcal{N}(STJu, LBu, Qu, rt), \mathcal{N}(ABIu, Qu, Lu, rt),\\ &\mathcal{N}(Lu, QBu, Qu, rt), \mathcal{N}(Ku, QBu, Lu, rt), \mathcal{N}(QBu, ABIBu, Lu, rt),\\ &\mathcal{N}(STJu, ABIBu, ABIu, rt), \mathcal{N}(STJu, QBu, ABIu, rt), \mathcal{N}(STJu, ABIBu, Qu, rt),\\ &\mathcal{N}(ABIu, QBu, LBu, rt), \mathcal{N}(LBu, Qu, Qu, rt), \mathcal{N}(QBu, ABIu, LBu, rt)\}\end{aligned}$$

Since $AB = BA$, $BQ = QB$ and $LB = BL$, we have $AB(Bu) = B(ABu) = Bu$, $Q(Bu) = B(Qu) = Bu$ and $L(Bu) = B(Lu) = Bu$. Thus $\mathcal{M}(u, Bu, u, t) > \mathcal{M}(u, Bu, u, rt)$ and $\mathcal{N}(u, Bu, u, t) < \mathcal{N}(u, Bu, u, rt)$ which is a contradiction. Thus $Bu = u$, Since

$$u = ABu \tag{4}$$

We have $u = Bu = Au$. By combining the above result, (1) , (2), (3) and (4). We get $Au = Bu = Su = Tu = Iu = Lu = Ju = Pu = Qu = Ku = u$. So, A, B, S, T, I, J, L, K, P and Q have a common fixed point u.

Uniqueness: Suppose let $v \neq u$, be another fixed point of A, B, S, T, I, J, L, K, P and Q then

$$\begin{aligned}\mathcal{M}(v, u, u, t) = \mathcal{M}(Pv, Qu, Qu, t) \geq \varphi\{&\mathcal{M}(STJv, ABIu, Kv, rt), \mathcal{M}(Kv, ABIu, ABIu, rt),\\ &\mathcal{M}(STJv, ABIu, Lu, rt), \mathcal{M}(STJv, Qu, Lu, rt), \mathcal{M}(STJv, Lu, Qu, rt),\\ &\mathcal{M}(ABIu, Qu, Lu, rt), \mathcal{M}(Lu, Qu, Qu, rt), \mathcal{M}(Kv, Qu, Lu, rt),\\ &\mathcal{M}(Qu, ABIu, Lu, rt), \mathcal{M}(STJv, ABIu, ABIu, rt), \mathcal{M}(STJv, Qu, ABIu, rt),\\ &\mathcal{M}(STJv, ABIu, Qu, rt), \mathcal{M}(ABIu, Qu, Lu, rt), \mathcal{M}(Lu, Qu, Qu, rt),\\ &\mathcal{M}(Qu, ABIu, Lu, rt)\} \quad \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{N}(v, u, u, t) = \mathcal{N}(Pv, Qu, Qu, rt) \leq \varphi'\{&\mathcal{N}(STJv, ABIu, Kv, rt), \mathcal{N}(Kv, ABIu, ABIu, rt),\\ &\mathcal{N}(STJv, ABIu, Lu, rt), \mathcal{N}(STJv, Qu, Lu, rt), \mathcal{N}(STJv, Lu, Qu, rt),\\ &\mathcal{N}(ABIu, Qu, Lu, rt), \mathcal{N}(Lu, Qu, Qu, rt), \mathcal{N}(Kv, Qu, Lu, rt),\\ &\mathcal{N}(Qu, ABIu, Lu, rt), \mathcal{N}(STJv, ABIu, ABIu, rt), \mathcal{N}(STJv, Qu, ABIu, rt),\\ &\mathcal{N}(STJv, ABIu, Qu, rt), \mathcal{N}(ABIu, Qu, Lu, rt), \mathcal{N}(Lu, Qu, Qu, rt),\\ &\mathcal{N}(Qu, ABIu, Lu, rt)\}\end{aligned}$$

Which is a contradiction. Therefore $v = u$ is common fixed point of A, B, S, T, I, J, L, K, P and Q. □

Corollary 3.2. *Let A, B, S, T, L, K, P and Q be self mappings of X satisfying the following conditions.*

[3.2.1] $P(X) \subseteq AB(X) \cup L(X)$ and $Q(X) \subseteq ST(X) \cup K(X)$ and $AB(X)$ or $ST(X)$ and $K(X)$ are complete IGFM subspace of X.

[3.2.2] $AB = BA$, $ST = TS$, $TP = PT$, $BQ = QB$, $LB = BL$ and $KT = TK$

[3.2.3] The (P, ST), (P, K) and (Q, AB), (Q, L) are weak compatible and (P, ST) or (Q, AB) and (Q, L) satisfies the property (E).

[3.2.4] If there exists a number $r > 1$ such that

$$\begin{aligned}\mathcal{M}(Px, Qy, Qz, t) \geq \varphi\{&\mathcal{M}(STx, ABy, Kx, rt), \mathcal{M}(Kx, ABy, ABz, rt), \mathcal{M}(STx, ABy, Lz, rt),\\ &\mathcal{M}(STx, Qy, Lz, rt), \mathcal{M}(STx, Ly, Qz, rt), \mathcal{M}(ABx, Qz, Lz, rt),\\ &\mathcal{M}(Lz, Qy, Qz, rt), \mathcal{M}(Kx, Qy, Lz, rt), \mathcal{M}(Qy, ABy, Lz, rt),\\ &\mathcal{M}(STx, ABy, ABz, rt), \mathcal{M}(STx, Qy, ABz, rt), \mathcal{M}(STx, ABy, Qz, rt),\\ &\mathcal{M}(ABz, Qy, Ly, rt), \mathcal{M}(Ly, Qy, Qz, rt), \mathcal{M}(Qy, ABz, Ly, rt)\} \quad \text{and}\\ \mathcal{N}(Px, Qy, Qz, t) \leq \varphi'\{&\mathcal{N}(STx, ABy, Kx, rt), \mathcal{N}(Kx, ABy, ABz, rt), \mathcal{N}(STx, ABy, Lz, rt),\\ &\mathcal{N}(STx, Qy, Lz, rt), \mathcal{N}(STx, Ly, Qz, rt), \mathcal{N}(ABx, Qz, Lz, rt),\\ &\mathcal{N}(Lz, Qy, Qz, rt), \mathcal{N}(Kx, Qy, Lz, rt), \mathcal{N}(Qy, ABy, Lz, rt),\\ &\mathcal{N}(STx, ABy, ABz, rt), \mathcal{N}(STx, Qy, ABz, rt), \mathcal{N}(STx, ABy, Qz, rt),\\ &\mathcal{N}(ABz, Qy, Ly, rt), \mathcal{N}(Ly, Qy, Qz, rt), \mathcal{N}(Qy, ABz, Ly, rt)\}\end{aligned}$$

for all $x, y, z \in X$ and $t > 0$. Then A, B, S, T, L, K, P and Q have a unique common fixed point in X.

Example 3.3. *Let $X = [0, 1]$ with the usual generalized metric D. Define, $\mathcal{M}(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|z-x|}$, $\mathcal{N}(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}$ for every x, y, z and $t > 4$, $\mathcal{M}(x, y, z, 0) = 0$; $\mathcal{N}(x, y, z, 0) = 1$ for all $x, y, z \in X$ clearly $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ is a complete generalized fuzzy metric spaces where $*$ and $\diamondsuit$ are defined $a * b = \min(a, b)$ and $a \diamondsuit b = \max(a, b)$. Let A, B, S, T, I, J, L, K, P and Q be defined as $Px = \frac{x}{12}$, $Ax = x$, $Bx = \frac{x}{2}$, $Ix = \frac{x}{4}$, $Lx = \frac{x}{6}$, $Qx = 0$, $Sx = \frac{x}{7}$, $Tx = \frac{x}{5}$, $Jx = \frac{x}{3}$ and $Kx = \frac{x}{9}$ for all $x, y, z \in X$. Then $P(x) = [0, \frac{1}{12}] \subset [0, \frac{1}{8}] \cup [0, \frac{1}{4}] = ABI(x) \cup L(x)$ and $Q(x) = \{0\} \subset [0, \frac{1}{105}] \cup [0, \frac{1}{9}] = STJ(x) \cup K(x)$, clearly, $AB = BA$, $AI = IA$, $AL = LA$, $BI = IB$, $BL = LB$, $IL = LI$, $QL = LQ$, $QI = IQ$, $QB = BQ$, $ST = TS$, $SJ = JS$, $SK = KS$, $TJ = JT$, $TK = KT$, $JK = KJ$, $PK = KP$, $PJ = JP$, $PT = TP$ moreover, the pairs (P, STJ), (P, K) and (Q, ABI), (Q, L) are weakly compatible and (P, STJ) or (Q, ABI) and (Q, L) satisfies the property (E) if $\lim_{n\to\infty} x_n = 0$ when $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} \mathcal{M}(Qx_n, u, u, t) = \lim_{n\to\infty} \mathcal{M}(ABIx_n, u, u, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(Qx_n, u, u, t) = \lim_{n\to\infty} \mathcal{N}(ABIx_n, u, u, t) = 0$ and $\lim_{n\to\infty} \mathcal{M}(Qx_n, u, u, t) = \lim_{n\to\infty} \mathcal{M}(Lx_n, u, u, t) = 1$ and $\lim_{n\to\infty} \mathcal{N}(Qx_n, u, u, t) = \lim_{n\to\infty} \mathcal{N}(Lx_n, u, u, t) = 0$ for some $u = 0 \in X$ and for every $t > 0$. If we take $K = 2$ and $t = 1$ then [3.1.4] of the main theorem is satisfied and 0 is the unique common fixed point of A, B, S, T, I, J, L, K, P and Q.*

References

[1] C.Alaca, D.Tukoglu and C.Yiliz, *Fixed ts in intuionistic fuzzy metric spaces*, Chaos, Solitons and Fractals, 29(2006), 1073-1078.

[2] K.Atanassov, *Intuionistic fuzzy Sets*, Fuzzy sets and Systems, 20(1986), 87-96.

[3] S.Banach, *Theories, lies, operations*, Laniaries Manograie Mathematyezene, warsaw, Poland, (1932).

[4] B.S.Choudhary, *A Unique common fixed Point theorem for sequence of self maps in menger spaces*, Bull. Korean mathe. Soc., 37(3)(2000), 569-575.

[5] A.George and P.Veeramani, *On Some results in fuzzy metric spaces*, Fuzzy sets and Systems, 64(1994), 395-399.

[6] O.Kramosil and J.Michalek, *Fuzzy metric and statistical metric spaces*, Ky-bernetics, 11(1975), 330-334.

[7] S.Mehra and M.Gugnani, *A Common fixed point for six mappings in an intuitionistic $\mathcal{M}$-fuzzy metric space*, Indian Journal of Mathematics, 51(1)(2009), 23-47.

[8] J.H.Park, *Intuionistic fuzzy metric spaces*, Chaos, Solitons and Fractals, 22(2004), 1039-11046.

[9] Ranjeeta Jain and N.Bajaj, *A Common Fixed Point for Eight Mappings in an Intuituionistic $\mathcal{M}$-Fuzzy metric space with Property E*, Global journal of science frontier research Mathematics and decision science, 13(2)(2013).

[10] S.Sedghi and N.Shobe, *Fixed point theorem in $\mathcal{M}$-fuzzy metric spaces with property (E)*, Advances in Fuzzy Mathematics, 1(1)(2006), 55-65.

[11] S.L.Singh and S.P.Singh, *A fixed point theorem*, Indian Jun. of Pure and App. Math., 11(1980), 1584-1586.

[12] D.Turkoglu, I.Altun and Y.J.Cho, *Common fixed points of compatible mappings of type (I) and type(II) in fuzzy metric spaces*, J. Fuzzy Math., 15(2007), 435-448.

[13] L.A.Zadeh, *Fuzzy sets*, Inform. and Control, 8(1965), 338-353.